Contest Theory

Incentive Mechanisms and Ranking Methods

Contests are prevalent in many areas, including sports, rent-seeking, patent races, innovation inducement, labor markets, scientific projects, crowdsourcing and other online services, and allocation of computer system resources. This book provides unified, comprehensive coverage of contest theory as developed in economics, computer science, and statistics, with a focus on online services applications, allowing professionals, researchers, and students to learn about the underlying theoretical principles and to test them in practice.

The book sets contest design in a game-theoretic framework that can be used to model a wide range of problems and efficiency measures such as total and individual output and social welfare, and it offers insight into how the structure of prizes relates to desired contest design objectives. Methods for rating the skills and ranking of players are presented, as are proportional allocation and similar allocation mechanisms, simultaneous contests, sharing utility of productive activities, sequential contests, and tournaments.

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To Sandra, Srđan, and Mirta

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CAMBRIDGE
UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment,
a department of the University of Cambridge.

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education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107033139

First published 2015

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication data
Vojnović, Milan
Contest theory : incentive mechanisms and ranking methods / Milan Vojnović,
Microsoft Research, Cambridge, UK.
pages cm
Includes bibliographical references and index.
ISBN 978-1-107-03313-9 (hardback : alk. paper)
1. Game theory. I. Title.
QA269.V634 2016
519.3 – dc23 2015027578

ISBN 978-1-107-03313-9 Hardback

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Preface

Contests are systems in which participants, whom I refer to as players, invest efforts in order to win one or more prizes. A distinctive feature of a contest is that each player invests effort but may not be awarded a prize. This makes the area of contest design a subset of *auction theory* where the aim is to design an auction that achieves a desired goal without necessarily restricting the design to one in which everybody pays. The area is also different from that of *mechanism design* where the goal is to design a mechanism that optimizes a given objective subject to the constraint that the mechanism is *truthful*, i.e., players truthfully report their private information. In general, no such constraint is imposed for a contest design problem, and in fact, many contest designs are non-truthful. Another important feature of a contest is that contestants are rewarded with respect to their *relative performance*, e.g., allocating an award to the best performing player or based on the rank of individual production outputs. This is different from traditional compensation schemes based on some estimate of absolute performance output. The theory of contest design has been developed over the last hundred years or so; in the early days it was predominantly studied in the areas of statistics, political economy and public choice, and the research was motivated by the need to understand and study various competitions, such as sport competitions, rent-seeking, lobbying, conflicts, arm races, R&D competitions, and, more recently, online marketplaces and resource allocation mechanisms. The development of the theory and experimental evaluation have been especially advanced over recent years in the areas of theoretical computer science and management sciences, fueled by the needs of various applications in the context of Internet online services. Here we find a wide variety of contests offering either monetary rewards or reputation. For example, soliciting solutions to tasks through open calls to large communities, so-called *crowdsourcing*, has emerged as a method of choice for solving a wide range of tasks, including web design, software development, algorithmic and data mining challenges, and various other tasks that require human intelligence.

This book was written to provide an exposition of some of the central concepts in contest design. It should be accessible to any senior-level undergraduate and graduate student equipped with a basic knowledge of mathematics and probability theory. It is

also written for a scientist or an engineer from any area where the aspects of contest design are of interest, including, but not limited to, computer science, economics, social sciences, operations research, and any engineering discipline. The book would be useful not only to those who are interested in contest theory and its development in its own right but also to those who care more about applications and want to learn some of their theoretical underpinnings. These theoretical foundations provide insights and guidelines for system design, and motivate the design of various hypotheses to be evaluated by experimental research. The goal was to put in one place the results developed by different communities over many years and, to make some of the domain-specific concepts in the areas of computer science and economics more widely accessible. The focus of this book is on principles that underpin various contest architectures that are of interest in applications, especially those that arise in the context of Internet online services. The book could be used as the main material to support a stand-alone course on the topic of contest theory, or parts of the book could be used to complement a course on a related subject. The book would also serve well as a research monograph because it provides a thorough overview of basic concepts and coverage of many references, and as such it would be a good starting point to pursue new research in the area.

