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## Symbol index

$\mathcal{A}$	Disk algebra	175
$\mathcal{A}(\mathbb{D})$	Disk algebra on $\mathbb{D}$	175
$\mathcal{A}(\mathbb{T})$	Disk algebra on $\mathbb{T}$	173
$ A $	Absolute value of $A$	278
$A^{1/2}$	Positive square root of $A$	260
$A^*$	Adjoint of operator $A$	40
$\langle \mathcal{M}, \mathcal{N} \rangle_{\mathcal{H}}$	Angle between the subspaces $\mathcal{M}$ and $\mathcal{N}$	55
$A_{\mathcal{M}_1 \rightarrow \mathcal{M}_2}$	Restricted mapping	43
$B_X^1$	Closed unit ball of $X$	215
$b_\alpha$	Blaschke factor with zero at $\alpha$	142
$C_\mu$	Cauchy transform of $\mu$	526
$\chi_n$	Monomial $\chi_n(z) = z^n$	4
$\mathcal{C}(\mathbb{T})$	Space of continuous functions on $\mathbb{T}$	4
$c_0$	Sequential Banach space	2
$(C)$	Carleson condition	612
$\mathbb{D}$	Open unit disk	xiv
$\mathbb{D}_e$	Set $\{z : 1 <  z  \leq \infty\}$	366
$D_A$	Defect operator associated to a contraction $A$	268
$\mathcal{D}_A$	Defect space associated to a contraction $A$	268
$D_{hyp}(z, r)$	Hyperbolic disk with center $z$ and radius $r$	123
$\delta_\alpha$	Dirac measure at $\alpha$	4
$d\mu/d\nu$	Derivative of $\mu$ with respect to $\nu$	104
$D_\mu$	Derivative of $\mu$ with respect to $m$	104
$ E $	Length of set $E$ with respect to $m$	3
$\mathfrak{e}_n$	Basis of sequence space $\ell^p$	2
$\partial_A$	Defect index associated to a contraction $A$	268
$\partial E$	Boundary points of $E$	66
$Ext(X)$	Set of extreme points of $B_X^1$	215

$\mathcal{F}(\mathcal{H}_1, \mathcal{H}_2)$	Family of finite-rank operators in $\mathcal{L}(\mathcal{H}_1, \mathcal{H}_2)$ . . . . .	60
$[f]$	Outer part of $f$ . . . . .	143
$\ f\ _\infty$	Norm of $f$ . . . . .	4
$\langle f, g \rangle_{L^2(\mu)}$	Inner product in $L^2(\mu)$ . . . . .	30
$\hat{\varphi}(n)$	Fourier coefficients of $\varphi$ . . . . .	5
$\mathcal{H}_1 \oplus \mathcal{H}_2$	Direct sum of $\mathcal{H}_1$ and $\mathcal{H}_2$ . . . . .	31
$\mathcal{H} \ominus \mathcal{M}$	Orthogonal complement of $\mathcal{M}$ in $\mathcal{H}$ . . . . .	34
$H(\mathbb{D})$	Space of analytic functions on $\mathbb{D}$ . . . . .	178
$H^\infty(\Omega)$	Space of bounded analytic functions on $H$ . . . . .	377
$H(\Omega)$	Space of analytic functions on $H$ . . . . .	377
$H^2_\perp$	Orthogonal complement of the Hardy space $H^2$ . . . . .	132
$H^p$	Hardy space . . . . .	126
$H^p(\mathbb{D})$	Hardy space on $\mathbb{D}$ . . . . .	124
$H^p_0(\mathbb{D})$	Functions in $H^p$ vanishing at the origin . . . . .	124
$H^p(\mathbb{T})$	Hardy space on $\mathbb{T}$ . . . . .	125
$H^p_0(\mathbb{T})$	Functions $f$ in $H^p(\mathbb{T})$ such that $\hat{f}(0) = 0$ . . . . .	126
$H^p(\mu)$	Generalized Hardy space . . . . .	183
(HS)	Helson–Szegő condition . . . . .	511
(HCR)	Corona hypothesis . . . . .	211
$i$	Inclusion of $H^2$ into $L^2$ . . . . .	133
$i_{\mathcal{M}}$	Inclusion of $\mathcal{M}$ into $\mathcal{H}$ . . . . .	43
$i_\varphi$	Inclusion of $H^2(\varphi)$ into $L^2(\varphi)$ . . . . .	185
$\text{ind} A$	Index of $A$ . . . . .	286
$J_\varphi$	Inclusion operator of $H^2$ into $L^2(\varphi)$ . . . . .	541
$J(B)$	Julia operator . . . . .	271
$K$	Cauchy operator on $L^1(\mathbb{T})$ . . . . .	535
$K_\varphi$	Cauchy operator on $L^2(\varphi)$ . . . . .	541
$K_\mu$	Cauchy operator on $L^1(\mu)$ . . . . .	535
$\mathcal{K}(\mathcal{H}_1, \mathcal{H}_2)$	Family of compact operators in $\mathcal{L}(\mathcal{H}_1, \mathcal{H}_2)$ . . . . .	62
$\ker A$	Kernel of operator $A$ . . . . .	9
$K_\Theta$	Co-invariant subspaces $K_\Theta$ in the Hilbert case . . . . .	369
$K^p_\Theta$	Model space . . . . .	369
$\tilde{k}$	Normalized reproducing kernel . . . . .	612
$k_z$	Reproducing kernel . . . . .	377
$k_z$	Cauchy kernel . . . . .	128
$\text{Lat}(A)$	Lattice of the closed invariant subspaces of operator $A$ . . . . .	49
$\lim_{z \rightarrow \zeta_0} \triangleleft$	Nontangential limit at $\zeta_0$ . . . . .	97
$\mathcal{L}(H)$	Space of bounded operators on $H$ . . . . .	9
$\mathcal{L}(H_1, H_2)$	Space of bounded operators from $H_1$ to $H_2$ . . . . .	9
$\text{Lin}(\mathcal{E})$	Linear manifold spanned by $\mathcal{E}$ . . . . .	2
$\log^+ t$	Positive part of logarithm . . . . .	115

