

# **Quantitative Methods of Data Analysis for the Physical Sciences** and **Engineering**

This book provides thorough and comprehensive coverage of most of the new and important quantitative methods of data analysis for college and graduate students and practitioners. In recent years, data analysis methods have exploded alongside advanced computing power, and an understanding of such methods is critical to getting the most out of data and for extracting signal from noise. The book excels in explaining difficult concepts through simple explanations and detailed explanatory illustrations. Most unique is the focus on confidence limits for power spectra and their proper interpretation, something rare or completely missing in other books. Likewise, there is a thorough discussion of how to assess uncertainty via use of Expectancy, and easy-to-apply and -understand Bootstrap method. The book is written so that descriptions of each method are as self-contained as possible. Many examples are presented to clarify interpretations, as are user tips in highlighted boxes.

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To the love of my life, my wife Rhonda





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#### **Preface**

This book is the outcome of a one-semester graduate class taught in the Department of Earth and Environmental Sciences at Columbia University, although the book could be used over two or even three semesters, if desired. I have taught this class since 1985, having taken over from a departing marine seismologist who had taught the course as one on Fourier analysis, the only topic that computers of the day were capable of performing, because of the development of the Fast Fourier Transform. However, at that time computers were rapidly becoming powerful enough to allow application of methods requiring more power and memory. New methods were sprouting yearly, and as the computers grew faster, previously sluggish methodologies were becoming realizable. At the time I started teaching the course, there were no textbooks (none!) that gave a thorough introduction to the primary methods. Numerical Recipes published in the early 1980s - did present a brief overview and the computer code necessary to run nearly every method, and it was a godsend. It occurred to me that my class notes should be converted to a book to fill this void. Over the last 30 years many other books have been published, but in my opinion there is still a need for an introductory-level book that spans a broad number of the most useful techniques. Regardless of its introductory nature, I have tried to give the reader a complete enough understanding to allow him or her to properly apply the methods while avoiding common pitfalls and misunderstandings.

I try to present the methods following a few fundamental themes: e.g., Principle of Maximum Likelihood for deriving optimal methods, and Expectancy for estimating uncertainty. I hope this makes these important themes better understood and the material easier to grasp.

#### **How to Use This Book**

This book is designed to fill many needs, according to the level of the student. Some, like myself, see the methods clearly if they understand their complete derivation, while others don't require that detailed understanding. In an effort to satisfy both readerships, I have placed complete derivations in boxes highlighted with 25 percent grayscale: these boxes are optional, and the reader is free, if preferred, to skip the box and go straight to the answer (all equations in derivation boxes are prefaced by "D" – for example, "D5.1"). There are student exercises at the end of each chapter, some of which require



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computing. I do not present code because it changes so quickly, but I do show some MATLAB code in the solution manual. Data for exercises requiring such can be found at www.cambridge.org/martinson. Most of the examples in the book are taken from the natural sciences, although they are presented so as to be understandable to anyone. Special user tips are included in boxes highlighted with 15 percent grayscale. I have attempted to make each chapter stand on its own (as far as possible), so the reader doesn't need to have read the entire book in order to understand material from previous chapters. This should make the book good for easy use and reference for a particular method.

Read it, practice, and when not sure what road to take, take all possible roads and then determine which is the most appropriate for your particular analysis. Then, maybe present several results explaining the differences, and why you favor the method you choose.



## **Acknowledgments**

As with all books evolving from a course, one must acknowledge the considerable input from students and teaching assistants. As any teacher knows, it is usually the one teaching who learns more than anyone - when first teaching this course there were numerous derivations that only were partly developed, then a "miracle occurred" that skipped some "intuitively obvious" steps to the final result. No such skipped steps occur within this book. Over the years, excellent questions from students that I could not answer on the spot forced me to fill in many aspects of the material. So I offer a heartfelt thanks to those who stumped me in class. In that same vein, I would appreciate hearing about any errors still present in the book. The class has benefited from some incredibly smart and motivated teaching assistants, and many of the exercises appearing at the end of chapters originated from them (special thanks go to Sharon Stammerjohn, Chen Chen, and Darren McKee, among many others). Unfortunately, as I transformed my class notes into a textbook, my wife, Rhonda, became a writer's widow for nearly a year - I can't thank her enough for all the support she has given me. And finally, but not least, thanks to my editor, Matt Lloyd, at Cambridge University Press, who was a constant source of improvements and encouragement!

Finally, that ubiquitous message accompanying all such books: any errors in the book are strictly mine. Oh, and the other statement: any views expressed in this book (and there are many) are entirely mine. Enjoy!

