

GEOMETRIC AND TOPOLOGICAL METHODS FOR QUANTUM FIELD THEORY

Based on lectures given at the renowned Villa de Leyva summer school, this book provides a unique presentation of modern geometric methods in quantum field theory. Written by experts, it enables readers to enter some of the most fascinating research topics in this subject.

Covering a series of topics on geometry, topology, algebra, number theory methods, and their applications to quantum field theory, the book covers topics such as Dirac structures, holomorphic bundles and stability, Feynman integrals, geometric aspects of quantum field theory and the standard model, spectral and Riemannian geometry, and index theory. This is a valuable guide for graduate students and researchers in physics and mathematics wanting to enter this interesting research field at the borderline between mathematics and physics.

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GEOMETRIC AND TOPOLOGICAL METHODS FOR QUANTUM FIELD THEORY

Proceedings of the 2009
Villa de Leyva Summer School

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CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press & Assessment

978-1-107-02683-4 — Geometric and Topological Methods for Quantum Field Theory

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Frontmatter

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UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment,
a department of the University of Cambridge.

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www.cambridge.org

Information on this title: www.cambridge.org/9781107026834

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First published 2013

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication data

Geometric and topological methods for quantum field theory : proceedings of the 2009

Villa de Leyva summer school / edited by Alexander Cardona,

Iván Contreras, Andrés F. Reyes-Lega.

pages cm

Includes bibliographical references and index.

ISBN 978-1-107-02683-4 (hardback)

1. Geometric quantization. 2. Quantum field theory – Mathematics.

I. Cardona, Alexander, editor of compilation. II. Contreras, Iván, 1985– editor of compilation.

III. Reyes-Lega, Andrés F., 1973– editor of compilation.

QC174.17.G46G46 2013

530.14'301516 – dc23 2012048560

ISBN 978-1-107-02683-4 Hardback

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