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Introduction

A tectonic earthquake is a unique, interesting and challenging natural phenomenon. At the same time the earthquake can cause death and huge material losses. The Emilia-Romagna, Italy (moment magnitude $M_w = 5.9$, May 20, 2012), Tohoku-Oki, Japan ($M_w = 9.0$, March 3, 2011), Christchurch, New Zealand ($M_w = 6.3$, February 22, 2011), Chile ($M_w = 8.8$, February 27, 2010), Port-au-Prince, Haiti ($M_w = 7.0$, January 12, 2010), L'Aquila, Italy ($M_w = 6.3$, April 6, 2009), Sichuan, China ($M_w = 7.9$, May 12, 2008), Pisco, Peru ($M_w = 8.0$, August 15, 2007), Kashmir, Pakistan ($M_w = 7.6$, October 8, 2005), Sumatra, Indonesia ($M_w = 9.1$, December 26, 2004), Bam, Iran ($M_w = 6.6$, December 26, 2003), Gujarat, India ($M_w = 7.7$, January 26, 2001), Izmit, Turkey ($M_w = 7.6$, August 17, 1999), Kobe, Japan ($M_w = 6.8$, January 17, 1995), and Northridge, California ($M_w = 6.7$, January 17, 1994) earthquakes are well-known examples of tragic and catastrophic events of the past two decades. Three of them belong to the largest earthquakes ever recorded. Some of them, however, indicate a troubling and important fact: an earthquake that kills and causes large material damage is not necessarily a big event in terms of released energy. The M_w 6.7 Northridge 1994 and M_w 6.8 Kobe 1995 earthquakes caused, at the time, unprecedented record economic losses in the USA and Japan, respectively, although they released (in the form of seismic waves) roughly 3000 times less energy than the M_w 9.0 Tohoku-Oki 2011 earthquake. In the long-term average, there are approximately 13 earthquakes in the magnitude range [7, 7.9] and 120 in the magnitude range [6, 6.9] per year. Any earthquake of this size can become a tragic and damaging event if it hits a densely populated area.

Apparently surprisingly, a significant part of the world's population lives in earthquake-prone areas: large populated areas are close to active seismogenic faults, and, moreover, large cities are often located at the surface of sediment-filled basins and valleys. The reasons why large human settlements developed in such areas relate to the geology, hydrology, climate and geography of the areas and regions. Both aspects of the locations of large cities, that is, being close to seismogenic faults and atop sediments, have strong impacts on the earthquake hazard and consequently also earthquake risk. Being close to a seismogenic fault obviously poses an earthquake threat. Also being atop a sediment-filled basin or valley can considerably increase the earthquake hazard. This is because seismic wave interference and resonant phenomena in sediment-filled basins and valleys can produce anomalously large

Cambridge University Press & Assessment
978-1-107-02881-4 — The Finite-Difference Modelling of Earthquake Motions
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earthquake motion at the Earth's surface and lead to so-called 'site effects': characteristics of the earthquake vibratory motion of the Earth's surface can attain locally anomalous values – e.g., amplitudes can be considerably amplified in the time or frequency domain, and strong motion can be significantly prolongated. The anomalous values can occur at frequencies at which buildings, constructions and industrial facilities can be damaged or destroyed. The greatest damage to buildings and constructions is often due to mutual resonance between the local geological and artificial structures. The September 19, 1985 Mexico quake is one of the best examples of the damaging potential of such effects. The epicentre was on the Pacific coast; however, the earthquake caused major damage in Ciudad de México – more than 350 km away from the epicentre. A major part of the Mexican capital sits on unconsolidated lake sediments and artificial land or, in other words, atop a very soft sedimentary basin. The interference and resonant phenomena in sediments led to disastrous effects. Hundreds of buildings were completely destroyed, hundreds partially collapsed or were seriously damaged. At least 10 000 people died.

In the worldwide long-term average, the number of earthquakes will not decrease. On the other hand, the density of population will increase in many areas. In industrialized nations the technological complexity of the populated areas will increase. This could bring more vulnerability to earthquakes if building codes are not either at the state-of-the-art level or actually enforced. In developing countries the increasing population means great and growing earthquake risk. Even relatively weak earthquakes will be capable of causing tremendous human losses and damage, and consequently significantly affect the economy of the region or the entire country.

Two natural scientific tasks for seismologists are, therefore, earthquake prediction and prediction of ground motion during future earthquakes at a site of interest. These tasks are also primary scientific responsibilities of seismologists towards society.

