

## Spectral Analysis for Univariate Time Series

Spectral analysis is widely used to interpret time series collected in diverse areas such as the environmental, engineering and physical sciences. This book covers the statistical theory behind spectral analysis and provides data analysts with the tools needed to transition theory into practical applications. Actual time series from oceanography, metrology, atmospheric science and other areas are used in running examples throughout, to allow clear comparison of how the various methods address questions of interest.

All major nonparametric and parametric spectral analysis techniques are discussed, with particular emphasis on the multitaper method, both in its original formulation involving Slepian tapers and in a popular alternative using sinusoidal tapers. The authors take a unified approach to quantifying the bandwidth of nonparametric spectral estimates, allowing for meaningful comparison among different estimates. An extensive set of exercises allows readers to test their understanding of both the theory and practical analysis of time series. The time series used as examples and R language code for recreating the analyses of the series are available from the book's web site.

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# Spectral Analysis for Univariate Time Series

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## Preface

Spectral analysis is one of the most widely used methods for interpreting time series and has been used in diverse areas including – but not limited to – the engineering, physical and environmental sciences. This book aims to help data analysts in applying spectral analysis to actual time series. Successful application of spectral analysis requires both an understanding of its underlying statistical theory and the ability to transition this theory into practice. To this end, we discuss the statistical theory behind all major nonparametric and parametric spectral analysis techniques, with particular emphasis on the multitaper method, both in its original formulation in Thomson (1982) involving Slepian tapers and in a popular alternative involving the sinusoidal tapers advocated in Riedel and Sidorenko (1995). We then use actual time series from oceanography, metrology, atmospheric science and other areas to provide analysts with examples of how to move from theory to practice.

This book builds upon our 1993 book *Spectral Analysis for Physical Applications: Multitaper and Conventional Univariate Techniques* (also published by Cambridge University Press). The motivations for considerably expanding upon this earlier work include the following.

- [1] A quarter century of teaching classes based on the 1993 book has given us new insights into how best to introduce spectral analysis to analysts. In particular we have greatly expanded our treatment of the multitaper method. While this method was a main focus in 1993, we now describe it in a context that more readily allows comparison with one of its main competitors (Welch’s overlapped segment averaging).
- [2] The core material on nonparametric spectral estimation is in Chapters 6 (“Periodogram and Other Direct Spectral Estimators”), 7 (“Lag Window Spectral Estimators”) and 8 (“Combining Direct Spectral Estimators”). These chapters now present these estimators in a manner that allows easier comparison of common underlying concepts such as smoothing, bandwidth and windowing.
- [3] There have been significant theoretical advances in spectral analysis since 1993, some of which are of particular importance for data analysts to know about. One that we have already mentioned is a new family of multitapers (the sinusoidal tapers) that was introduced in the mid-1990s and that has much to recommend its use. Another is a new bandwidth measure that allows nonparametric spectral analysis methods to be meaningfully compared. A third is bandwidth selection for smoothing periodograms.

- [4] An important topic that was not discussed in the 1993 book is computer-based simulation of time series. We devote Chapter 11 to this topic, with particular emphasis on simulating series whose statistical properties agree with those dictated by nonparametric and parametric spectral analyses of actual time series.
- [5] We used software written in Common Lisp to carry out spectral analysis for all the time series used in our 1993 book. Here we have used the popular and freely available R software package to do all the data analysis and to create the content for almost all the figures and tables in the book. We do *not* discuss this software explicitly in the book, but we make it available as a supplement so that data analysts can replicate and build upon our use of spectral analysis. The website for the book gives access to the R software and to information about software in other languages – see “Data, Software and Ancillary Material” on page xx for details.

Finally, a key motivation for us to undertake an expansion has been the gratifying response to the 1993 book from its intended audience.

The following features of this book are worth noting.

