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978-1-107-02686-5 - Turbulence in Rotating, Stratified and Electrically Conducting Fluids
P. A. Davidson
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# TURBULENCE IN ROTATING, STRATIFIED AND ELECTRICALLY CONDUCTING FLUIDS

There are two recurring themes in astrophysical and geophysical fluid mechanics: waves and turbulence. This book investigates how turbulence responds to rotation, stratification or magnetic fields, identifying common themes, where they exist, as well as the essential differences which inevitably arise between different classes of flow.

The discussion is developed from first principles, making the book suitable for graduate students as well as professional researchers. The author focusses first on the fundamentals and then progresses to such topics as the atmospheric boundary layer, turbulence in the upper atmosphere, turbulence in the core of the Earth, zonal winds in the giant planets, turbulence within the interior of the Sun, the solar wind, and turbulent flows in accretion discs. The book will appeal to engineers, geophysicists, astrophysicists and applied mathematicians who are interested in naturally occurring turbulent flows.

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For Sarah Elizabeth and James Alexander

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## PART III TURBULENCE IN THE PRESENCE OF BODY FORCES



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# Preface

> I love deadlines. I especially love the swooshing sound they make as they go flying by.
>
> *Douglas Adams*

This is a book I had in mind to write for some years, but self-imposed deadlines came and went to little avail. It was not until late 2008, inspired by an Isaac Newton Institute programme on turbulence, that lethargy finally gave way to action.

There are two recurring themes in astrophysical and geophysical fluid mechanics: waves and turbulence. These flows are generally subject to a background rotation, strong stratification, or an ambient magnetic field, and in all three cases this allows the fluid to sustain internal wave motion. Moreover, such flows are almost invariably turbulent, and the turbulence is often central to their behaviour, allowing accretion discs to feed mass to young or dying stars, triggering explosions on the surface of the Sun, diffusing heat, momentum and pollutants across the atmospheric boundary layer, and generating the terrestrial magnetic field deep within the interior of the Earth. Sometimes the waves and turbulence coexist with little interaction, but more commonly there is an interplay between the two. For example, in some flows the turbulence excites waves which, in turn, reshape the structure of the turbulence by dispersing the energy held in vortices. Conversely, at times internal waves grow and become unstable, initiating new turbulence. On yet other occasions the turbulence displays almost no wave-like properties, despite a background rotation or stratification. There appears to be a multitude of possibilities.

Understanding this two-way interaction between waves and turbulence, where and when it occurs, has proven to be a formidable challenge. When the turbulence is very weak (relative to the wave motion) there are well-established mathematical techniques that can be brought to bear on the problem, but unfortunately turbulence in nature is rarely weak, and so we have few mathematical formalisms at our disposal. As with conventional turbulence, much rests on dimensional analysis, heuristic physical arguments, and careful numerical or physical experiments. Moreover, the nature and extent of this wave–turbulence interaction varies markedly from case to case, being quite different for, say, internal gravity waves, inertial waves maintained by the Coriolis force, and Alfvén waves which travel along magnetic field lines. In the case of rapidly rotating turbulence, some progress has been



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made and it is, perhaps, possible to rationalise the observed anisotropic structuring of the large eddies in terms of inertial wave propagation, though there are many details still to be resolved. And in magnetohydrodynamic turbulence the observed distribution of energy across the various scales can now be explained in terms of the interaction of Alfvén waves with turbulent eddies. In stratified turbulence, however, the significance of gravity waves, and the manner in which they interact with the turbulence, is still poorly understood, and indeed in some instances it is, perhaps, not terribly helpful to try and interpret events in terms of wave–turbulence interactions.

Any author embarking on a book on geophysical and astrophysical turbulence is immediately faced with a number of problems, not the least of which is that many of the central issues remain unresolved, or at least only partially understood. There is disagreement, for example, as to why rapidly rotating turbulence is dominated by cyclonic columnar vortices, or why strongly stratified turbulence takes the form of flat, pancake-like eddies (at least at the large scales). So this is a story without an ending. A second difficulty is that many diverse communities study such flows (meteorologists, oceanographers, astrophysicists . . . ) and these communities have tended to develop their own language and ways of conceiving the phenomena. Communication between these groups is not always straightforward. Yet, despite all these difficulties, it seems natural to seek to provide an overview, if only a partial one, of these distinct yet closely related areas of study.

Given the difficulty of the subject matter, the open-ended nature of the problem (or rather problems), and the difficulties of language, prudence dictates that any text on the subject must have modest aims. Certainly this book makes no claims for completeness; indeed, entire books could be (have been) devoted to, say, turbulent motion in accretion discs, or in the Sun, or in the atmospheric boundary layer. Rather, our aim here is to take a step back and provide an account of how turbulence responds to rotation, stratification and magnetic fields, identifying common themes where they exist, as well as the essential differences which inevitably arise. In order to counter the issue of language, it was decided to develop the entire subject more or less from first principles, and so the book starts with extended chapters on the theory of rotating fluids, stratified flows, and magnetohydrodynamics, all in the absence of turbulence. This constitutes Part I of the text. Turbulence too tends to be shrouded in its own language and mysteries, and so turbulence theory is also introduced and developed from first principles (Part II of the book). It is not until we reach Part III of this text that we arrive at the core of the problem, where turbulence is combined with rotation, stratification and magnetic fields. Here we encounter the recurring difficulty that many of the central questions remain unanswered, and that often there are competing explanations for the observed phenomena. I have tried to pick my way carefully through this minefield, mentioning controversies where they exist, and avoiding topics and theories that seem likely to date rather quickly. While I hope the outcome is broadly satisfactory, I have lived long enough to be quite familiar with my own imperfections, and so I beg the reader to be indulgent if, at times, they find the balance is not to their taste.

It is a pleasure to acknowledge the help of friends and colleagues. Over the years I have benefited from many interesting discussions on turbulence with Julian Hunt, Yukio Kaneda,

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Per-Åge Krogstad and Keith Moffatt. Kate Graham helped with some of the figures, Jim Riley introduced me to the mysteries of geophysical turbulence, Uli Christensen was kind enough to share his thoughts on recent geodynamo simulations, and Alex Schekochihin helped guide me through the labyrinth of spectral theories of MHD turbulence. David Tranah of Cambridge University Press was a delight to work with and helped shape this book. Finally, I have been blessed with a long-suffering wife who has patiently endured those unreasonably long Sunday silences which inevitably accompanied the writing of this book.

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