MULTIPHASE FLOW WITH SOLID PARTICLES

Multiphase flow is an area of fluid dynamics that describes interactions between two or more phases of matter and is relevant across a wide range of industrial processes and natural environmental systems, from the transport of natural resources to volcanic ash flow. This book covers the topic in detail, providing clear explanations of the underlying physics behind the complex behaviour of solid particles in fluids. The forces involved in particle–fluid interactions are first used to describe the interactions between the particles, and the fundamentals of contact mechanics are then outlined and applied to model interparticle collisions. The book is illustrated with frequent worked examples and algorithms, enabling the reader to develop the required tools for simulating the flow of fluids with solid particles. This self-contained text will appeal to physicists, applied mathematicians, and mechanical engineers working in this important area of research.

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Contents

Nomenclature

1 Basic Concepts and Scope of the Book 1

2 Flow of a Single Particle 6
  2.1 Introduction 6
  2.2 Drag Force 7
  2.3 Added Mass Force 15
  2.4 Basset Force 17
  2.5 Lift Forces 20
  2.6 Drag Torque 28
  2.7 Brownian Motion 30
  2.8 Rarefied Gases 32
  2.9 Thermophoretic Force 35
  2.10 Convective Heat Exchange 38
  2.11 Radiative Heat Exchange 42

3 Introduction to Contact and Impact Mechanics 44
  3.1 Introduction 44
  3.2 Normal Contact: Elastic Deformation 45
  3.3 Normal Contact: Dissipation 54
  3.4 Normal Contact: Plastic Deformation 69

4 Tangential Contact 86
  4.1 Introduction 86
  4.2 Tangential Loading 86
  4.3 Spherical Particles in Contact 90
  4.4 Collisions Dynamics with Tangential Forces 95
## Contents

4.5 The Linear Spring Model: Analytic Solutions 107
4.6 Tangential Forces and Dissipation in Normal Direction 109
4.7 Other Issues 111

5 Adhesion 115
5.1 Introduction 115
5.2 Surface Forces 115
5.3 Adhesion between Bodies 121
5.4 Collisions Dynamics with Adhesion 133
5.5 Collisions Dynamics with Adhesion and Dissipation 134
5.6 Effect of Roughness 136
5.7 Other Models 137

6 Coefficient of Restitution 139
6.1 Definition 139
6.2 Experiments 142
6.3 Theoretical Relations: Dissipative Forces 149
6.4 Theoretical Relations: Plastic Deformation 157
6.5 Selected Issue: Granules 160
6.6 Selected Issue: Collisions of Nanoparticles 162

7 Heat Conduction between Particles 170
7.1 Introduction 170
7.2 Elastic Collisions 172
7.3 Model by Ben-Ammar et al. 175
7.4 Further Extension 175

8 Hard-Sphere Model 177
8.1 Introduction 177
8.2 Hard-Sphere Model: The Main Steps 178
8.3 Relations for Impulse $J$ 180
8.4 Summary of the Models 183
8.5 Extension of the Model to Account for Adhesive Collisions 188
8.6 Final Relations for the Post-Collisional Velocities 191
8.7 Agglomeration 192
8.8 Final Algorithm and Illustration of the Model 194
8.9 Further Investigation and Experimental Validation 196
8.10 More Discussion on the Two-Parameter Hard-Sphere Model for Non-Adhesive Collisions 199
Contents

8.11 Combination with the Soft-Sphere Model for Non-Adhesive Collisions 200

9 Discrete Particle Simulations: A Summary of the Model 206
  9.1 Governing Equations 207
  9.2 Collision Detection 208

10 Multiphase Systems 216
  10.1 Volume Fraction and Particle Spacing 216
  10.2 Response Time 220
  10.3 Phase Coupling 224
  10.4 Suspension Viscosity 227
  10.5 Turbulent Dispersion and Preferential Concentration 231
  10.6 Particle Size Distribution 242
  10.7 Collision Frequency and Collision Frequency Function 249
  10.8 Flows through a Bed of Particles 256