Seismologists still cannot predict the time, place and size of future earthquakes. Even more interestingly, we still do not know whether such prediction is possible in principle and will be possible technically. This is because we still do not have answers to important questions regarding the processes of the long-term preparation and nucleation of earthquakes. We still do not know enough about seismogenic faults and the Earth's interior at depths where earthquakes are being prepared. This is mainly because we cannot simply install sensors and instruments at those depths and places. In other words, a classical direct controlled physical experiment aiming to measure these processes is impossible – at least from an economic viewpoint at present. Direct measurements are practically restricted to the Earth's surface, and almost all information about the rupture process and structure of the Earth's interior is encoded in instrument records of earth motion (seismograms) during earthquakes. Consequently, our knowledge of the earthquake source and the Earth's interior has to be confronted with the seismograms.

Hereby, we come to the role of theoretical and numerical-modelling methods. They are irreplaceable tools in earthquake research – in investigating preparation and nucleation of earthquakes, the rupture process on the fault, radiation of seismic waves, seismic wave propagation in the Earth's interior, and earthquake motion of the Earth's surface.

No matter whether seismologists can or cannot timely predict earthquake occurrence, they must predict earthquake ground motion during potential future earthquakes in densely populated areas and sites of special importance, e.g., sites of nuclear power plants, big dams and key industrial facilities. Even if the timely prediction of earthquake occurrence were physically possible and technically feasible, seismologists must predict what can or will happen during a future earthquake. This is vital for land-use planning, designing new buildings and reinforcing existing ones. It is also extremely important for undertaking actions that could help mitigate losses during future earthquakes.

Prediction of the earthquake ground motion for a given area or site might be based on an empirical approach if sufficient earthquake recordings at the site or physically relevant for the site were available. In most cases, however, there is a severe lack of data. Consequently, it is the theory and numerical simulations that have to be applied.

Although we still need to better understand processes in the Earth and considerably better know the Earth's interior and seismogenic faults, the present state of our knowledge and the capabilities of modern seismic arrays impose stringent requirements on the theoretical and computational models. For example, considering computational models of surface local geological structures, it is necessary to include nonplanar interfaces between layers – possibly with large contrasts in values of material parameters, gradients in P-wave and S-wave speeds, density and quality factors inside layers, P-wave to S-wave speed ratio possibly as large as 5 and more in the soft surface sediments, and often also free-surface topography. In particular, the rheology of the medium has to allow for realistic broad-band attenuation. Realistic strong ground motion simulations should also account for the possibility of nonlinear behaviour in soft soils.

There are no exact (analytical) solutions for such realistic models. Only approximate computational methods are able to account for the geometrical and rheological complexity of the sufficiently realistic models. The most important aspects of all methods are accuracy and computational efficiency (in terms of computer memory and time). These two aspects are in most cases contradictory. It is, however, the reasonable balance between the accuracy and computational efficiency in the case of complex realistic structures that makes the numerical-modelling methods and, more specifically, so-called domain (in the spatial sense) numerical methods dominant among all approximate methods.

A variety of domain numerical methods have been developed in application to earthquake motion during the past few decades. The best known are the (time-domain) finite-difference, finite-element, Fourier pseudo-spectral, spectral-element and discontinuous Galerkin methods. Both theoretical analyses and numerical experience show that none of these methods can be chosen as the universally best (in terms of accuracy and computational efficiency) method for all important problems in earthquake research, that is, for all medium-wavefield configurations. Each method has its advantages and disadvantages, which often depend on the particular application.

Moreover, recent experience from two international comparative numerical exercises for the Grenoble valley, France, and the Mygdonian basin near Thessaloniki, Greece (ESG 2006 and E2VP, respectively), show that at least two different but comparably accurate methods should be used in order to obtain a reliable numerical prediction of earthquake ground motion for a site of interest.

Two decades of 3D earthquake motion modelling, mainly in California, the SCEC comparative exercises, ESG 2006 and E2VP confirmed that, despite development of alternative and new methods, the FD time-domain method has an important and, without hesitation and exaggeration, irreplaceable position and role among recent time-domain numerical-modelling methods in earthquake research.

It is important to say that the term ‘finite-difference method’(FDM) in the numerical modelling of earthquake motion may represent one out of a large number of various FD schemes and codes. The schemes may considerably differ from each other in several methodological aspects. Consequently, the schemes and the numerical results obtained by different schemes may differ considerably in accuracy and computational efficiency.

