
Contents for Volume 1

	<i>Preface</i>	<i>page xvii</i>
1	Normed linear spaces and their operators	1
	1.1 Banach spaces	1
	1.2 Bounded operators	9
	1.3 Fourier series	14
	1.4 The Hahn–Banach theorem	15
	1.5 The Baire category theorem and its consequences	21
	1.6 The spectrum	26
	1.7 Hilbert space and projections	30
	1.8 The adjoint operator	40
	1.9 Tensor product and algebraic direct sum	45
	1.10 Invariant subspaces and cyclic vectors	49
	1.11 Compressions and dilations	52
	1.12 Angle between two subspaces	54
	Notes on Chapter 1	57
2	Some families of operators	60
	2.1 Finite-rank operators	60
	2.2 Compact operators	62
	2.3 Subdivisions of spectrum	65
	2.4 Self-adjoint operators	70
	2.5 Contractions	77
	2.6 Normal and unitary operators	78
	2.7 Forward and backward shift operators on ℓ^2	80
	2.8 The multiplication operator on $L^2(\mu)$	83
	2.9 Doubly infinite Toeplitz and Hankel matrices	86
	Notes on Chapter 2	92

3	Harmonic functions on the open unit disk	96
	3.1 Nontangential boundary values	96
	3.2 Angular derivatives	98
	3.3 Some well-known facts in measure theory	101
	3.4 Boundary behavior of $P\mu$	106
	3.5 Integral means of $P\mu$	110
	3.6 Boundary behavior of $Q\mu$	112
	3.7 Integral means of $Q\mu$	113
	3.8 Subharmonic functions	116
	3.9 Some applications of Green's formula	117
	Notes on Chapter 3	120
4	Hardy spaces	122
	4.1 Hyperbolic geometry	122
	4.2 Classic Hardy spaces H^p	124
	4.3 The Riesz projection P_+	130
	4.4 Kernels of P_+ and P_-	135
	4.5 Dual and predual of H^p spaces	137
	4.6 The canonical factorization	141
	4.7 The Schwarz reflection principle for H^1 functions	148
	4.8 Properties of outer functions	149
	4.9 A uniqueness theorem	154
	4.10 More on the norm in H^p	157
	Notes on Chapter 4	163
5	More function spaces	166
	5.1 The Nevanlinna class \mathcal{N}	166
	5.2 The spectrum of b	171
	5.3 The disk algebra \mathcal{A}	173
	5.4 The algebra $\mathcal{C}(\mathbb{T}) + H^\infty$	181
	5.5 Generalized Hardy spaces $H^p(\nu)$	183
	5.6 Carleson measures	187
	5.7 Equivalent norms on H^2	198
	5.8 The corona problem	202
	Notes on Chapter 5	211
6	Extreme and exposed points	214
	6.1 Extreme points	214
	6.2 Extreme points of $L^p(\mathbb{T})$	217
	6.3 Extreme points of H^p	219
	6.4 Strict convexity	224
	6.5 Exposed points of $\mathfrak{B}(\mathcal{X})$	227
	6.6 Strongly exposed points of $\mathfrak{B}(\mathcal{X})$	230
	6.7 Equivalence of rigidity and exposed points in H^1	232