Structure of the Book

Chapter 1 provides an introduction and preview. Throughout the book, we examine various contest architectures and study their equilibrium properties under two standard informational assumptions: (i) a game with complete information where the abilities of the players are common knowledge and (ii) a game with incomplete information where the abilities of the players are private information. Both these assumptions are of interest for modeling contests that arise in practice. Several quantities are of interest in equilibrium, including the total effort, maximum individual effort, and social efficiency. Chapter 2 begins by considering one of the most basic contest designs – the standard all-pay contest – where the entire prize is awarded to the player who invests the largest effort. This simple contest design already provides us with an abundance of interesting results and serves well to introduce and study basic concepts of equilibria. Chapter 3 takes one step forward in considering a natural extension of awarding one or more placement prizes depending on the rank of invested efforts. Here an interesting question is how to allocate a prize purse so as to optimize a given objective. We find conditions under which it is optimal to allocate only the first place prize. Chapter 4 considers a class of smooth allocation of prizes, where a prize is allocated according to a smooth function of the invested efforts. This class of contests includes as a special case the well-known Tullock contest and, in particular, the allocation proportional to invested efforts. In Chapter 5, we consider systems of simultaneous contests where each player has a choice to invest his or her effort in one of several simultaneous contests. This serves as a natural model of crowdsourcing systems, which are now in prevalent use in the context of Internet online services. Chapter 6 covers sequential contests where, for example, players make sequential effort investments competing for one prize, or multiple prizes are awarded in a sequential

manner. Chapter 7 gives an account of utility sharing and social welfare, where the efforts invested by the players amount to a utility of production, which is shared among the contributors according to a given allocation mechanism. Our goal in this chapter is to gain some understanding of the social efficiency of equilibria under simple utility sharing mechanisms. Chapter 8 studies the design of single-elimination tournaments with respect to various objectives. It discusses designing a tournament plan that specifies the seeding and schedule of matches and presents reasoning about which tournament plan is better. In Chapter 9, we discuss the main principles of rating systems for estimation of the skills of players; these systems have been in active use for the rating of chess players and in other sport competitions, rating of players in online computer games, and rating of coders in competition-based software development platforms. Chapter 10 covers the area of ranking and aggregation of judgments. The Appendix provides a review of various mathematical concepts that are used in the book.

Presentation Style

The presentation style is standard exposition structured around theorems, which helps highlight the main results. Most of the theorems are presented with proofs. A discussion of insights and implications of a theorem is usually presented following the proof of the theorem. An effort was made to keep the complexity of notation at a minimum level, while still allowing for some level of mathematical precision. Throughout the book, simple drawings are used to quickly explain or support some of the key ideas. The main results of each chapter are highlighted in a summary section near the end of each chapter. This is followed by selected exercises that vary in difficulty. Some of the exercises are simple and serve the purpose of checking basic understanding. Others are more involved, and usually their aim is to cover some known interesting results that did not fit in the main text. Each chapter ends with bibliographical notes that not only refer to the sources used in the content of the chapter but also put the results in a historical context and provide pointers to related references.

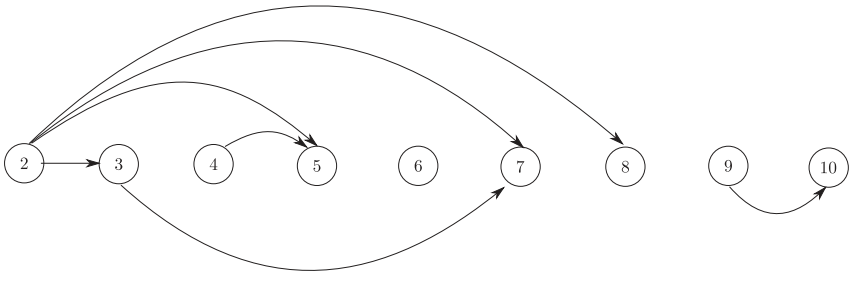
Use of the Book in Courses

This book could be used as the main material to teach a stand-alone course on the theory of contests as part of various programs in computer science, economics, electrical engineering, mathematics, and statistics. It could also be used to support parts of a course on a related subject, such as courses on economics and computation, online marketplaces, and special topics in economics, game theory, and statistics. The book contains a substantial amount of material and can well support a one-semester course. The delivery of the course can be tailored to specific audiences by the choice of presentation style and putting more emphasis on one type of applications than another. The students are expected to have some prior knowledge of basic real analysis and some elements of probability theory. The course could be delivered as part of a senior-level undergraduate program or a graduate program.

This book was used as the main reference for the course "Contest Theory," a 16-lecture course of a master's program in mathematics at the University of Cambridge. The content covered in the course varied from one year to the next. The core material that was covered included standard all-pay contests, rank-order allocation of prizes, smooth allocation of prizes, and simultaneous contests, as well as basic principles of rating systems. Each of these topics was taught using a subset of the material of the corresponding book chapter. The selection of these topics allowed there to be a flow of thought throughout the course. The students were given a good exposition of various notions of strategic equilibria and their efficiency for games of concern. The lectures on rating systems gave students some exposure to standard probabilistic models and statistical inference methods that underlie the design of popular rating systems.

