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SPECTRAL THEORY AND ITS APPLICATIONS

Bernard Helffer's graduate-level introduction to the basic tools of spectral analysis is illustrated by numerous examples from the theory of Schrödinger operators and various branches of physics, including statistical mechanics, superconductivity, fluid mechanics, and kinetic theory. The later chapters also introduce the theory of non-self-adjoint operators, with an emphasis on the role of pseudospectra.

The author's focus on applications, along with exercises and examples, enables readers to connect theory with practice so that they will develop a good understanding of how the abstract spectral theory can be applied. The final chapter provides various problems that have been the subject of active research in recent years and will challenge the reader's understanding of the material covered.

Bernard Helffer is a Professor in the Department of Mathematics at Université Paris-Sud. He has published more than 200 papers in mathematics and mathematical physics and authored five books. In 2011 he was awarded the Prix de l'État by the French Academy of Sciences.

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Spectral Theory and its Applications

BERNARD HELFFER

Université Paris-Sud



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