The most advanced FD schemes can be more than competitive, for many important configurations, with other modern methods: at the same level of accuracy they can be computationally more efficient. For some configurations, other methods can be more appropriate.

More than four decades of development of the FDM in application to seismic wave propagation and earthquake motion, and the present state of FD theory suggest that there is room for further improvements, and that the future will bring even more accurate, efficient and competitive schemes for geometrically and rheologically complex realistic problem configurations.

In this book we focus on the FDM as applied to modelling earthquake motion and earthquake ground motion prediction. Obviously, the included material also reflects our contributions to the methodology of FD modelling. Due to the chosen focus and limited extent, we do not cover all aspects of FD modelling. At the same time, we believe that the book brings material that will be found useful by those who are not familiar with the method (students, professionals, researchers) and also those who develop and apply numerical modelling in their earthquake research or investigations of elastic wave propagation in complex media (e.g., oil exploration, shallow geophysics, machine-induced vibrations).

Part I

Mathematical-physical model

2

Basic mathematical-physical model

In this chapter we briefly present the basics of the mathematical-physical model necessary for the explanations and elaborations in the following chapters. For more detailed expositions of the theory of earthquakes, seismic wave propagation and earthquake ground motion we refer to some of the recent monographs and fundamental textbooks. For a general introduction to seismology – Aki and Richards (2002), Pujol (2003), Shearer (2009), Stein and Wysession (2003), Lay and Wallace (1995), Kennett (2001), Beroza and Kanamori (2009), Dziewonski and Romanowicz (2009); for earthquake sources – Kostrov and Das (2005), Scholz (2002), Ohnaka (2013); for theory of seismic wave propagation – Ben-Menahem and Singh (2000) and Carcione (2007); for global seismic wave propagation – Dahlen and Tromp (1998); for full waveform modelling and inversion – Fichtner (2011); for geotechnical earthquake engineering – Kramer (1996); and for waves and vibrations in soils caused by earthquakes, traffic, shocks and construction works – Semblat and Pecker (2009).

2.1 Medium

In order to reasonably numerically simulate seismic wave propagation and earthquake motion in the Earth we need an adequate model of the medium inside a target domain (volume) of the Earth. We should clearly distinguish geological models, physical models and discrete (or grid) models.

In general, a physical model of a medium is described by 3D distributions of all material parameters that determine seismic wave propagation and earthquake motion. Being focused on seismic and earthquake motion in near-surface local structures, in most cases the real material can be modelled as a heterogeneous linear viscoelastic isotropic continuum. Models of the medium may comprise both spatially smooth and discontinuous variations of material parameters. The model has to properly account for attenuation due to anelasticity of the Earth's real material. A perfectly elastic medium or oversimplified description of attenuation is not sufficient. A reasonable rheological viscoelastic model is necessary in order to account for the realistic dependence of attenuation on frequency and its spatial variations.

So far, the least addressed aspect in numerical modelling of earthquake motion in near-surface local structures is the (an)isotropy of the real material. We know, in general, that there are true isotropic materials and true anisotropic materials. The question is, what can be seen in seismic records? For example, anisotropy of the Earth's upper mantle is clearly observed in seismic records, and numerical modelling of seismic waves at the regional and global scales has to assume inherent physical anisotropy. We are not in such a situation in numerical modelling of earthquake motion in near-surface local structures.

Although the real medium and its physical model may consist of truly isotropic materials, the mathematical-physical and grid representations of wave propagation in such a medium may be anisotropic. We may speak, for instance, of an equivalent anisotropic medium in the case of a low-frequency approximation (wavelengths much larger than the characteristic size of heterogeneity) for wave propagation in heterogeneous isotropic media (Backus 1962, Helbig 1984).

The usual physical model of the medium is specified by 3D spatial distributions of the P-wave and S-wave speeds (V_P or α, and V_S or β, respectively) at some frequency, density (ρ), and P-wave and S-wave quality factors as functions of frequency ($Q_P(\omega)$ and $Q_S(\omega)$, respectively).

Soft sediments near the free surface may behave in a nonlinear fashion. The stress–strain relation is not linear but nonlinear hysteretic. In the simplest (but still tremendously demanding) case the medium has to be represented by a rheological elastoplastic model. This poses a major complication for 3D modelling of earthquake motion. At present, reasonable 3D numerical modelling with possibly nonlinear behaviour of part of the whole model is still a challenge for numerical modellers.

Plastic deformation in the close vicinity of a rupturing fault is another example of nonlinear behaviour that is not trivial to model numerically.

