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978-1-107-02927-9 - The Skew-Normal and Related Families
Adelchi Azzalini and Antonella Capitanio
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The Skew-Normal and Related Families

Interest in the skew-normal and related families of distributions has grown enormously over recent years, as theory has advanced, challenges of data have grown and computational tools have become more readily available. This comprehensive treatment, blending theory and practice, will be the standard resource for statisticians and applied researchers. Assuming only basic knowledge of (non-measure-theoretic) probability and statistical inference, the book is accessible to the wide range of researchers who use statistical modelling techniques.

Guiding readers through the main concepts and results, the book covers both the probability and the statistics sides of the subject, in the univariate and multivariate settings. The theoretical development is complemented by numerous illustrations and applications to a range of fields including quantitative finance, medical statistics, environmental risk studies and industrial and business efficiency. The authors' freely available R package `sn`, available from CRAN, equips readers to put the methods into action with their own data.

ADELCHI AZZALINI was Professor of Statistics in the Department of Statistical Sciences at the University of Padua until his retirement in 2013. Over the last 15 years or so, much of his work has been dedicated to the research area of this book. He is regarded as the pioneer of this subject due to his 1985 paper on the skew-normal distribution; in addition, several of his subsequent papers, some of which have been written jointly with Antonella Capitanio, are considered to represent fundamental steps. He is the author or co-author of three books, over 70 research papers and four packages written in the R language.

ANTONELLA CAPITANIO is Associate Professor of Statistics in the Department of Statistical Sciences at the University of Bologna. She began working on the skew-normal distribution about 15 years ago, co-authoring with Adelchi Azzalini a series of papers, related to the skew-normal and skew-elliptical distributions, which have provided key results in this area.

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with the collaboration of
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Preface

Since about the turn of the millennium, the study of parametric families of probability distributions has received new, intense interest. The present work is an account of one approach which has generated a great deal of activity.

The distinctive feature of the construction to be discussed is to start from a symmetric density function and, by suitable modification of this, generate a set of non-symmetric distributions. The simplest effect of this process is represented by skewness in the distribution so obtained, and this explains why the prefix ‘skew’ recurs so often in this context. The focus of this construction is not, however, skewness as such, and we shall not discuss the quintessential nature of skewness and how to measure it. The target is instead to study flexible parametric families of continuous distributions for use in statistical work. A great deal of those in standard use are symmetric, when the sample space is unbounded. The aim here is to allow for possible departure from symmetry to produce more flexible and more realistic families of distributions.

The concentrated development of research in this area has attracted the interest of both scientists and practitioners, but often the variety of proposals and the existence of related but different formulations bewilders them, as we have been told by a number of colleagues in recent years. The main aim of this work is to provide a key to enter this theme. Besides its role as an introductory text for the newcomer, we hope that the present book will also serve as a reference work for the specialist.

This is not the first book covering this area: there exists a volume, edited by Marc Genton in 2004, which has been very beneficial to the dissemination of these ideas, but since its publication many important results have appeared and the state of the art is now quite different. Even today a definitive stage of development of this field has not been reached, if one assumes for a moment that such a state can ever be achieved, but we feel that the material is now sufficiently mature to also be fruitfully used for routine work of non-specialists.

The general framework and the key concepts of our development are formulated in Chapter 1. Subsequent chapters develop specific directions, in the univariate and in the multivariate case, and discuss why other directions are given lesser importance or even neglected. Some people may find it surprising that quite ample space is given to univariate distributions, considering that the context of multivariate distributions is where the new proposals appear more significant. However, besides its interest *per se*, the univariate case facilitates the exposition of many concepts, even when their main relevance is in the multivariate context.

There is a noticeable difference in the more articulate expository style of Chapters 1 to 6 compared with the briefer – even meagre one might say – summaries employed in Chapters 7 and 8, which deal with more specific themes. One reason for this choice is the greater importance given to the exposition of the basic concepts, recalling our main target in writing the book, and certain applied topics do not require a detailed discussion after the foundations of the construction are in place. Moreover, some of the more specialized or advanced topics are still in an evolutionary state, and any attempt to arrange them in an organized system is likely to become obsolete quite rapidly.

Chapters 1 to 6 are organized with a set of complements each, dealing with some more specialized topics. At first reading or if a reader is interested in getting a grasp of the key concepts only, these complements can be skipped without hindrance to understanding the core parts. At the end of these chapters there are sets of problems of varied levels of difficulty. As a rule of thumb, the harder ones are those with a reference at the end.

The development of this work has greatly benefited from the generous help of Giuliana Regoli, who has dedicated countless hours to examining and discussing with us many mathematical aspects. Obviously, any remaining errors are our own responsibility. We are also grateful to Elvezio Ronchetti, Marco Minozzo and Chris Adcock for comments on aspects of robustness, time series and quantitative finance, respectively, and to Marc Genton for several remarks on the nearly final draft. Even if in a less tangible form, our views on this research area have benefited from interactions with people of the ‘skew community’, with whom we have shared our enthusiasm during these years. It has been a stimulating and rewarding enterprise.

Adelchi Azzalini and Antonella Capitanio
February 2013