6.8	Properties of rigid functions	235
6.9	Strongly exposed points of H^1	246
	Notes on Chapter 6	254
7	More advanced results in operator theory	257
7.1	The functional calculus for self-adjoint operators	257
7.2	The square root of a positive operator	260
7.3	Möbius transformations and the Julia operator	269
7.4	The Wold–Kolmogorov decomposition	274
7.5	Partial isometries and polar decomposition	275
7.6	Characterization of contractions on $\ell^2(\mathbb{Z})$	281
7.7	Densely defined operators	282
7.8	Fredholm operators	286
7.9	Essential spectrum of block-diagonal operators	291
7.10	The dilation theory	298
7.11	The abstract commutant lifting theorem	306
	Notes on Chapter 7	310
8	The shift operator	314
8.1	The bilateral forward shift operator Z_μ	314
8.2	The unilateral forward shift operator S	321
8.3	Commutants of Z and S	328
8.4	Cyclic vectors of S	333
8.5	When do we have $H^p(\mu) = L^p(\mu)$?	336
8.6	The unilateral forward shift operator S_μ	342
8.7	Reducing invariant subspaces of Z_μ	351
8.8	Simply invariant subspaces of Z_μ	353
8.9	Reducing invariant subspaces of S_μ	360
8.10	Simply invariant subspaces of S_μ	361
8.11	Cyclic vectors of Z_μ and S^*	363
	Notes on Chapter 8	372
9	Analytic reproducing kernel Hilbert spaces	376
9.1	The reproducing kernel	376
9.2	Multipliers	381
9.3	The Banach algebra $\mathfrak{Mult}(\mathcal{H})$	383
9.4	The weak kernel	386
9.5	The abstract forward shift operator $S_{\mathcal{H}}$	390
9.6	The commutant of $S_{\mathcal{H}}$	392
9.7	When do we have $\mathfrak{Mult}(\mathcal{H}) = H^\infty$?	394
9.8	Invariant subspaces of $S_{\mathcal{H}}$	396
	Notes on Chapter 9	396

10	Bases in Banach spaces	399
	10.1 Minimal sequences	399
	10.2 Schauder basis	403
	10.3 The multipliers of a sequence	411
	10.4 Symmetric, nonsymmetric and unconditional basis	414
	10.5 Riesz basis	422
	10.6 The mappings $J_{\mathfrak{X}}$, $V_{\mathfrak{X}}$ and $\Gamma_{\mathfrak{X}}$	425
	10.7 Characterization of the Riesz basis	430
	10.8 Bessel sequences and the Feichtinger conjecture	435
	10.9 Equivalence of Riesz and unconditional bases	440
	10.10 Asymptotically orthonormal sequences	442
	Notes on Chapter 10	449
11	Hankel operators	454
	11.1 A matrix representation for H_{φ}	454
	11.2 The norm of H_{φ}	457
	11.3 Hilbert's inequality	462
	11.4 The Nehari problem	466
	11.5 More approximation problems	470
	11.6 Finite-rank Hankel operators	473
	11.7 Compact Hankel operators	475
	Notes on Chapter 11	478
12	Toeplitz operators	481
	12.1 The operator $T_{\varphi} \in \mathcal{L}(H^2)$	481
	12.2 Composition of two Toeplitz operators	487
	12.3 The spectrum of T_{φ}	490
	12.4 The kernel of T_{φ}	494
	12.5 When is T_{φ} compact?	499
	12.6 Characterization of rigid functions	500
	12.7 Toeplitz operators on $H^2(\mu)$	503
	12.8 The Riesz projection on $L^2(\mu)$	506
	12.9 Characterization of invertibility	511
	12.10 Fredholm Toeplitz operators	515
	12.11 Characterization of surjectivity	518
	12.12 The operator $X_{\mathcal{H}}$ and its invariant subspaces	520
	Notes on Chapter 12	522
13	Cauchy transform and Clark measures	526
	13.1 The space $\mathfrak{R}(\mathbb{D})$	526
	13.2 Boundary behavior of C_{μ}	533
	13.3 The mapping K_{μ}	534
	13.4 The operator $K_{\varphi} : L^2(\varphi) \rightarrow H^2$	541
	13.5 Functional calculus for S_{φ}	545

