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978-1-107-02911-8 - Superstring Theory Volume 1: Introduction 25th Anniversary Edition

Michael B. Green, John H. Schwarz and Edward Witten

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Superstring Theory

Volume 1: Introduction

25th Anniversary Edition

Twenty-five years ago, Michel Green, John Schwarz, and Edward Witten wrote two volumes on string theory. Published during a period of rapid progress in this subject, these volumes were highly influential for a generation of students and researchers. Despite the immense progress that has been made in the field since then, the systematic exposition of the foundations of superstring theory presented in these volumes is just as relevant today as when first published.

A self-contained introduction to superstrings, Volume 1 begins with an elementary treatment of the bosonic string, before describing the incorporation of additional degrees of freedom: fermionic degrees of freedom leading to supersymmetry, and internal quantum numbers leading to gauge interactions. A detailed discussion of the evaluation of tree-approximation scattering amplitudes is also given.

Featuring a new Preface setting the work in context in light of recent advances, this book is invaluable for graduate students and researchers in general relativity and elementary particle theory.

MICHAEL B. GREEN is the Lucasian Professor of Mathematics at the University of Cambridge, JOHN H. SCHWARZ is the Harold Brown Professor of Theoretical Physics at the California Institute of Technology, and EDWARD WITTEN is the Charles Simonyi Professor of Mathematical Physics at the Institute for Advanced Study. Each of them has received numerous honors and awards.

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TO OUR PARENTS

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Frontmatter

[More information](#)

Contents

	<i>Preface to the 25th Anniversary Edition</i>	ix
1	Introduction	1
1.1	The early days of dual models	1
1.1.1	The Veneziano amplitude and duality	6
1.1.2	High-energy behavior of the Veneziano model	10
1.1.3	Ramifications of the Veneziano model	12
1.2	Dual models of everything	13
1.2.1	Duality and the graviton	14
1.2.2	Unification in higher dimensions	14
1.2.3	Supersymmetry	16
1.3	String theory	18
1.3.1	The massless point particle	18
1.3.2	Generalization to strings	21
1.3.3	Constraint equations	23
1.4	String interactions	27
1.4.1	Splitting of strings	29
1.4.2	Vertex operators	32
1.4.3	Use of vertex operators	35
1.4.4	Evaluation of the scattering amplitude	38
1.4.5	The mass of the graviton	40
1.5	Other aspects of string theory	42
1.5.1	Gravitational Ward identities	42
1.5.2	Open strings	46
1.5.3	Internal symmetries of open strings	47
1.5.4	Recovery of the Veneziano amplitude	49
1.5.5	Comparison with QCD	51
1.5.6	Unitarity and gravity	52
1.6	Conclusion	55
2	Free bosonic strings	57
2.1	The classical bosonic string	57
2.1.1	String action and its symmetries	59
2.1.2	The free string in Minkowski space	61
2.1.3	Classical covariant gauge fixing and field equations	62
2.2	Quantization – old covariant approach	74

Cambridge University Press

978-1-107-02911-8 - Superstring Theory Volume 1: Introduction 25th Anniversary Edition

Michael B. Green, John H. Schwarz and Edward Witten

Frontmatter

[More information](#)