- [1] We provide a large number of exercises (over 300 in all), some of which are embedded within the chapters, and others, at the ends of the chapters. The embedded exercises challenge readers to verify certain theoretical results in the main text, with solutions in an Appendix that is available on the website for the book (see page xx). The exercises at the end of the chapters are suitable for use in a classroom setting (solutions are available only for instructors). These exercises both expand upon the theory presented and delve into the practical considerations behind spectral analysis.
- [2] We use actual time series to illustrate various spectral analysis methods. We do so to encourage data analysts to carefully consider the link between spectral analysis and what questions this technique can address about particular series. In some instances we use the same series with different techniques to allow analysts to compare how well various methods address questions of interest.
- [3] We provide a large number of “Comments and Extensions” (C&Es) to the main material. These C&Es appear at the ends of sections when appropriate and provide interesting supplements to the main material; however, readers can skip the C&Es without compromising their ability to follow the main material later on (we have set the C&Es in a slightly smaller font to help differentiate them from the main material). The C&Es cover a variety of ancillary – but valuable – topics such as the Lomb–Scargle periodogram, jackknifing of multitaper spectral estimates, the method of surrogate time series, a periodogram based upon the discrete cosine transform (and its connection to Albert Einstein!) and the degree to which windows designed for one purpose can be used for another.
- [4] At the end of most chapters, we provide a comprehensive summary of that chapter. The summaries allow readers to check their understanding of the main points in a chapter and to review the content of a previous chapter when tackling a later chapter. The comprehensive subject index at the end of the book will aid in finding details of interest.

We also note that “univariate” is part of the title of the book because a volume on multivariate spectral analysis is in progress.

Books do not arise in isolation, and ours is no exception. With a book that is twenty-five years in the making, the list of editors, colleagues, students, readers of our 1993 book, friends and relatives who have influenced this book in some manner is so long that thanking a select few individuals here will only be at the price of feeling guilty both now and later on about not thanking many others. Those who are on this list know who you are. We propose to thank you with a free libation of your choice upon our first post-publication meeting

*Preface*

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(wherever this might happen – near Seattle or London or both or elsewhere!). We do, however, want to explicitly acknowledge financial support through EPSRC Mathematics Platform grant EP/I019111/1. We also thank Stan Murphy (posthumously), Bob Spindel and Jeff Simmen (three generations of directors of the Applied Physics Laboratory, University of Washington) for supplying ongoing discretionary funding without which this and the 1993 book would not exist.

Finally, despite our desire for a book needing no errata list, past experience says this will not happen. Readers are encouraged to contact us about blemishes in the book so that we can make others aware of them (our email addresses are listed with our signatures).

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## Conventions and Notation

- *Important conventions*

(14)	refers to the single displayed equation on page 14
(3a), (3b)	refers to different displayed equations on page 3
Figure 2	refers to the figure on page 2
Table 214	refers to the table on page 214
Exercise [8]	refers to the embedded exercise on page 8 (see the Appendix on the website for an answer)
Exercise [1.3]	refers to the third exercise at the end of Chapter 1
$a$ , $\mathbf{a}$ and $\mathbf{A}$	refer to a scalar, a vector and a matrix/vector
$S(\cdot)$	refers to a function
$S(f)$	refers to the value of the function $S(\cdot)$ at $f$
$\{h_t\}$	refers to a sequence of values indexed by $t$
$h_t$	refers to a single value of a sequence
$\alpha$ and $\hat{\alpha}$	refer to a parameter and an estimator thereof

In the following lists, the numbers at the end of the brief descriptions are page numbers where more information about – or an example of the use of – an abbreviation or symbol can be found.