Contents

xi

13.6	Toeplitz operators with symbols in $L^2(\mathbb{T})$	551
13.7	Clark measures μ_α	555
13.8	The Cauchy transform of μ_α	562
13.9	The function ρ	563
	Notes on Chapter 13	564
14	Model subspaces K_Θ	567
14.1	The arithmetic of inner functions	567
14.2	A generator for K_Θ	570
14.3	The orthogonal projection P_Θ	576
14.4	The conjugation Ω_Θ	579
14.5	Minimal sequences of reproducing kernels in K_B	580
14.6	The operators J and M_Θ	583
14.7	Functional calculus for M_Θ	589
14.8	Spectrum of M_Θ and $\varphi(M_\Theta)$	593
14.9	The commutant lifting theorem for M_Θ	602
14.10	Multipliers of K_Θ	607
	Notes on Chapter 14	608
15	Bases of reproducing kernels and interpolation	611
15.1	Uniform minimality of $(k_{\lambda_n})_{n \geq 1}$	611
15.2	The Carleson–Newman condition	612
15.3	Riesz basis of reproducing kernels	618
15.4	Nevanlinna–Pick interpolation problem	621
15.5	H^∞ -interpolating sequences	623
15.6	H^2 -interpolating sequences	624
15.7	Asymptotically orthonormal sequences	627
	Notes on Chapter 15	638
	<i>References</i>	641
	<i>Symbol index</i>	669
	<i>Author index</i>	673
	<i>Subject index</i>	677

Contents for Volume 2

Preface

- 16 The spaces $\mathcal{M}(A)$ and $\mathcal{H}(A)$**
- 16.1 The space $\mathcal{M}(A)$
 - 16.2 A characterization of $\mathcal{M}(A) \subset \mathcal{M}(B)$
 - 16.3 Linear functionals on $\mathcal{M}(A)$
 - 16.4 The complementary space $\mathcal{H}(A)$
 - 16.5 The relation between $\mathcal{H}(A)$ and $\mathcal{H}(A^*)$
 - 16.6 The overlapping space $\mathcal{M}(A) \cap \mathcal{H}(A)$
 - 16.7 The algebraic sum of $\mathcal{M}(A_1)$ and $\mathcal{M}(A_2)$
 - 16.8 A decomposition of $\mathcal{H}(A)$
 - 16.9 The geometric definition of $\mathcal{H}(A)$
 - 16.10 The Julia operator $\mathfrak{J}(A)$ and $\mathcal{H}(A)$
- Notes on Chapter 16
- 17 Hilbert spaces inside H^2**
- 17.1 The space $\mathcal{M}(u)$
 - 17.2 The space $\mathcal{M}(\bar{u})$
 - 17.3 The space $\mathcal{H}(b)$
 - 17.4 The space $\mathcal{H}(\bar{b})$
 - 17.5 Relations between different $\mathcal{H}(\bar{b})$ spaces
 - 17.6 $\mathcal{M}(\bar{u})$ is invariant under S and S^*
 - 17.7 Contractive inclusion of $\mathcal{M}(\varphi)$ in $\mathcal{M}(\bar{\varphi})$
 - 17.8 Similarity of S and $S_{\mathcal{H}}$
 - 17.9 Invariant subspaces of $Z_{\bar{u}}$ and $X_{\bar{u}}$
 - 17.10 An extension of Beurling's theorem
- Notes on Chapter 17