Dependence Graph

Most of the chapters can be read individually because they are self-contained. However, there are some dependencies of which the reader should be aware, especially, a novice reader. Figure 0.1 depicts dependencies between individual chapters. Chapter 2 contains results about standard all-pay contests, which are invoked in several subsequent chapters. Chapter 3 is concerned with a generalization of the standard all-pay contest to one or more prizes. Hence, it would help the reader to go through Chapter 2 first. Chapter 4 can be read independently of any other chapter because it covers a class of prize allocation mechanisms introduced in the given chapter. Chapter 5 covers a generalization to a system of simultaneous contests and, in particular, a system of simultaneous all-pay contests, so there is some dependency with Chapter 2. Chapters 9 and 10 are somewhat distinct from the other chapters – the focus in these



2. standard all-pay contest
3. rank-order allocation of prizes
4. smooth allocation of prizes
5. simultaneous contests
6. utility sharing and social welfare
7. sequential contests
8. tournaments
9. rating systems
10. ranking and aggregation of judgments

Figure 0.1. Dependence graph.

two chapters is on statistical estimation of skills and ranking of the players and not so much on consideration of their strategic behavior.

Acknowledgments

I would like to express my gratitude to several people who provided useful feedback for specific chapters of this book, including Dan Alistarh, Sofia Ceppi, Subhasish M. Chowdhury, Thore Graepel, Tobias Harks, Ian Kash, Thành Nguyen, David Parkes, Ella Segev, Vasilis Syrgkanis, and several anonymous reviewers.

I am very grateful to my collaborators who worked with me on projects that involved studying various models of contests. In particular, I would like to mention Dominic DiPalantino, Nan Li, Thành Nguyen, and Vasilis Syrgkanis who as part of their internship worked with me at Microsoft Research. They provided me with a great deal of inspiration and a source of knowledge.

I owe a special debt to James Norris for providing me with the opportunity to teach a course at the University of Cambridge and the faculty board for appointing me as an Affiliated Lecturer at the university. I am indebted to Frank Kelly who recommended the course for Part III of Mathematical Tripos, a one-year master's program in mathematics whose tradition and prestige are world renowned. Felix Fischer, Frank Kelly, James Norris, and Richard Weber kindly shared with me their teaching experience, which helped me to adjust to a new and challenging environment. This was especially useful to someone like me whose previous teaching experience was limited to computer science and engineering programs, where ordinarily one more often uses a presentation slide deck than a piece of chalk and blackboard. Felix Fischer was my de facto mentor, guiding me throughout with useful advice and feedback, showing me how to maneuver through the university system, and acting as a checker for my course exam sheets. Last but not least, the book benefited greatly from the feedback of students, both in class during lectures and individually. The course was attended by truly inspiring and bright young mathematicians with a wide range of backgrounds and interests.

I am indebted to Lauren Cowles, my book editor at Cambridge University Press, who helped me throughout the book production process with moral support, soliciting anonymous feedback for individual chapters, and taking a personal interest by reading some of the chapters and providing me with her own reviews. The initial book proposal, review, and contract agreement were handled by Ada Brunstein who at that time was with Cambridge University Press – I thank her and Cambridge University Press for sharing a view of a need for this book, taking the proposal through a successful review process, and, finally, signing a contract. I would also like to thank the book copy-editor, Gail Naron Chalew, for her meticulous reviews, and the production project manager, Minaketan Dash, for handling everything so kindly and professionally.

The idea of writing this book was born in late 2011. It was an ambitious and demanding, but rewarding journey. It allowed me to focus on and learn a great deal of new things, exploring far beyond my initial knowledge. I am grateful to Microsoft Research for providing me with a stimulating work environment that helped toward putting this book together. This book was written in my office, at home, in cafe bars, at airports, on planes, and in hotels in many different countries while on trips of

business and pleasure. Specific parts of the book evoke personal memories of the various moments and places when they were written.

The production of this book would not have been possible without continued support, interest, and encouragement from my family – my wife and two children. They shared a great deal of the book writing project with me through all the time I was tied up to a desk, often during weekends and while on holidays, which deprived us of many other things we could be doing together.

Since this book is as much about reasoning about investments of efforts, I end with the hope that the reader will find the end product worthy of all the effort put into its production.

Milan Vojnović
May, 2015