2.2 Governing equation: equation of motion

Consider a material volume V of continuum with surface S. Material parameters are continuous functions of spatial coordinates inside V. Consider an arbitrary volume Ω with surface S^{Ω} inside volume V. Let $\vec{n}^{\Omega}$ be a normal vector to surface S^{Ω} pointing from the interior of volume Ω outward. Let $\vec{f}(x_k, t)$ be the density of the body force acting in volume Ω and $\vec{T}^{\Omega}(x_k, t)$ the traction acting at surface S^{Ω}. Here $x_k; k \in \{1, 2, 3\}$ are Cartesian coordinates and t is time. The configuration is shown in Fig. 2.1. Let $\vec{u}\,(u_1, u_2, u_3)$ or, in an alternative notation, $\vec{u}\,\left(u_x, u_y, u_z\right)$, be the displacement vector. Let ε_{ij} be the strain tensor,

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right); \quad i, j \in \{1, 2, 3\} \tag{2.1}$$

and σ_{ij} the stress tensor. We briefly introduce the basic forms of the equations of motion for the considered configuration.

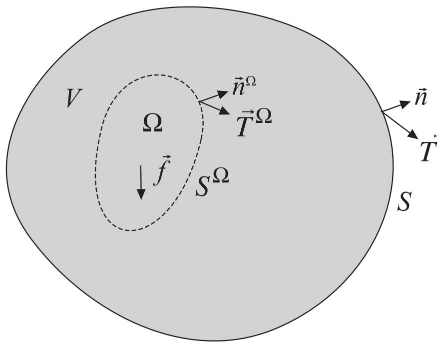

Figure 2.1 Material volume V of a smooth continuum bounded by surface S. External traction $\vec{T}$ acts at surface S, body force $\vec{f}$ acts in volume V. Volume Ω with surface S^{Ω} is a testing volume considered in the derivation of the equation of motion.

In the following formulations, the traction vector appears explicitly, which means the possible imposition of the Neumann boundary condition on a surface. The possible application of the Dirichlet boundary condition (prescribed displacement) does not explicitly appear in the formulations.

2.2.1 Strong form

An application of Newton's second law to volume Ω gives

$$\frac{d}{dt}\int_{\Omega}\rho\frac{\partial u_i}{\partial t}dV = \int_{S^{\Omega}} T_i^{\Omega} dS + \int_{\Omega} f_i dV \tag{2.2}$$

Throughout the text dV and dS will be used for volume and surface elements, respectively. Because Ω and S^{Ω} move with particles, the particle mass ρdV does not change with time. The equation can be written as

$$\int_{\Omega}\rho\frac{\partial^2 u_i}{\partial t^2}dV = \int_{S^{\Omega}} T_i^{\Omega} dS + \int_{\Omega} f_i dV \tag{2.3}$$

At surface S^{Ω}, traction T_i^{Ω} is related to the stress tensor σ_{ij}:

$$T_i^{\Omega} = \sigma_{ij} n_j^{\Omega} \tag{2.4}$$

In Eq. (2.4) and hereafter we assume the Einstein summation convention for repeated indices. Assuming continuity of the stress tensor throughout volume Ω, Gauss's divergence theorem can be applied to the surface integral:

$$\int_{S^{\Omega}} T_i^{\Omega} dS = \int_{S^{\Omega}} \sigma_{ij} n_j^{\Omega} dS = \int_{\Omega} \frac{\partial \sigma_{ij}}{\partial x_j} dV \tag{2.5}$$

Equation (2.3) can be then written as

$$\int_{\Omega}\left(\rho\frac{\partial^2 u_i}{\partial t^2}-\frac{\partial\sigma_{ij}}{\partial x_j}-f_i\right)dV=0 \tag{2.6}$$

Equation (2.6) is valid for any volume Ω inside V. Assume that the integrand is greater than 0 at some point inside V. Because the integrand is continuous throughout V, it is possible to find such a volume Ω (containing that point) for which the integrand is greater than 0. This, however, would be in contradiction with Eq. (2.6). Consequently,

$$\rho\frac{\partial^2 u_i}{\partial t^2}-\frac{\partial\sigma_{ij}}{\partial x_j}-f_i=0 \tag{2.7}$$

everywhere in V. Equation (2.7) together with the boundary condition at surface S,

$$T_i=\sigma_{ij}n_j \tag{2.8}$$

represent a strong formulation for the considered problem. The formulation requires continuity of displacement and its first spatial and temporal derivatives.

