Ordinal Definability and Recursion Theory: The Cabal Seminar, Volume III

The proceedings of the Los Angeles Caltech-UCLA "Cabal Seminar" were originally published in the 1970s and 1980s. *Ordinal Definability and Recursion Theory* is the third in a series of four books collecting the seminal papers from the original volumes together with extensive unpublished material, new papers on related topics and discussion of research developments since the publication of the original volumes.

Focusing on the subjects of "HOD and its Local Versions" (Part V) and "Recursion Theory" (Part VI), each of the two sections is preceded by an introductory survey putting the papers into present context. These four volumes will be a necessary part of the book collection of every set theorist.

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PART VI: RECURSION THEORY

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PREFACE

This book continues the series of volumes containing reprints of the papers in the original Cabal Seminar volumes of the Springer *Lecture Notes in Mathematics* series [Cabal i, Cabal ii, Cabal iii, Cabal iv], unpublished material, and new papers. The first volume, [Cabal I], contained papers on games, scales and Suslin cardinals. The second volume, [Cabal II], contained papers on Wadge degrees and pointclasses and projective ordinals. In this volume, we continue with Parts V and VI of the project: *Ordinal definability in models of determinacy* and *Recursion theory*. As in our first two volumes, each of the parts contains an introductory survey (written by John Steel for Part V and by Leo Harrington and Ted Slaman for Part VI) putting the papers into a present-day context.

In addition to the reprinted papers, this volume contains a paper by Kechris and Martin (*On the theory of* Π^1_3 *sets of reals, II*) that dates back to the period of the original Cabal publications but was not included in the old volumes. Neeman contributed a new paper, *An inner models proof of the Kechris–Martin theorem*, related to this paper. Steel and Woodin contributed two new papers (*A theorem of Woodin on mouse sets*, authored by Steel, and **HOD** *as a core model*, jointly) with recent results that fit well with the topics of Part V. There is also a new paper by Marks, Slaman and Steel (*Martin's conjecture, arithmetic equivalence, and countable Borel equivalence relations*) that contains earlier, unpublished, as well as new results related to the theme of Part VI. Table 1 gives an overview of the papers in this volume with their original references.

As emphasized in our first two volumes, our project is not to be understood as a historical edition of old papers. In the retyping process, we uniformized and modernized notation and numbering of sections and theorems. As a consequence, references to papers in the old Cabal volumes will not always agree with references to their reprinted versions. In this volume, references to papers that already appeared in reprinted form will use the new numbering. In order to help the reader to easily cross-reference old and new numberings, we provide a list of changes after the preface.

TABLE 1

The typing and design were partially funded by the *Marie Curie Research Training Site* GLoRiClass (MEST-CT-2005-020841) of the European Commission. Infrastructure was provided by the Institute for Logic, Language and Computation (ILLC) of the *Universiteit van Amsterdam*. Many people were involved in typing, laying out, and proofreading the papers. We should like to

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ORIGINAL NUMBERING

Numbering in the reprints may differ from the original numbering. Where numbering differs, the original designation is listed on the left, with the corresponding number in the reprint listed on the right. In rare cases where an item numbered in the reprint had neither a number nor a name in the original, we have indicated that with a '—'.

Partially playful universes, Becker, [CABAL i, pp. 55–90]

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Theorem 2	Theorem 2.1
Theorem 3	Theorem 2.2
Definition	Definition 2.3
Lemma 4	Lemma 2.4
Definition	Definition 2.5
Definition	Definition 2.6
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Diagram 1	Figure 1
Diagram 2	Figure 2
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Definition	Definition 10.3

Theorem 29	Theorem 10.4	Claim	Claim 11.12
Definition	Definition 11.1	Corollary 39	Corollary 11.13
Lemma 30	Lemma 11.2	Theorem 40	Theorem 11.14
Theorem 31	Theorem 11.3	Definition	Definition 11.15
Corollary 32	Corollary 11.4	Definition	Definition 11.16
Corollary 33	Corollary 11.5	Theorem 41	Theorem 11.17
Theorem 34	Theorem 11.6	Definition	Definition 11.18
Lemma 35	Lemma 11.7	Lemma 42	Lemma 11.19
Definition	Definition 11.8	Theorem 43	Theorem 11.20
Lemma 36	Lemma 11.9	Theorem 44	Theorem 11.21
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Ordinal games and playful models, Moschovakis, [Cabal ii, pp. 169–201]