- *Abbreviations used frequently*

ACF	autocorrelation function	27
ACLS	approximate conditional least squares	549
ACS	autocorrelation sequence	27
ACVF	autocovariance function	27
ACVS	autocovariance sequence	27
AIC	Akaike's information criterion	494
AICC	AIC corrected for bias	495, 576



*Conventions and Notation*

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AR( $p$ )	$p$ th-order autoregressive process	33, 446
ARMA( $p, q$ )	autoregressive moving average process of order ( $p, q$ )	35
BLS	backward least squares	477
C&Es	Comments and Extensions	24
CI	confidence interval	204
CPDF	cumulative probability distribution function	23
dB	decibels, i.e., $10 \log_{10}(\cdot)$	13
DCT-II	discrete cosine transform of type II	184, 217
DFT	discrete Fourier transform	74, 92
DPSS	discrete prolate spheroidal sequence (Slepian sequence)	87, 155
DPSWF	discrete prolate spheroidal wave function	87
ECLS	exact conditional least squares	549
EDOFs	equivalent degrees of freedom	264
EULS	exact unconditional least squares	549
FBLs	forward/backward least squares	477, 555
FFT	fast Fourier transform	92, 94
FIR	finite impulse response	147
FLS	forward least squares	476
FPE	final prediction error	493
GCV	generalized cross-validated	309
GSSM	Gaussian spectral synthesis method	605
Hz	Hertz: 1 Hz = 1 cycle per second	
IID	independent and identically distributed	31
IIR	infinite impulse response	147
LS	least squares	475
LTI	linear time-invariant	132
MA( $q$ )	$q$ th-order moving average process	32
MLE	maximum likelihood estimator or estimate	480
MSE	mean square error	167
MSLE	mean square log error	296
NMSE	normalized mean square error	296
OLS	ordinary least squares	409
PACS	partial autocorrelation sequence	462
PDF	probability density function	24
PSWF	prolate spheroidal wave function	64
RV	random variable	3
SDF	spectral density function	111
SS	sum of squares	467–8, 476–7
SVD	singular value decomposition	565
WOSA	Welch's (or weighted) overlapped segment averaging	414
ZMNL	zero-memory nonlinearity	634

• *Non-Greek notation used frequently*

$A_l$	real-valued amplitude associated with $\cos(2\pi f_l t \Delta_t)$ . . . . .	35, 515
$b(\cdot)$	bias . . . . .	192, 239, 378
$b^{(B)}(\cdot)$	broad-band bias . . . . .	378
$b^{(L)}(\cdot)$	local bias . . . . .	378
$b_k(f)$	weight associated with $k$ th eigenspectrum at frequency $f$ . . . . .	386
$b_W(\cdot)$	bias due to smoothing window only . . . . .	256
$B_l$	real-valued amplitude associated with $\sin(2\pi f_l t \Delta_t)$ . . . . .	35, 515
$B_{\mathcal{H}}$	bandwidth of spectral window $\mathcal{H}$ . . . . .	194
$B_S$	spectral bandwidth . . . . .	292, 297
$B_T$	bandwidth measure for $\{X_t\}$ with dominantly unimodal SDF . . . . .	300
$\tilde{B}_T$	approximately unbiased estimator of $B_T$ . . . . .	300
$B_U$	bandwidth of spectral window $\mathcal{U}_m$ . . . . .	256
$B_W$	Jenkins measure of smoothing window bandwidth . . . . .	251
$\{c_\tau\}$	inverse Fourier transform of $C(\cdot)$ (cepstrum if properly scaled) . . . . .	301
$C(\cdot)$	log spectral density function . . . . .	301
$\hat{C}^{(D)}(\cdot)$	log of direct spectral estimator . . . . .	301
$\hat{C}_m^{(LW)}(\cdot)$	smoothed log of direct spectral estimator . . . . .	303
$C_h$	variance inflation factor due to tapering . . . . .	259, 262
$C_l$	complex-valued amplitude associated with $\exp(i2\pi f_l t \Delta_t)$ . . . . .	108, 519
$d(\cdot, \cdot)$	Kullback–Leibler discrepancy measure (general case) . . . . .	297
$dZ(\cdot)$	orthogonal increment . . . . .	109
$D_l$	real-valued amplitude associated with $\cos(2\pi f_l t \Delta_t + \phi_l)$ or with $\exp(i[2\pi f_l t \Delta_t + \phi_l])$ . . . . .	35, 511, 517
$D_N$	$N \times N$ diagonal matrix . . . . .	375
$\mathcal{D}_N(\cdot)$	Dirichlet’s kernel . . . . .	17
$\vec{e}_t(k)$	observed forward prediction error . . . . .	467
$\overleftarrow{e}_t(k)$	observed backward prediction error . . . . .	467
$f_k$	$k/(N \Delta_t)$ , member of grid of Fourier frequencies . . . . .	171, 515
$f'_k$	$k/(N' \Delta_t)$ , member of arbitrary grid of frequencies . . . . .	171
$\tilde{f}_k$	$k/(2N \Delta_t)$ , member of grid twice as fine as Fourier frequencies . . . . .	181
$f_l$	frequency of a sinusoid . . . . .	35, 511
$f_N$	$1/(2 \Delta_t)$ , Nyquist frequency . . . . .	82, 122, 512
$F_t(\cdot)$	cumulative probability distribution function . . . . .	23
$\mathcal{F}(\cdot)$	Fejér’s kernel . . . . .	174, 236
$g$	Fisher’s test statistic for simple periodicity . . . . .	539
$g_F$	critical value for Fisher’s test statistic $g$ . . . . .	540
$g(\cdot)$	real- or complex-valued function . . . . .	53
$g_p(\cdot)$	periodic function . . . . .	48
$\{g_u\}$	impulse response sequence of a digital filter . . . . .	143
$g \star g^*(\cdot)$	autocorrelation of deterministic function $g(\cdot)$ . . . . .	72
$g \star h^*(\cdot)$	cross-correlation of deterministic functions $g(\cdot)$ and $h(\cdot)$ . . . . .	72
$\{g \star h_t\}$	convolution of sequences $\{g_t\}$ and $\{h_t\}$ . . . . .	99

