

Contents

Preface	vii
To the student	ix
Chapter 1. Numbers, sets, and functions	1
1.1. The natural numbers, integers, and rational numbers	1
1.2. Sets	6
1.3. Functions	9
More exercises	11
Chapter 2. The real numbers	15
2.1. The complete ordered field of real numbers	15
2.2. Consequences of completeness	17
2.3. Countable and uncountable sets	19
More exercises	21
Chapter 3. Sequences	23
3.1. Convergent sequences	23
3.2. New limits from old	25
3.3. Monotone sequences	27
3.4. Series	28
3.5. Subsequences and Cauchy sequences	32
More exercises	35
Chapter 4. Open, closed, and compact sets	39
4.1. Open and closed sets	39

4.2. Compact sets	41
More exercises	42
Chapter 5. Continuity	45
5.1. Limits of functions	45
5.2. Continuous functions	47
5.3. Continuous functions on compact sets and intervals	49
5.4. Monotone functions	51
More exercises	53
Chapter 6. Differentiation	55
6.1. Differentiable functions	55
6.2. The mean value theorem	59
More exercises	60
Chapter 7. Integration	63
7.1. The Riemann integral	63
7.2. The fundamental theorem of calculus	67
7.3. The natural logarithm and the exponential function	69
More exercises	71
Chapter 8. Sequences and series of functions	73
8.1. Pointwise and uniform convergence	73
8.2. Power series	76
8.3. Taylor series	80
8.4. The trigonometric functions	83
More exercises	87
Chapter 9. Metric spaces	91
9.1. Examples of metric spaces	91
9.2. Convergence and completeness in metric spaces	95
More exercises	99
Chapter 10. The contraction principle	103
10.1. The contraction principle	103
10.2. Picard's theorem	107
More exercises	111
Index	113