$\log^- t$	Opposite of the negative part of logarithm	115
$\ell^p$	Sequential Banach space	7
$m$	Normalized Lebesgue measure on $\mathbb{T}$	3
$m(A)$	Minimum of $W(A)$	71
$M(A)$	Maximum of $W(A)$	71
$M_\varphi$	Multiplication operator	83
$\mathfrak{Mult}(H)$	Space of multipliers of $H$	383
$\mathcal{M}(\mathbb{T})$	Borel measures on $\mathbb{T}$	4
$\mathcal{M}^+(\mathbb{T})$	Positive Borel measures on $\mathbb{T}$	4
$\mathcal{M}^\perp$	Orthogonal complement of $\mathcal{M}$	34
$ \mu $	Total variation of $\mu$	4
$\ \mu\ $	Norm of $\mu$	4
$\hat{\mu}(n)$	Fourier coefficients of $\mu$	5
$\tilde{\mu}$	Hilbert transform of measure $\mu$	113
$\mu \ll \nu$	$\mu$ is absolutely continuous with respect to $\nu$	102
$\mu \perp \nu$	$\mu$ is singular with respect to $\nu$	103
$\mu_\lambda$	Clark measure associated to $\bar{\lambda}b$	556
$\mathcal{N}$	Nevanlinna class	167
$\mathcal{N}^+$	Smirnov class	168
$\Omega_\mu$	Domain of definition of $C_\mu$	526
$\pi$	Canonical projection of $\mathcal{L}(H)$ onto $\mathcal{L}(H)/\mathcal{K}(H)$	63
$\mathcal{P}$	Space of trigonometric polynomials	4
$\mathcal{P}_+$	Space of analytic polynomials	4
$\mathcal{P}_{0+}$	Space of analytic polynomials vanishing at the origin	4
$P_+$	Riesz projection	131
$P_-$	Orthogonal complement of the Riesz projection	132
$P_\varphi$	Generalized Riesz projection	185
$P_{\mathcal{M}}$	Orthogonal projection on $\mathcal{M}$	34
$\mathbf{P}_{\mathcal{M}}$	Restricted orthogonal projection on $\mathcal{M}$	43
$\mathbf{P}_{\mathcal{N} \rightarrow \mathcal{M}}$	Restricted injection	44
$\mathbf{P}_\Theta$	Restricted orthogonal projection $\mathbf{P}_{K_\Theta}$	577
$P_\Theta$	Projection from $L^2(\mathbb{T})$ onto $K_\Theta$	577
$P_{\mathcal{M} \parallel \mathcal{N}}$	Skew projection	54
$P_z$	Poisson kernel	107
$P_\mu$	Poisson integral of $\mu$	107
$\tilde{\varphi}$	Hilbert transform of function $\varphi \in L^1(\mathbb{T})$	113
$\psi(S_\varphi)$	Multiplication by $\psi$ on $H^2(\varphi)$	548
$Q_z$	Conjugate Poisson kernel	107
$Q_\mu$	Conjugate Poisson integral of $\mu$	107
$Q_w$	Difference quotient operator	327
$\mathcal{R}(A)$	Range of operator $A$	10
$r(x)$	Spectral radius of $x$	26