## Conventions and Notation

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$g * h(\cdot)$	convolution of functions $g(\cdot)$ and $h(\cdot)$ . . . . .	67
$G(\cdot)$	Fourier transform of $g(\cdot)$ or transfer function . . . . .	54, 97, 136, 141
$G_p(\cdot)$	Fourier transform of $\{g_t\}$ . . . . .	74, 99
$\{G_n\}$	Fourier transform of $g_p(\cdot)$ . . . . .	49, 96
$\{G_t\}$	stationary <i>Gaussian</i> process . . . . .	201, 445
$\{h_t\}$	data taper . . . . .	186
$\{h_{k,t}\}$	$k$ th-order data taper for multitaper estimator . . . . .	352, 357, 392
$H(\cdot)$	Fourier transform of data taper $\{h_t\}$ . . . . .	186
$\{H_t\}$	Gaussian autoregressive process . . . . .	445
$\mathcal{H}(\cdot)$	spectral window of direct spectral estimator . . . . .	186
$\mathcal{H}_k(\cdot)$	spectral window of $k$ th eigenspectrum . . . . .	352
$\overline{\mathcal{H}}(\cdot)$	spectral window of basic multitaper estimator . . . . .	353
$\tilde{\mathcal{H}}(\cdot)$	spectral window of weighted multitaper estimator . . . . .	353
$\mathcal{HT}\{\cdot\}$	Hilbert transform . . . . .	114, 562, 579
$J(\cdot)$	scaled Fourier transform of tapered time series . . . . .	186, 544
$K_{\max}$	maximum number of usable Slepian multitapers . . . . .	357
$\text{KL}(\cdot)$	Kullback–Leibler discrepancy measure (special case) . . . . .	297
$L\{x(\cdot)\}$	continuous parameter filter acting on function $x(\cdot)$ . . . . .	133
$L\{x_t\}$	discrete parameter filter acting on sequence $\{x_t\}$ . . . . .	141
$L_N$	$N \times N$ lower triangular matrix with 1's on diagonal . . . . .	464
$m$	parameter controlling smoothing in lag window estimator . . . . .	247
$N$	sample size . . . . .	2, 163
$N'$	integer typically greater than or equal to $N$ . . . . .	179, 237
$N_B$	number of blocks in WOSA . . . . .	414
$N_S$	block size in WOSA . . . . .	414
$p$	order of an autoregressive process or a proportion . . . . .	33, 189, 204
$\mathbf{P}[A]$	probability that the event $A$ will occur . . . . .	23
$\mathcal{P}_k$	normalized cumulative periodogram . . . . .	215
$q$	order of a moving average process . . . . .	32, 503
$\mathbf{Q}$	weight matrix in quadratic spectral estimator . . . . .	374
$Q_\nu(p)$	$p \times 100\%$ percentage point of $\chi_\nu^2$ distribution . . . . .	265
$\{r_\tau\}$	inverse Fourier transform of $R(\cdot)$ . . . . .	212
$R$	signal-to-noise ratio . . . . .	526, 532
$R(\eta)$	correlation of direct spectral estimators at $f$ and $f + \eta$ . . . . .	212
$\{R_t\}$	residual process . . . . .	550
$\{s_\tau\}$	autocovariance sequence (ACVS) . . . . .	27, 29
$\{s_\tau^{(\text{BL})}\}$	ACVS for band-limited white noise . . . . .	379
$\{\hat{s}_\tau^{(\text{D})}\}$	ACVS estimator, inverse Fourier transform of $\{\hat{S}^{(\text{D})}(\cdot)\}$ . . . . .	188
$\{\hat{s}_\tau^{(\text{P})}\}$	“biased” estimator of ACVS . . . . .	166
$\{\hat{s}_\tau^{(\text{U})}\}$	“unbiased” estimator of ACVS . . . . .	166
$s(\cdot)$	autocovariance function (ACVF) . . . . .	27
$S(\cdot)$	spectral density function (SDF) . . . . .	111
$S_\eta(\cdot)$	SDF of (possibly) colored noise . . . . .	519

