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General Editors

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202 Canonical Ramsey Theory on Polish Spaces

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Canonical Ramsey Theory on Polish Spaces

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Contents

Preface

We wrote this book to lay the foundation to an area of descriptive set theory parallel to the canonical Ramsey theory in finite combinatorics. The book develops the theory of analytic and Borel equivalence relations, a subject that has received a great deal of attention over the past two decades, and connects it with the abstract methods of forcing and generic extensions.

The subject grew quickly from near nonexistence in 2010 to a large body of interconnected results of considerable sophistication by the end of 2012. An entirely new landscape was created, with a number of satisfactory general results as well as many avenues open for further investigation.

The target audience consists of graduate students and researchers with basic experience in descriptive set theory and forcing. We hope that the book will help them to sharpen the understanding of Borel and analytic equivalence relations.

We would like to thank Bohuslav Balcar, Clinton Conley, Benjamin Miller, Christian Rosendal and Sławek Solecki for many helpful comments and suggestions. We are also indebted to Adrian Mathias for allowing us to include Theorem 8.17 in this book.

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