Predictive Modeling Applications in Actuarial Science

Volume II: Case Studies in Insurance

Predictive modeling uses data to forecast future events. It exploits relationships between explanatory variables and the predicted variables from past occurrences to predict future outcomes. Forecasting financial events is a core skill that actuaries routinely apply in insurance and other risk-management applications. Predictive Modeling Applications in Actuarial Science emphasizes lifelong learning by developing tools in an insurance context, providing the relevant actuarial applications, and introducing advanced statistical techniques that can be used to gain a competitive advantage in situations with complex data.

Volume II examines applications of predictive modeling. Where Volume I developed the foundations of predictive modeling, Volume II explores practical uses for techniques, focusing especially on property and casualty insurance. Readers are exposed to a variety of techniques in concrete, real-life contexts that demonstrate their value, and the overall value of predictive modeling, for seasoned practicing analysts as well as those just starting out.

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The International Series on Actuarial Science, published by Cambridge University Press in conjunction with the Institute and Faculty of Actuaries, contains textbooks for students taking courses in or related to actuarial science, as well as more advanced works designed for continuing professional development or for describing and synthesizing research. The series is a vehicle for publishing books that reflect changes and developments in the curriculum, that encourage the introduction of courses on actuarial science in universities, and that show how actuarial science can be used in all areas where there is long-term financial risk.

A complete list of books in the series can be found at www.cambridge.org/statistics. Recent titles include the following:

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  Erik Bølviken

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  David C.M. Dickson, Mary R. Hardy & Howard R. Waters

- *Risk Modelling in General Insurance*  
  Roger J. Gray & Susan M. Pitts

- *Financial Enterprise Risk Management*  
  Paul Sweeting

- *Regression Modeling with Actuarial and Financial Applications*  
  Edward W. Frees

- *Predictive Modeling Applications in Actuarial Science, Volume I: Predictive Modeling Techniques*  
  Edited by Edward W. Frees, Richard A. Derrig & Glenn Meyers

- *Nonlife Actuarial Models*  
  Yiu-Kuen Tse

- *Generalized Linear Models for Insurance Data*  
  Piet De Jong & Gillian Z. Heller
PREDICTIVE MODELING APPLICATIONS IN ACTUARIAL SCIENCE

Volume II: Case Studies in Insurance

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Preface

In January 1983, the North American actuarial education societies (the Society of Actuaries and the Casualty Actuarial Society) announced that a course based on regression and time series would be part of their basic educational requirements. Since that announcement, a generation of actuaries has been trained in these fundamental applied statistical tools. This two-set volume builds on this training by developing the fundamentals of predictive modeling and providing corresponding applications in actuarial science, risk management, and insurance.

The series is written for practicing actuaries who wish to get a refresher on modern-day data-mining techniques and predictive modeling. Almost all of the international actuarial organizations now require continuing education of their members. Thus, in addition to responding to competitive pressures, actuaries will need materials like these books for their own continuing education. Moreover, it is anticipated that these books could be used for seminars that are held for practicing actuaries who wish to get professional accreditation (known as VEE, or validated by educational experience).

Volume I lays out the foundations of predictive modeling. Beginning with reviews of regression and time series methods, this book provides step-by-step introductions to advanced predictive modeling techniques that are particularly useful in actuarial practice. Readers will gain expertise in several statistical topics, including generalized linear modeling and the analysis of longitudinal, two-part (frequency/severity) and fat-tailed data. Thus, although the audience is primarily professional actuaries, the book exhibits a “textbook” approach, and so this volume will also be useful for continuing professional development.

An international author team (seven countries, three continents) developed Volume I, published in 2014. You can more learn more about Volume I at http://research.bus.wisc.edu/PredModelActuaries

Volume II examines applications of predictive models, focusing on property and casualty insurance, primarily through the use of case studies. Case studies provide a learning experience that is closer to real-world actuarial work than can be provided
by traditional self-study or lecture/work settings. They can integrate several analysis techniques or, alternatively, can demonstrate that a technique normally used in one practice area could have value in another area. Readers can learn that there is no unique correct answer. Practicing actuaries can be exposed to a variety of techniques in contexts that demonstrates their value. Academic actuaries and students see that there are valid applications for the theoretical material presented in Volume I. As with Volume I, we have extensive sample data and statistical code on the series website so that readers can learn by doing.

The first three chapters of Volume II focus on applications of the generalized linear model (GLM), arguably the workhorse of predictive modeling in actuarial applications. Chapter 1, by Ernesto Schirmacher, gives an overview of the use of GLMs in pricing strategy, focusing on private passenger automobile. Dan Tevet’s Chapter 2 reinforces this discussion by examining insurance for motorcycles, emphasizing the comparison between frequency-severity and pure premium models. In Chapter 3, Greg Taylor and James Sullivan demonstrate how to use GLM techniques in loss reserving. Although the two books in the series are written independently, readers with access to the first book will appreciate these three chapters more deeply after reviewing the foundations in Chapters 2–6 of Volume I.

Chapters 4 and 5 provide extensions of the generalized linear model. Like Chapter 1, in Chapter 4, Peng Shi and James Guszcza also examine pricing strategies for personal automobile insurance. However, they show how to price insurance when more than one type of coverage, such as third-party liability and personal injury protection, is available in the database; by taking advantage of the multivariate nature of claims, they are able to incorporate dependencies among coverages in their pricing structure. For another approach to incorporating dependencies, in Chapter 5, Mona S. A. Hammad and Galal A. H. Harby use multilevel models. They provide a unique and interesting case study of group health insurance in the Egyptian market. Chapters 8 and 16 of Volume I provide an introduction to mixed and multilevel modeling.

Chapters 6 and 7 describe applications of unsupervised predictive modeling methods. Most predictive modeling tools require that one or more variables be identified as “dependent variables” or the “outcome of interest,” and other variables are used to explain or predict them; this is known as a supervised predictive model. In contrast, unsupervised models treat all variables alike and do not require this identification. Chapter 12 of Volume I, by Louise Frances, introduced unsupervised learning with a focus on common methods of dimension reduction, principal components/factor analysis, and clustering. Chapter 6 of this volume, by Ji Yao, builds on this introduction with an application in insurance ratemaking. Louise A. Frances, the author of Chapter 7 of this volume, also follows up with two advanced unsupervised learning techniques, a variation of principal components known as PRIDIT, and a
(unsupervised) variation of random forests, a tree-based data-mining method. She applies these techniques to help identify predictors of claims that are fraudulent or questionable.

Chapters 8 through 11 show how to use predictive modeling techniques in problems that are currently receiving substantial attention in actuarial science and insurance risk modeling. In Chapter 8, Glenn Meyers shows how to take the output of a Bayesian Monte Carlo Markov chain (MCMC) stochastic loss reserve model and calculate the predictive distribution of the estimates of the expected loss over a finite time horizon. Luyang Fu and Xianfang (Frank) Liu, in Chapter 9, compare GLM modeling to finite mixture models to study claims triaging and high-deductible pricing using workers’ compensation data.

Chapter 10, by Mohamad A. Hindawi and Claudine H. Modlin, provides a framework for managing claim escalation. This chapter also discusses claims triaging and text mining, using penalized regression techniques, such as elastic net, to help with variable selection. In Chapter 11, Udi Makov and Jim Weiss describe how to analyze data collected from policyholders’ vehicles via telematics to help determine motor vehicle premium rates. Data collected via telematics are volatile and voluminous, and actuaries and data scientists must take particular care when applying predictive modeling techniques.
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