vi

Contents

2.2.1	Commutation relations and mode expansions	75
2.2.2	Virasoro algebra and physical states	79
2.2.3	Vertex operators	86
2.3	Light-cone gauge quantization	92
2.3.1	Light-cone gauge and Lorentz algebra	93
2.3.2	Construction of transverse physical states	100
2.3.3	The no-ghost theorem and the spectrum-generating algebra	102
2.3.4	Analysis of the spectrum	113
2.3.5	Asymptotic formulas for level densities	116
2.4	Summary	119
3	Modern covariant quantization	121
3.1	Covariant path-integral quantization	121
3.1.1	Faddeev–Popov ghosts	122
3.1.2	Complex world-sheet tensor calculus	124
3.1.3	Quantization of the ghosts	127
3.2	BRST quantization	131
3.2.1	Construction of BRST charge	131
3.2.2	Covariant calculation of the Virasoro anomaly	139
3.2.3	Virasoro, conformal and gravitational anomalies	144
3.2.4	Bosonization of ghost coordinates	148
3.3	Global aspects of the string world sheet	156
3.4	Strings in background fields	165
3.4.1	Introduction of a background space-time metric	165
3.4.2	Weyl invariance	167
3.4.3	Conformal invariance and the equations of motion	172
3.4.4	String-theoretic corrections to general relativity	177
3.4.5	Inclusion of other modes	178
3.4.6	The dilaton expectation value and the string coupling constant	182
3.5	Summary	184
4	World-sheet supersymmetry in string theory	185
4.1	The classical theory	186
4.1.1	Global world-sheet supersymmetry	188
4.1.2	Superspace	190
4.1.3	Constraint equations	194
4.1.4	Boundary conditions and mode expansions	197
4.2	Quantization – the old covariant approach	199
4.2.1	Commutation relations and mode expansions	200
4.2.2	Super-Virasoro algebra and physical states	203
4.2.3	Boson-emission vertex operators	207

Cambridge University Press

978-1-107-02911-8 - Superstring Theory Volume 1: Introduction 25th Anniversary Edition

Michael B. Green, John H. Schwarz and Edward Witten

Frontmatter

[More information](#)*Contents*

vii

4.3	Light-cone gauge quantization	210
	4.3.1 The light-cone gauge	210
	4.3.2 No-ghost theorem and the spectrum-generating algebra	214
	4.3.3 The GSO conditions	218
	4.3.4 Locally supersymmetric form of the action	224
	4.3.5 Superstring action and its symmetries	228
4.4	Modern covariant quantization	231
	4.4.1 Faddeev–Popov ghosts	231
	4.4.2 BRST symmetry	235
	4.4.3 Covariant computation of the Virasoro anomaly	236
4.5	Extended world-sheet supersymmetry	237
	4.5.1 The $N = 2$ theory	237
	4.5.2 The $N = 4$ theory	243
4.6	Summary	244
4.A	Super Yang–Mills theories	244
5	Space-time supersymmetry in string theory	249
5.1	The classical theory	249
	5.1.1 The superparticle	249
	5.1.2 The supersymmetric string action	253
	5.1.3 The local fermionic symmetry	255
	5.1.4 Type I and type II superstrings	258
5.2	Quantization	259
	5.2.1 Light-cone gauge	260
	5.2.2 Super-Poincaré algebra	266
5.3	Analysis of the spectrum	270
	5.3.1 Open superstrings	270
	5.3.2 Closed superstrings	277
5.4	Remarks concerning covariant quantization	280
5.5	Summary	281
5.A	Properties of $SO(2n)$ groups	282
5.B	The $\text{spin}(8)$ Clifford algebra	288
6	Nonabelian gauge symmetry	291
6.1	Open strings	291
	6.1.1 The Chan–Paton method	292
	6.1.2 Allowed gauge groups and representations	298
6.2	Current algebra on the string world sheet	300
6.3	Heterotic strings	305
	6.3.1 The $SO(32)$ theory	307
	6.3.2 The $E_8 \times E_8$ theory	312
6.4	Toroidal compactification	317

Cambridge University Press

978-1-107-02911-8 - Superstring Theory Volume 1: Introduction 25th Anniversary Edition

Michael B. Green, John H. Schwarz and Edward Witten

Frontmatter

[More information](#)