References 262
Index 276
Nomenclature

\(\alpha\) damping parameter in the model by Tsuji et al.
\(\alpha_f\) volume fraction of the fluid phase
\(\alpha_p\) volume fraction of the particle phase
\(\alpha_r\) radiative absorptivity of the particle material
\(\alpha_{max}\) particle packing limit
\(\tilde{\rho}_f\) bulk density of the fluid phase
\(\tilde{\rho}_p\) bulk density of the particle phase
\(\bar{d}_n\) particle number average size
\(\bar{R}_s\) curvature during recovery
\(\bar{u}_z\) surface deformation along z-axis
\(\beta\) coefficient of tangential restitution, \(v_s/v_{s0}\)
\(\beta_0\) limiting coefficient of tangential restitution
\(\beta_{ij}\) collision frequency function (kernel)
\(\omega\) particle angular velocity
\(\Delta p\) pressure loss
\(\Delta t\) time step
\(\delta\) indentation (deformation)
\(\delta_{c}\) maximum indentation
\(\delta_{f}\) final indentation
\(\delta_p\) indentation that leads to the fully plastic deformation
\(\delta_r\) indentation during recovery
\(\delta_t\) tangential indentation
\(\delta_y\) indentation that leads to the elastic–plastic deformation
\(\dot{E}\) energy flux
\(\dot{M}\) momentum flux
\(\dot{Q}\) heat transfer rate
\(\epsilon\) dissipation rate of turbulent kinetic energy per unit mass
\(\epsilon_l\) energy constant in the Lennard-Jones potential
### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>surface energy</td>
</tr>
<tr>
<td>$\gamma_n$</td>
<td>damping parameter in the extended linear spring and dashpot model</td>
</tr>
<tr>
<td>$\gamma_{adh}$</td>
<td>$2\gamma$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>dimensionless indentation in the model by Tsuji et al.</td>
</tr>
<tr>
<td>$\hat{v}$</td>
<td>dimensionless velocity in the model by Tsuji et al.</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>$4/3\pi R_s^{1/2}$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>mean free path of molecules</td>
</tr>
<tr>
<td>$\lambda_n$</td>
<td>damping parameter in the non-linear spring and dashpot model</td>
</tr>
<tr>
<td>$\lambda_o$</td>
<td>damping parameter in the linear spring and dashpot model</td>
</tr>
<tr>
<td>$\mu_f$</td>
<td>fluid dynamic viscosity</td>
</tr>
<tr>
<td>$\mu_m$</td>
<td>suspension viscosity</td>
</tr>
<tr>
<td>$\mu_o$</td>
<td>parameter in the linear spring and dashpot model</td>
</tr>
<tr>
<td>$\mu_r$</td>
<td>relative viscosity</td>
</tr>
<tr>
<td>$\mu_T$</td>
<td>Tabor’s parameter</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson coefficient</td>
</tr>
<tr>
<td>$\psi_f$</td>
<td>fluid kinematic viscosity</td>
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<tr>
<td>$\Phi$</td>
<td>flow potential</td>
</tr>
<tr>
<td>$\Phi_s$</td>
<td>sphericity</td>
</tr>
<tr>
<td>$\Pi_e$</td>
<td>energy coupling parameter</td>
</tr>
<tr>
<td>$\Pi_m$</td>
<td>momentum coupling parameter</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>fluid density</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>particle material density</td>
</tr>
<tr>
<td>$\rho_w$ or $\rho_s$</td>
<td>molecular density (wall or sphere)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan–Boltzmann constant</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>lattice constant</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>standard deviation</td>
</tr>
<tr>
<td>$\sigma_n^2$</td>
<td>number variance</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>absorption coefficient</td>
</tr>
<tr>
<td>$\sigma_r$, $\sigma_\theta$, $\sigma_z$</td>
<td>normal stress components</td>
</tr>
<tr>
<td>$\tau_f$</td>
<td>fluid characteristic time</td>
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<tr>
<td>$\tau_o$</td>
<td>shear stress maximum value</td>
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<tr>
<td>$\tau_t$</td>
<td>thermal response time</td>
</tr>
<tr>
<td>$\tau_v$</td>
<td>momentum response time</td>
</tr>
<tr>
<td>$\tau_w$</td>
<td>wall shear</td>
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<tr>
<td>$\tau_{ij}$</td>
<td>shear stress tensor</td>
</tr>
<tr>
<td>$F_D$</td>
<td>drag force</td>
</tr>
<tr>
<td>$F_M$</td>
<td>Magnus force</td>
</tr>
<tr>
<td>$F_S$</td>
<td>Saffman force</td>
</tr>
<tr>
<td>$F_{am}$</td>
<td>added mass force</td>
</tr>
<tr>
<td>$F_{th}$</td>
<td>thermophoretic force</td>
</tr>
</tbody>
</table>
Nomenclature

\( J \) \hspace{1cm} \text{impulse}

\( J_n \) \hspace{1cm} \text{impulse along the normal direction}

\( J_t \) \hspace{1cm} \text{impulse along the tangential direction}

\( M \) \hspace{1cm} \text{moment of momentum}

\( n \) \hspace{1cm} \text{unit vector (normal direction)}

\( r \) \hspace{1cm} \text{particle location vector}

\( T \) \hspace{1cm} \text{torque}

\( t \) \hspace{1cm} \text{unit vector (tangential direction)}

\( u \) \hspace{1cm} \text{fluid velocity}

\( u_r \) \hspace{1cm} \text{relative velocity}

\( v \) \hspace{1cm} \text{particle velocity}

\( v_{12:0} \) \hspace{1cm} \text{relative pre-collisional velocity}

\( v_{12} \) \hspace{1cm} \text{relative post-collisional velocity}

\( v_{s0} \) \hspace{1cm} \text{relative velocity along the normal direction}

\( v_{t0} \) \hspace{1cm} \text{relative velocity along the tangential direction}