$S^{(I)}(\cdot)$	integrated spectrum or spectral distribution function	110
$S^{(BL)}(\cdot)$	SDF of band-limited white noise	379
$\hat{S}^{(AMT)}(\cdot)$	adaptive multitaper spectral estimator	389
$\hat{S}^{(D)}(\cdot)$	direct spectral estimator	186
$\hat{S}^{(DCT)}(\cdot)$	DCT-based periodogram	217
$\hat{S}^{(DS)}(\cdot)$	discretely smoothed direct spectral estimator	246
$\hat{S}_m^{(DSP)}(\cdot)$	discretely smoothed periodogram	307
$\hat{S}_m^{(LW)}(\cdot)$	lag window spectral estimator	247
$\hat{S}^{(MT)}(\cdot)$	basic multitaper spectral estimator	352
$\hat{S}_k^{(MT)}(\cdot)$	$k$ th eigenspectrum for multitaper estimator	352
$\hat{S}^{(P)}(\cdot)$	periodogram (special case of $\hat{S}^{(D)}(\cdot)$ )	170, 188
$\tilde{S}^{(P)}(\cdot)$	periodogram of shifted time series or rescaled periodogram	184, 222, 240
$\hat{S}^{(PC)}(\cdot)$	postcolored spectral estimator	198
$\hat{S}^{(Q)}(\cdot)$	quadratic spectral estimator	374
$\hat{S}^{(WMT)}(\cdot)$	weighted multitaper spectral estimator	352
$\hat{S}^{(WOSA)}(\cdot)$	WOSA spectral estimator	414
$\hat{S}^{(YW)}(\cdot)$	Yule–Walker spectral estimator	451
$t$	actual time (continuous) or a unitless index (discrete)	22, 74
$t_\lambda$	critical value for Siegel’s test statistic	541
$T_\lambda$	Siegel’s test statistic	541
$U_k(\cdot; N, W)$	discrete prolate spheroidal wave function of order $k$	87
$U_m(\cdot)$	spectral window of $\hat{S}^{(LW)}(\cdot)$	255
$\mathbf{v}_k(N, W)$	vector with portion of DPSS, order $k$	87
$\{v_{m,\tau}\}$	nontruncated version of lag window $\{w_{m,\tau}\}$	247
$V_m(\cdot)$	design window (Fourier transform of $\{v_{m,\tau}\}$ )	247
$\{w_{m,\tau}\}$	lag window (truncated version of $\{v_{m,\tau}\}$ )	247
$\text{width}_a\{\cdot\}$	autocorrelation width	73
$\text{width}_e\{\cdot\}$	equivalent width	58
$\text{width}_{hp}\{\cdot\}$	half-power width	192
$\text{width}_v\{\cdot\}$	variance width	60, 192
$W$	DPSS half-bandwidth, regularization half-bandwidth	65, 377
$W_m(\cdot)$	smoothing window	247–8
$x_0, \dots, x_{N-1}$	time series realization or deterministic series	2
$X_0, \dots, X_{N-1}$	sequence of random variables	3
$\{X_t\}$	real-valued discrete parameter stochastic process	22
$\{X(t)\}$	real-valued continuous parameter stochastic process	22
$\{X_{j,t}\}$	$j$ th real-valued discrete parameter stochastic process	23
$\{X_j(t)\}$	$j$ th real-valued continuous parameter stochastic process	23
$\bar{X}$	sample mean (arithmetic average) of $X_0, \dots, X_{N-1}$	164
$\bar{X}_t(k)$	best (forward) linear predictor of $X_t$ given $X_{t-1}, \dots, X_{t-k}$	452
$\bar{X}_t^{\leftarrow}(k)$	best (backward) linear predictor of $X_t$ given $X_{t+1}, \dots, X_{t+k}$	455
$\{Y_t\}$	real-valued discrete parameter stochastic process (refers to an AR process in Chapter 9)	23, 445