$\rho(x)$	Resolvent of $x$ . . . . .	26
$\rho(z, w)$	Hyperbolic distance between the two points $z, w \in \mathbb{D}$ . . . . .	122
$\mathcal{R}(\varphi)$	Range of $\varphi$ . . . . .	385
$\mathcal{R}_e^\mu(\varphi)$	Essential range of $\varphi$ with respect to $\mu$ . . . . .	85
$\mathcal{R}_e(\varphi)$	Essential range of $\varphi$ with respect to $m$ . . . . .	86
$R_\lambda(x)$	Resolvent operator . . . . .	26
$S$	Forward shift operator . . . . .	321
$S_\mu$	Forward shift operator on $H^2(\mu)$ . . . . .	342
$S_w$	Forward shift operator on $H^2(w \, dm)$ . . . . .	342
$S_C(\zeta_0)$	Stoltz's domain anchored at $\zeta_0$ . . . . .	96
$S(\zeta, h)$	Carleson disc . . . . .	189
$S_\sigma$	Generalization of the singular inner function . . . . .	147
$\text{sgn } z$	Projection of $z$ on $\mathbb{T}$ . . . . .	541
$S_{\mathcal{H}}$	Forward shift operator on $\mathcal{H}$ . . . . .	390
$S_{H^2(\mu)}$	Forward shift operator on $H^2(\mu)$ . . . . .	314
$S_{L^2(\mu)}$	Forward shift operator on $L^2(\mu)$ . . . . .	314
$\text{Span}(\mathcal{E})$	Closure of $\text{Lin}(\mathcal{E})$ . . . . .	2
$\sigma(b)$	Spectrum of $b \in H^\infty$ . . . . .	171
$\sigma(x)$	Spectrum of $x$ . . . . .	26
$\sigma_a(A)$	Approximate spectrum of operator $A$ . . . . .	66
$\sigma_c(A)$	Continuous spectrum of operator $A$ . . . . .	66
$\sigma_{\text{ess}}(A)$	Essential spectrum of operator $A$ . . . . .	66
$\sigma_{\text{ess}}^\ell(A)$	Essential left-spectrum of operator $A$ . . . . .	66
$\sigma_p(A)$	Point spectrum of operator $A$ . . . . .	66
$\sigma_n(\varphi)$	$n$ th Fejér sum of $\varphi$ . . . . .	14
$\text{supp}(\mu)$	Support of $\mu$ . . . . .	4
$\mathbb{T}$	Unit circle . . . . .	xiv, 3
(T)	Thin condition . . . . .	634
$T_\varphi$	Toeplitz operator . . . . .	481
$T_\varphi^{\mathcal{K}}$	Toeplitz operator on Cauchy kernels . . . . .	503
$T_\varphi^{\mathcal{P}}$	Toeplitz operator on polynomials . . . . .	503
$\tilde{W}(A)$	Numerical range of operator $A$ . . . . .	74
$W(\zeta, h)$	Carleson boxes . . . . .	195
$W_I$	Carleson window . . . . .	196
$\hat{x}$	Image of $x \in B$ in the second dual $B^{**}$ . . . . .	10
$x_1 \oplus x_2$	Element of $H_1 \oplus H_2$ . . . . .	31
$x_1 \otimes x_2$	Tensor product of $x_1$ and $x_2$ . . . . .	45
$x_n \xrightarrow{w} x$	Sequence $(x_n)_n$ tends weakly to $x$ . . . . .	35
$\mathcal{X}^*$	Dual space of $\mathcal{X}$ . . . . .	10
$\mathcal{X}^{**}$	Second dual of $\mathcal{X}$ . . . . .	10
$Z_w$	Forward shift operator on $L^2(w \, dm)$ . . . . .	315
$Z$	Forward shift operator on $L^2(\mathbb{T})$ . . . . .	315

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Emmanuel Fricain and Javad Mashreghi

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