\( Kn \) \hspace{1cm} \text{Knudsen number}

\( Nu \) \hspace{1cm} \text{Nusselt number}

\( Pr_f \) \hspace{1cm} \text{fluid Prandtl number}

\( Re_g \) \hspace{1cm} \text{shear Reynolds number}

\( Re_p \) \hspace{1cm} \text{particle Reynolds number}

\( Re_R \) \hspace{1cm} \text{rotational Reynolds number}

\( St \) \hspace{1cm} \text{Stokes number}

\( f_m \) \hspace{1cm} \text{particle mass frequency}

\( F_n \) \hspace{1cm} \text{cumulative number distribution function}

\( f_n \) \hspace{1cm} \text{particle number frequency}

\( \varepsilon \) \hspace{1cm} \text{particle radiative emissivity}

\( A \) \hspace{1cm} \text{surface area}

\( a_c \) \hspace{1cm} \text{maximum contact radius}

\( A_H \) \hspace{1cm} \text{Hamaker constant}

\( A_p \) \hspace{1cm} \text{projected area}

\( a_p \) \hspace{1cm} \text{contact radius that leads to the fully plastic deformation}

\( A_v \) \hspace{1cm} \text{viscoelastic coefficient (model by Brilliantov et al.)}

\( a_r \) \hspace{1cm} \text{contact radius that leads to the elastic–plastic deformation}

\( b_1 \) \hspace{1cm} \text{coefficient used when modelling elastic–plastic deformations}

\( b_2 \) \hspace{1cm} \text{coefficient used when modelling elastic–plastic deformations}

\( C \) \hspace{1cm} \text{Cunningham correction factor}

\( C_D \) \hspace{1cm} \text{drag force coefficient}

\( c_f \) \hspace{1cm} \text{fluid heat capacity}

\( C_M \) \hspace{1cm} \text{lift force coefficient}
### Nomenclature

- $c_n$: damping parameter
- $c_p$: particle heat capacity
- $C_R$: torque coefficient
- $C_S$: Saffman lift force coefficient
- $C_{ij}$: correlation coefficient
- $d$: particle diameter
- $D_o$: gap between surfaces on the molecular level
- $E$: Young’s modulus
- $e$: coefficient of restitution
- $E_e$: effective Young’s modulus
- $E_k$: total kinetic energy
- $E_{loss}$: mechanical energy loss
- $f$: friction coefficient
- $F_B$: Basset force magnitude
- $f_m$: mass density distribution function
- $f_n$: number density distribution function
- $g$: gravitational acceleration
- $h$: heat transfer coefficient
- $I$: particle moment of inertia
- $K_1, K_2, S$: parameters in Walton and Braun’s model
- $k_B$: Boltzmann’s constant
- $k_f$: fluid thermal conductivity
- $k_n$: spring constant
- $k_p$: particle thermal conductivity
- $k_t$: tangential stiffness (spring constant)
- $L_B$: distance covered due to Brownian motion
- $L_{ave}$: average distance between particles
- $M$: molecular weight
- $m$: particle mass
- $m_f$: mass of the displaced fluid
- $n$: number density
- $N_{ij}$: number of collisions between particles from classes $i$ and $j$
- $P$: contact force
- $p$: fluid pressure or contact pressure
- $P_c$: maximum contact force
- $P_f$: dissipative force (models by Kuwabara and Kono, and Brilliantov et al.)
- $p_m$: average contact pressure
- $P_n$: contact force (normal direction)
- $p_o$: maximum value of the contact pressure
### Nomenclature

- $P_p$: contact force that leads to the fully plastic deformation
- $P_t$: contact force (tangential direction)
- $P_y$: contact force that leads to the elastic–plastic deformation
- $p_y$: contact pressure that leads to elastic–plastic deformation
- $Q$: heat
- $q_w$: heat flux
- $R$: particle radius
- $R$: universal gas constant
- $R_e$: effective radius
- $R_g$: specific gas constant
- $T$: temperature
- $t$: time
- $t_c$: total duration of collision
- $t_d$: collision time
- $T_f$: fluid temperature
- $T_p$: particle temperature
- $t_p$: compression duration
- $t_r$: recovery duration
- $U$: elastic energy
- $u_x, u_y, u_z$: displacement coordinates
- $V$: fluid volume
- $V$: potential energy (Chapter 5)
- $v_1$: post-collisional relative velocity
- $v_o$: initial relative velocity
- $W$: work
- $W_p$: work that leads to the fully plastic deformation
- $W_r$: work during recovery
- $W_y$: work that leads to the elastic–plastic deformation
- $Y$: yield stress in tension/compression