$\{Z_t\}$	complex-valued discrete parameter stochastic process	23, 29
$\{Z(t)\}$	complex-valued continuous parameter stochastic process	23
$\{Z(f)\}$	orthogonal process	109
• <i>Greek notation used frequently</i>		
$\alpha$	intercept term in linear model, scalar, level of significance or exponent of power law	39, 43, 215, 327
$\alpha^2(N)$	fraction of sequence's energy lying in $0, \dots, N - 1$	85
$\alpha^2(T)$	fraction of function's energy lying in $[-T/2, T/2]$	62
$\beta$	slope term in linear model	39
$\beta^{(B)}\{\cdot\}$	indicator of broad-band bias in $\hat{S}^{(Q)}(\cdot)$	379
$\beta^{(L)}\{\cdot\}$	indicator of magnitude of local bias in $\hat{S}^{(Q)}(\cdot)$	378
$\beta_W$	Grenander's measure of smoothing window bandwidth	251
$\beta^2(W)$	fraction of function's energy lying in $[-W, W]$	62, 85
$\beta_{\mathcal{H}}^2$	indicator of bias in $\hat{S}^{(D)}(\cdot)$	391
$\gamma$	quadratic term in linear model or Euler's constant	46, 210
$\Gamma(\cdot)$	gamma function	440
$\Gamma$	covariance matrix (typically for AR( $p$ ) process)	450, 464
$\Delta_f$	spacing in frequency	206
$\Delta_t$	spacing in time (sampling interval)	74, 81–2, 122
$\{\epsilon_t\}$	white noise or innovation process	32, 446
$\vec{\epsilon}_t(k)$	forward prediction error: $X_t - \vec{X}_t(k)$	453
$\overleftarrow{\epsilon}_t(k)$	backward prediction error: $X_t - \overleftarrow{X}_t(k)$	455
$\eta$	equivalent degrees of freedom of a time series	298
$\{\eta_t\}$	zero mean stationary noise process (not necessarily white)	518
$\theta(\cdot)$	phase function corresponding to transfer function $G(\cdot)$	136
$\theta$	coefficient of an MA(1) process	43
$\theta_{q,1}, \dots, \theta_{q,q}$	coefficients of an MA( $q$ ) process	32
$\vartheta_{q,0}, \dots, \vartheta_{q,q}$	coefficients of an MA( $q$ ) process ( $\vartheta_{q,0} = 1$ and $\vartheta_{q,j} = -\theta_{q,j}$ )	594
$\lambda$	constant to define different logarithmic scales	301
$\lambda_k(c)$	eigenvalue associated with PSWF, order $k$	64
$\lambda_k(N, W)$	eigenvalue associated with DPSWF, order $k$	86
$\mu$	expected value of a stationary process	27
$\nu$	degrees of freedom associated with RV $\chi_\nu^2$	202, 264
$\{\rho_\tau\}$	autocorrelation sequence (ACS)	27
$\rho(\cdot)$	autocorrelation function (ACF)	27
$\sigma^2$	variance	27
$\sigma_\epsilon^2$	white noise variance or innovation variance	32, 404
$\sigma_\eta^2$	variance of noise process $\{\eta_t\}$	519
$\sigma_k^2$	mean square linear prediction error for $\vec{X}_t(k)$ or $\overleftarrow{X}_t(k)$	453, 455
$\sigma_p^2$	innovation variance for an AR( $p$ ) process	446
$\hat{\sigma}_p^2$	estimator of $\sigma_p^2$	485
$\bar{\sigma}_p^2$	Burg estimator of $\sigma_p^2$	468