2.2.2 *Weak form*

Alternatively to the application of Newton's second law to the material volume V we can apply the principle of virtual work. Consider a fixed state of continuum at some time and its virtual (arbitrary, infinitesimal) deformation. Let δu_i be the corresponding virtual displacement. Then the virtual deformation is characterized by the virtual strain tensor $\delta\varepsilon_{ij}$:

$$\delta\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial}{\partial x_j}\delta u_i+\frac{\partial}{\partial x_i}\delta u_j\right) \tag{2.9}$$

Because the virtual displacements are assumed in a fixed state of continuum, they do not affect displacements and accelerations of continuum particles in this state. The principle states that during virtual deformation the work done by external forces has to be equal to the sum of the increment of energy of deformation and the work of inertial forces:

$$\int_S T_i\delta u_i dS+\int_V f_i\delta u_i dV=\int_V \sigma_{ij}\delta\varepsilon_{ij}dV+\int_V \rho\frac{\partial^2 u_i}{\partial t^2}\delta u_i dV \tag{2.10}$$

Functions δu_i are arbitrary; they are equivalent to weight functions. Therefore, we replace δu_i by w_i in Eqs. (2.9) and (2.10). Then, due to symmetry of the stress tensor,

$$\sigma_{ij}\delta\varepsilon_{ij}=\frac{1}{2}\left(\sigma_{ij}\frac{\partial w_i}{\partial x_j}+\sigma_{ij}\frac{\partial w_j}{\partial x_i}\right)=\sigma_{ij}\frac{\partial w_i}{\partial x_j} \tag{2.11}$$

Equation (2.10) can be written as

$$\int_V\left(\rho\frac{\partial^2 u_i}{\partial t^2}-f_i\right)w_i dV+\int_V \sigma_{ij}\frac{\partial w_i}{\partial x_j}dV=\int_S T_i w_i dS \tag{2.12}$$

Equation (2.12) is called the weak form of the equation of motion. This is because the requirement of continuity of displacement and its first spatial derivatives in the strong form is replaced here by a weaker requirement of continuity of displacement and the weight functions.

2.2.3 Integral strong form

Integration by parts of the last term on the left hand side (l.h.s.) of Eq. (2.12) yields

$$\int_V \left(\rho \frac{\partial^2 u_i}{\partial t^2} - f_i \right) w_i dV + \int_V \frac{\partial}{\partial x_j} \left(\sigma_{ij} w_i \right) dV - \int_V \frac{\partial \sigma_{ij}}{\partial x_j} w_i dV = \int_S T_i w_i dS \qquad (2.13)$$

and, using Gauss's divergence theorem,

$$\int_V \left(\rho \frac{\partial^2 u_i}{\partial t^2} - f_i \right) w_i dV + \int_S \sigma_{ij} n_j w_i dS - \int_V \frac{\partial \sigma_{ij}}{\partial x_j} w_i dV = \int_S T_i w_i dS \qquad (2.14)$$

Assembling the volume and surface integrals together gives

$$\int_V \left(\rho \frac{\partial^2 u_i}{\partial t^2} - \frac{\partial \sigma_{ij}}{\partial x_j} - f_i \right) w_i dV = \int_S \left(T_i - \sigma_{ij} n_j \right) w_i dS \qquad (2.15)$$

In Eq. (2.15) we can specify the boundary condition for traction at surface S by specifying values of T_i. We can call Eq. (2.15) the integral strong form of the equation of motion (we adopted this term based on our personal communication with Robert J. Geller). While being integral, the form requires continuity of the first derivative of displacement. These two features clearly distinguish it from the (differential) strong form and the integral weak form.

2.2.4 Concluding remark

In principle, any of the three forms can be the basis for discretization aiming in an FD scheme. Most of the developed FD schemes are based on the differential strong form – likely due to its apparent relative simplicity. Depending on the problem configuration, one of the two other forms may be found more suitable. The weak form is the basis for the traditional FEM, the more recent spectral-element method and the discontinuous Galerkin method. These methods will be briefly characterized in Chapter 5.

2.3 Constitutive law: stress–strain relation

In order to solve the equation of motion we need a constitutive law that specifies the relation between the stress and strain tensors, and consequently also the relation between the stress tensor and displacement vector. We will consider three types of continuum – linear elastic, linear viscoelastic and nonlinear elastoplastic. The linear elastic continuum is the simplest type of continuum. It is useful for a simple introduction of many important concepts and approaches but is incapable of accounting for attenuation of seismic waves