- 18 The structure of $\mathcal{H}(b)$ and $\mathcal{H}(\bar{b})$**
- 18.1 When is $\mathcal{H}(b)$ a closed subspace of H^2 ?
 - 18.2 When is $\mathcal{H}(b)$ a dense subset of H^2 ?
 - 18.3 Decomposition of $\mathcal{H}(b)$ spaces
 - 18.4 The reproducing kernel of $\mathcal{H}(b)$
 - 18.5 $\mathcal{H}(b)$ and $\mathcal{H}(\bar{b})$ are invariant under $T_{\bar{\varphi}}$
 - 18.6 Some inhabitants of $\mathcal{H}(b)$
 - 18.7 The unilateral backward shift operators X_b and $X_{\bar{b}}$
 - 18.8 The inequality of difference quotients
 - 18.9 A characterization of membership in $\mathcal{H}(b)$
- Notes on Chapter 18
- 19 Geometric representation of $\mathcal{H}(b)$ spaces**
- 19.1 Abstract functional embedding
 - 19.2 A geometric representation of $\mathcal{H}(b)$
 - 19.3 A unitary operator from \mathbb{K}_b onto \mathbb{K}_{b^*}
 - 19.4 A contraction from $\mathcal{H}(b)$ to $\mathcal{H}(b^*)$
 - 19.5 Almost conformal invariance
 - 19.6 The Littlewood Subordination Theorem revisited
 - 19.7 The generalized Schwarz–Pick estimates
- Notes on Chapter 19
- 20 Representation theorems for $\mathcal{H}(b)$ and $\mathcal{H}(\bar{b})$**
- 20.1 Integral representation of $\mathcal{H}(\bar{b})$
 - 20.2 \mathbf{K}_ρ intertwines S_ρ^* and $X_{\bar{b}}$
 - 20.3 Integral representation of $\mathcal{H}(b)$
 - 20.4 A contractive antilinear map on $\mathcal{H}(b)$
 - 20.5 Absolutely continuity of the Clark measure
 - 20.6 Inner divisors of the Cauchy transform
 - 20.7 V_b intertwines S_μ^* and X_b
 - 20.8 Analytic continuation of $\mathcal{H}(b)$ functions
 - 20.9 Multipliers of $\mathcal{H}(b)$
 - 20.10 Multipliers and Toeplitz operators
 - 20.11 Comparison of measures
- Notes on Chapter 20
- 21 Angular derivatives of $\mathcal{H}(b)$ functions**
- 21.1 Derivative in the sense of Carathéodory
 - 21.2 Angular derivatives and Clark measures
 - 21.3 Derivatives of Blaschke products
 - 21.4 Higher derivatives of b
 - 21.5 Approximating by Blaschke products
 - 21.6 Reproducing kernels for derivatives
 - 21.7 An interpolation problem

- 21.8 Derivatives of $\mathcal{H}(b)$ functions
 Notes on Chapter 21
- 22 Bernstein-type inequalities**
- 22.1 Passage between \mathbb{D} and \mathbb{C}_+
 22.2 Integral representations for derivatives
 22.3 The weight $w_{p,n}$
 22.4 Some auxiliary integral operators
 22.5 The operator $T_{p,n}$
 22.6 Distances to the level sets
 22.7 Carleson-type embedding theorems
 22.8 A formula of combinatorics
 22.9 Norm convergence for the reproducing kernels
 Notes on Chapter 22
- 23 $\mathcal{H}(b)$ spaces generated by a nonextreme symbol b**
- 23.1 The pair (a, b)
 23.2 Inclusion of $\mathcal{M}(u)$ into $\mathcal{H}(b)$
 23.3 The element f^+
 23.4 Analytic polynomials are dense in $\mathcal{H}(b)$
 23.5 A formula for $\|X_b f\|_b$
 23.6 Another representation of $\mathcal{H}(b)$
 23.7 A characterization of $\mathcal{H}(b)$
 23.8 More inhabitants of $\mathcal{H}(b)$
 23.9 Unbounded Toeplitz operators and $\mathcal{H}(b)$ spaces
 Notes on Chapter 23
- 24 Operators on $\mathcal{H}(b)$ spaces with b nonextreme**
- 24.1 The unilateral forward shift operator S_b
 24.2 A characterization of $H^\infty \subset \mathcal{H}(b)$
 24.3 Spectrum of X_b and X_b^*
 24.4 Comparison of measures
 24.5 The function F_λ
 24.6 The operator W_λ
 24.7 Invariant subspaces of $\mathcal{H}(b)$ under X_b
 24.8 Completeness of the family of difference quotients
 Notes on Chapter 24
- 25 $\mathcal{H}(b)$ spaces generated by an extreme symbol b**
- 25.1 A unitary map between $\mathcal{H}(\bar{b})$ and $L^2(\rho)$
 25.2 Analytic continuation of $f \in \mathcal{H}(\bar{b})$
 25.3 Analytic continuation of $f \in \mathcal{H}(b)$
 25.4 A formula for $\|X_b f\|_b$
 25.5 S^* -cyclic vectors in $\mathcal{H}(b)$ and $\mathcal{H}(\bar{b})$
 25.6 Orthogonal decompositions of $\mathcal{H}(b)$