$\hat{\sigma}_p^2$	Yule–Walker estimator of $\sigma_p^2$ .....	451, 458
$\Sigma$	covariance matrix .....	28
$\tau$	lag value .....	27
$\phi_l$	phase of a sinusoid .....	35, 511
$\phi$	coefficient of an AR(1) process .....	44
$\phi_{p,1}, \dots, \phi_{p,p}$	coefficients of an AR( $p$ ) process .....	33, 446
$\hat{\phi}_{p,1}, \dots, \hat{\phi}_{p,p}$	estimators of AR( $p$ ) coefficients .....	485
$\bar{\phi}_{p,1}, \dots, \bar{\phi}_{p,p}$	Burg estimators of AR( $p$ ) coefficients .....	466, 505
$\tilde{\phi}_{p,1}, \dots, \tilde{\phi}_{p,p}$	Yule–Walker estimators of AR( $p$ ) coefficients .....	451, 458, 505
$\varphi_{2p,1}, \dots, \varphi_{2p,2p}$	coefficients of a pseudo-AR( $2p$ ) process .....	553
$\Phi_p, \hat{\Phi}_p, \tilde{\Phi}_p$	$[\phi_{p,1}, \dots, \phi_{p,p}]^T, [\hat{\phi}_{p,1}, \dots, \hat{\phi}_{p,p}]^T, [\tilde{\phi}_{p,1}, \dots, \tilde{\phi}_{p,p}]^T$ .....	450, 485, 451
$\Phi^{-1}(p)$	$p \times 100\%$ percentage point of standard Gaussian distribution .....	265
$\chi_\nu^2$	chi-square RV with $\nu$ degrees of freedom .....	37, 202
$\psi(\cdot)$	digamma function .....	210, 296
$\psi'(\cdot)$	trigamma function .....	296
$\psi_k(\cdot; c)$	prolate spheroidal wave function (PSWF), order $k$ .....	64
$\omega$	angular frequency .....	8, 119