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Measurable cardinals in playful models, Becker & Moschovakis, [Cabal ii, pp. 203–214]

Lemma 4.3	Lemma 4.1	Lemma 4.5	Lemma 4.3
Lemma 4.4	Lemma 4.2	Theorem 4.6	Theorem 4.4

Introduction to Q-*theory*, Kechris, Martin, & Solovay, [Cabal iii, pp. 199–282]

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Remark	Remark 2.6	Sublemma 2	Sublemma 14.13
Remark	Remark 7.3	Sublemma 3	Sublemma 14.14
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Remark	Remark 13.8	Fact	Fact 15.6
Claim	Claim 14.8	Claim	Claim 15.17

On recursion in E *and semi-Spector classes*, Kolaitis, [Cabal i, pp. 209–243]

Theorem 1.7	Theorem 1.1	Fact 2	Fact 1.3
Fact 1	Fact 1.2	Theorem 1.9	Theorem 1.4

Theorem 1.11	Theorem 1.5	Theorem 4.2	Theorem 4.1
—	Table 1	Theorem 4.3	Theorem 4.2
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Lemma 2.7	Lemma 2.4	Corollary 4.11	Corollary 4.6
Lemma 2.8	Lemma 2.5	Theorem 4.12	Theorem 4.7
Theorem 2.9	Theorem 2.6	Theorem 4.14	Theorem 4.8
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Theorem 3.8	Theorem 3.4	Theorem 5.2	Theorem 5.1
Theorem 3.10	Theorem 3.5		

On Spector classes, Kechris, [CABAL i, pp. 245–277]

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Trees and degrees, Odifreddi, [Cabal ii, pp. 235–271]

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Definition	Definition 1.4	Lemma 4.3	Lemma 4.4
Proposition 1.3	Proposition 1.5	Lemma 4.4	Lemma 4.5
Proposition 1.4	Proposition 1.6	Lemma 4.5	Lemma 4.6
Definition	Definition 2.1	Theorem 4.6	Theorem 4.7
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Lemma 2.2	Lemma 2.3	Theorem 4.8	Theorem 4.9
Theorem 2.3	Theorem 2.4	Theorem 4.9	Theorem 4.10
Definition	Definition 2.5	Theorem 4.10	Theorem 4.11
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Lemma 2.5	Lemma 2.7	Theorem 4.12	Theorem 4.13
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Theorem 2.8	Theorem 2.11	Lemma 5.3	Lemma 5.4
Open Problem	Open Problem 2.12	Theorem 5.4	Theorem 5.5
Lemma 2.9	Lemma 2.13	Theorem 5.5	Theorem 5.6
Theorem 2.10	Theorem 2.14	Definition	Definition 5.7
Definition	Definition 2.15	Theorem 5.6	Theorem 5.8
Lemma 2.11	Lemma 2.16	Definition	Definition 5.9
Theorem 2.12	Theorem 2.17	Theorem 5.7	Theorem 5.10
Theorem 2.13	Theorem 2.18	Definition	Definition 7.1
Definition	Definition 3.1	Theorem 7.1	Theorem 7.2
Lemma 3.1	Lemma 3.2	Theorem 7.2	Theorem 7.3
Lemma 3.2	Lemma 3.3	Theorem 7.3	Theorem 7.4
Definition	Definition 3.4	Theorem 7.4	Theorem 7.5
Lemma 3.3	Lemma 3.5	Theorem 7.5	Theorem 7.6
Theorem 3.4	Theorem 3.6	Open Problem	Open Problem 7.7
Theorem 3.5	Theorem 3.7	Theorem 7.6	Theorem 7.8
Theorem 3.6	Theorem 3.8	Theorem 7.7	Theorem 7.9
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Definable functions on degrees, Slaman & Steel, [Cabal iv, pp. 37–55]

—	§1	Theorem	Theorem 2.4
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Π^1_2 *monotone inductive definitions*, Martin, [Cabal ii, pp. 215–233]

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Lemma A.12	Lemma 2.20	Corollary	Corollary 6.4