viii

Contents

6.4.1	Compactification on a circle	318
6.4.2	Fermionization	321
6.4.3	Bosonized description of the heterotic string	323
6.4.4	Vertex operator representations	329
6.4.5	Formulas for the cocycles	335
6.4.6	The full current algebra	337
6.4.7	The E_8 and $\text{spin}(32)/Z_2$ lattices	337
6.4.8	The heterotic string spectrum	339
6.5	Summary	343
6.A	Elements of E_8	344
6.B	Modular forms	350
7	Tree amplitudes	353
7.1	Bosonic open strings	354
7.1.1	The structure of tree amplitudes	355
7.1.2	Decoupling of ghosts	362
7.1.3	Cyclic symmetry	364
7.1.4	Examples	371
7.1.5	Tree-level gauge invariance	374
7.1.6	The twist operator	377
7.2	Bosonic closed strings	378
7.2.1	Construction of tree amplitudes	379
7.2.2	Examples	381
7.2.3	Relationship to open-string trees	389
7.3	Superstrings in the RNS formulation	390
7.3.1	Open-string tree amplitudes in the bosonic sector	391
7.3.2	The F_1 picture	393
7.3.3	Examples	395
7.3.4	Tree amplitudes with one fermion line	399
7.3.5	Fermion-emission vertices	401
7.4	Superstrings in the supersymmetric formulation	411
7.4.1	Massless particle vertices	411
7.4.2	Open-string trees	418
7.4.3	Closed-string trees	422
7.4.4	Heterotic-string trees	423
7.5	Summary	428
7.A	Coherent-state methods and correlation functions	429
	<i>Bibliography</i>	435
	<i>Index</i>	465

Cambridge University Press

978-1-107-02911-8 - Superstring Theory Volume 1: Introduction 25th Anniversary Edition

Michael B. Green, John H. Schwarz and Edward Witten

Frontmatter

[More information](#)

Preface to the 25th Anniversary Edition

In the twenty-five years since the original publication of these two Volumes, there have been numerous developments in string theory. The curious twists and turns that marked its pre-1987 evolution have continued apace, and current research makes contact with a wide range of areas of mathematics and physics. In the following we will mention briefly some of these developments and then explain why we believe that these volumes are still useful.

Major insights into the non-perturbative structure of string theory followed from the discovery of non-perturbative duality symmetries of super-string theory. This led to the realization that the myriad of apparently distinct superstring theories that arise in ten or fewer dimensions actually are different perturbative approximations to the same underlying theory, which has come to be known as M-theory. Furthermore, M-theory has eleven-dimensional supergravity as another semiclassical limit. The understanding of these interconnections was aided by the simultaneous discovery of the properties of a family of dynamical objects called p -branes, which are extended objects that fill p spatial dimensions, as opposed to the 1 dimension of the string. p -branes can be viewed as solitons that are generalizations of the magnetic monopoles of conventional quantum field theory and the black holes of general relativity. Indeed, these discoveries have stimulated impressive advances in understanding the quantum and thermodynamic properties of large classes of black holes.

An important outcome of these considerations has been striking progress in understanding the nonperturbative structure of the quantum field theories that arise from string theory in various limits. Most notably, it has led to the gauge/gravity correspondence, also known as holographic duality, according to which a quantum theory of gravity in a D -dimensional asymptotically anti-de Sitter spacetime is equivalent to a $(D - 1)$ -dimensional local quantum field theory. The most studied example is an equivalence between four-dimensional maximally supersymmetric $U(N)$ Yang–Mills field theory and type IIB superstring theory in five-dimensional anti-de Sitter space times a five-dimensional sphere with N units of Ramond–Ramond flux.

In 1987, the main focus of this field was the application of string perturbation theory to describe the fundamental forces and particles. This remains a focus, but nowadays nonperturbative methods are used as well. The possibilities for using string theory to construct models of particle

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Frontmatter

[More information](#)

physics that are at least semi-realistic have proliferated since 1987, and some physicists, in part because of clues coming from cosmological observations, have come to suspect that the “landscape” of string theory possibilities is actually the way the Universe works. In addition, nowadays, string theory is applied to other types of problems in theoretical physics, including the modeling of heavy ion collisions and the theory of quantum critical points relevant to condensed matter physics. From a contemporary point of view, the gauge/gravity correspondence means that string theory and quantum gravity are inevitable parts of the description of strongly coupled quantum systems.

Many of the developments of the last quarter century, which of course are not covered in these two volumes, have been described in more recent books and review articles. These more recent developments are rooted to a large extent in the basic material covered here. Given the immense breadth and volume of recent research, it is extremely difficult for newer books and articles to present all the details of the more basic underlying material. For this reason, we feel that the material contained in these volumes remains of value and we hope it will continue to provide stimulus for further research in this rich field – wherever it eventually may lead.

Michael B. Green

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