• *Standard mathematical symbols*

e	base for natural logarithm (2.718282...) .....	17
i	$\sqrt{-1}$ .....	17
$\log(\cdot), \log_{10}(\cdot)$	log base e, log base 10	
$\approx$	approximately equal to	
$\doteq$	equal at given precision, e.g., $\pi \doteq 3.1416$	
$\stackrel{\text{def}}{=}$	equal by definition .....	23–4
$\stackrel{\text{d}}{=}$	equal in distribution .....	203
$\stackrel{\text{ms}}{=}$	equal in mean square sense .....	49
$E\{\cdot\}$	expectation operator .....	24
$\text{var}\{\cdot\}$	variance operator .....	24–5, 27
$\text{cov}\{\cdot, \cdot\}$	covariance operator .....	24–5, 27
$\text{corr}\{\cdot, \cdot\}$	correlation operator .....	27
$z^*$	complex conjugate of $z$ .....	25
$\Re(z)$	real part of complex-valued number $z$ .....	61, 551
$\Im(z)$	imaginary part of complex-valued number $z$ .....	551
$\arg(z)$	argument of complex-valued number $z$ .....	54
*	convolution operator .....	67, 96, 98–9, 101
*	cross-correlation operator .....	72, 96, 98–9, 101
$\longleftrightarrow$	Fourier transform pair relationship .....	69, 96–7, 99–100
$\mathbf{a}^T$ and $\mathbf{Q}^T$	transpose of vector $\mathbf{a}$ and of matrix $\mathbf{Q}$ .....	28, 374
$\mathbf{Z}^H$ and $\mathbf{Q}^H$	Hermitian transpose of vector $\mathbf{Z}$ and of matrix $\mathbf{Q}$ .....	374
$\mathbf{I}_N$	$N \times N$ identity matrix .....	366
$\text{tr}\{\mathbf{Q}\}$	trace of matrix $\mathbf{Q}$ .....	376

## Conventions and Notation

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$ \mathbf{I}_N $	determinant of matrix $\mathbf{I}_N$ . . . . .	480
$\mathbf{R}^\#$	generalized inverse of matrix $\mathbf{R}$ . . . . .	566
$\langle \cdot, \cdot \rangle$	inner product . . . . .	470
$\ \cdot\ ^2$	squared norm . . . . .	476
$\mathbb{R}$	set of all real-valued numbers, i.e., $\{t : -\infty < t < \infty\}$ . . . . .	22
$\mathbb{Z}$	set of all integers, i.e., $\{\dots, -2, -1, 0, 1, 2, \dots\}$ . . . . .	17
$L^2(\cdot)$	set of square integrable functions over specified domain . . . . .	54–5
$\in, \notin$	contained in, not contained in	
$\delta_n$	Kronecker delta function . . . . .	44, 65
$\delta(\cdot)$	Dirac delta function . . . . .	74, 120
$\text{sinc}(\cdot)$	sinc function, i.e., $\sin(\pi t)/(\pi t)$ . . . . .	63
$[x]$	greatest integer $\leq x$ , e.g., $[\pi] = 3$ and $[3] = 3$ . . . . .	7–8
$(a)_+$	positive part, i.e., $\max\{a, 0\}$ . . . . .	541
$\text{mod}$	modulo operator, e.g., $5 \text{ mod } 4 = 1$ . . . . .	45, 101
$O(\cdot)$	$f(x) = O(g(x))$ as $x \rightarrow 0$ if $ f(x)/g(x)  \leq C$ for constant $C$ . . . . .	175

## Data, Software and Ancillary Material

The website for this book is currently at

<http://faculty.washington.edu/dbp/sauts.html>

(alternatively go to [www.cambridge.org/9781107028142](http://www.cambridge.org/9781107028142) – this is maintained by Cambridge University Press and should have both a description of the book and a link to the current location for the book’s website). The website gives access to

- an Appendix with answers to all the exercises embedded within Chapters 1 to 11;
- almost all the time series used as examples in the book;
- software in the R language for recreating the content in the bulk of the figures and tables in each chapter (and updates on the status of software in other languages);
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