- 25.7 The closure of $\mathcal{H}(\bar{b})$ in $\mathcal{H}(b)$
 25.8 A characterization of $\mathcal{H}(b)$
 Notes on Chapter 25
- 26 Operators on $\mathcal{H}(b)$ spaces with b extreme**
 26.1 Spectrum of X_b and X_b^*
 26.2 Multipliers of $\mathcal{H}(b)$ spaces, extreme case, part I
 26.3 Comparison of measures
 26.4 Further characterizations of angular derivatives for b
 26.5 Model operator for Hilbert space contractions
 26.6 Conjugation and completeness of difference quotients
 Notes on Chapter 26
- 27 Inclusion between two $\mathcal{H}(b)$ spaces**
 27.1 A new geometric representation of $\mathcal{H}(b)$ spaces
 27.2 The class $\mathcal{I}nt(V_{b_1}, V_{b_2})$
 27.3 The class $\mathcal{I}nt(\mathcal{S}_{b_1}, \mathcal{S}_{b_2})$
 27.4 Relations between different $\mathcal{H}(b)$ spaces
 27.5 The rational case
 27.6 Coincidence between $\mathcal{H}(b)$ and $\mathcal{D}(\mu)$ spaces
 Notes on Chapter 27
- 28 Topics regarding inclusions $\mathcal{M}(a) \subset \mathcal{H}(\bar{b}) \subset \mathcal{H}(b)$**
 28.1 A sufficient and a necessary condition for $\mathcal{H}(\bar{b}) = \mathcal{H}(b)$
 28.2 Characterizations of $\mathcal{H}(\bar{b}) = \mathcal{H}(b)$
 28.3 Multipliers of $\mathcal{H}(b)$, extreme case, part II
 28.4 Characterizations of $\mathcal{M}(a) = \mathcal{H}(b)$
 28.5 Invariant subspaces of S_b when $b(z) = (1+z)/2$
 28.6 Characterization of $\overline{\mathcal{M}(a)}^b = \mathcal{H}(b)$
 28.7 Characterization of the closedness of $\mathcal{M}(a)$ in $\mathcal{H}(b)$
 28.8 Boundary eigenvalues and eigenvectors of S_b^*
 28.9 The space $\mathcal{H}_0(b)$
 28.10 The spectrum of S_0
 Notes on Chapter 28
- 29 Rigid functions and strongly exposed points of H^1**
 29.1 Admissible and special pairs
 29.2 Rigid functions of H^1 and $\mathcal{H}(b)$ spaces
 29.3 Dimension of $\mathcal{H}_0(b)$
 29.4 S_b -invariant subspaces of $\mathcal{H}(b)$
 29.5 A necessary condition for nonrigidity
 29.6 Strongly exposed points and $\mathcal{H}(b)$ spaces
 Notes on Chapter 29

- 30 Nearly invariant subspaces and kernels of Toeplitz operators**
- 30.1 Nearly invariant subspaces and rigid functions
 - 30.2 The operator R_f
 - 30.3 Extremal functions
 - 30.4 A characterization of nearly invariant subspaces
 - 30.5 Description of kernels of Toeplitz operators
 - 30.6 A characterization of surjectivity for Toeplitz operators
 - 30.7 The right inverse of a Toeplitz operator
- Notes on Chapter 30
- 31 Geometric properties of sequences of reproducing kernels**
- 31.1 Completeness and minimality in $\mathcal{H}(b)$ spaces
 - 31.2 Spectral properties of rank one perturbation of X_b^*
 - 31.3 Orthonormal bases in $\mathcal{H}(b)$ spaces
 - 31.4 Riesz sequences of reproducing kernels in $\mathcal{H}(b)$
 - 31.5 The invertibility of distortion operator and Riesz bases
 - 31.6 Riesz sequences in $H^2(\mu)$ and in $\mathcal{H}(\bar{b})$
 - 31.7 Asymptotically orthonormal sequences and bases in $\mathcal{H}(b)$
 - 31.8 Stability of completeness and AOB
 - 31.9 Stability of Riesz bases
- Notes on Chapter 31

References

Symbol Index

Subject Index