

Index

- absolute value, 5
- absolutely convergent series, 30
- addition, 2
- additive
 - identity, 2
 - inverse, 2
- algebraic limit theorem, 25, 47
- algebraic number, 22
- alternating harmonic series, 31
- alternating series test, 31
- antiderivative, 68
- Archimedean property, 17
- arcsine, 87
- associativity, 2
- axiom of completeness, 16

- ball
 - closed, 100
 - open, 95
- Banach fixed point theorem, 104
- bijection, 10
- bijjective function, 10
- binary expansion, 37
- Bolzano-Weierstrass theorem, 33
- bound
 - greatest lower, 16
 - least upper, 16
 - lower, 15
 - upper, 15
- boundary, 43
- bounded
 - function, 53
 - sequence, 25
 - set, 15, 101

- cancellation law, 12

- for functions, 13
- Cantor set, 43
- cardinality, 19
- Cauchy criterion, 33
- Cauchy sequence, 33, 97
- Cauchy-Schwarz inequality, 92
- centre, 77
- chain rule, 57
- change of variables, 72
- closed
 - ball, 100
 - interval, 4
 - set, 40, 99
- closure, 42
- codomain, 9
- coefficient, 77
- commutativity, 2
- compact
 - metric space, 101
 - set, 41, 101
- comparison test, 29
- complement, 7
- complete metric space, 97
- completeness, axiom of, 16
- composition, 9
- concave function, 61
- conditionally convergent series, 30
- continuous function, 47, 103
 - at a point, 47, 103
- continuously differentiable function, 55
- contraction, 103
- contraction principle, 104
- contrapositive, 2
- convergent
 - sequence, 23, 95
 - series, 28, 76

- converse, 2
 - convex function, 61
 - cosine, 84
 - countable set, 20
 - countably infinite set, 20
 - cover, open, 101
 - critical point, 58

 - Darboux's theorem, 58
 - De Morgan's laws, 7
 - decimal expansion, 37
 - decreasing
 - function, 51
 - sequence, 27
 - degenerate interval, 4
 - degree, 48
 - dense set, 18
 - derivative, 55
 - differentiable function, 55
 - at a point, 55
 - Dini's theorem, 88
 - discrete space, 92
 - disjoint sets, 7
 - distance function, 91
 - distributivity, 2
 - divergent
 - sequence, 23
 - series, 28
 - domain, 9

 - e , 69, 70, 80
 - element, 6
 - empty set \emptyset , 7
 - equinumerous sets, 19
 - equivalent metrics, 93
 - error function, 81
 - Euclidean metric, 92
 - Euclidean norm, 92
 - Euler's constant γ , 72
 - even function, 61
 - eventually constant sequence, 95
 - expansion
 - binary, 37
 - decimal, 37
 - to a base, 37
 - exponential function, 70
 - extreme point, 58
 - extreme value theorem, 49

 - family of sets, 8
 - fibre, 9
 - field, 2
 - ordered, 3
 - function, 9
 - bijective, 10
 - bounded, 53
 - above, 53
 - below, 53
 - locally, 54
 - concave, 61
 - continuous, 47, 103
 - at a point, 47, 103
 - uniformly, 49
 - convex, 61
 - decreasing, 51
 - strictly, 51
 - differentiable, 55
 - at a point, 55
 - continuously, 55
 - even, 61
 - exponential, 70
 - identity, 9
 - increasing, 51
 - strictly, 51
 - injective, 10
 - integrable, 64
 - inverse, 11
 - monotone, 51
 - strictly, 51
 - odd, 61
 - one-to-one, 10
 - onto, 10
 - periodic, 86
 - polynomial, 48
 - rational, 48
 - Riemann integrable, 64
 - surjective, 10
 - trigonometric, 33, 83–87
- fundamental theorem of calculus, 67
- geometric series, 29
 - graph, 11
 - greatest lower bound, 16

 - harmonic series, 29
 - Heine-Borel theorem, 41

 - identity
 - additive, 2
 - multiplicative, 2
 - identity function, 9
 - image
 - inverse, 9
 - of a function, 9
 - of a subset, 9
 - of an element, 9
 - improper integral, 71
 - increasing
 - function, 51
 - sequence, 27
 - indefinite integral, 68
 - index set, 8
 - induced metric, 95
 - induction, 1

- inductive set, 22
- inductively defined sequence, 27
- inequality
 - Cauchy-Schwarz, 92
 - triangle, 5, 91
- infimum, 16
- inflection point, 89
- initial value problem, 107
- injection, 10
- injective function, 10
- inner product, 92
- integer, 1
- integrable function, 64
- integral, 64
 - improper, 71
 - indefinite, 68
 - lower, 64
 - upper, 64
- integral test, 72
- integration by parts, 72
- interior, 43
- intermediate value theorem, 50
- intersection, 7, 8
- interval, 4
 - closed, 4
 - degenerate, 4
 - nondegenerate, 4
 - of convergence, 78
 - open, 78
- open, 4
- inverse
 - additive, 2
 - multiplicative, 2
- inverse function, 11
- inverse function theorem, 57
- inverse image, 9
- inverse sine, 87
- isolated point, 47

- L^1 metric, 93
- L^2 metric, 93
- L^∞ metric, 93
- Lagrange's remainder theorem, 81
- least upper bound, 16
- L'Hôpital's rule, 60
- limit
 - inferior, 38
 - of a function, 45, 54
 - of a sequence, 23, 95
 - superior, 37
- limit comparison test, 30
- limit point, 45
- Lipschitz condition, 108
- locally bounded function, 54
- logarithm (natural), 69
- lower
 - bound, 15
- integral, 64
 - sum, 63
- Maclaurin series, 81
- map, mapping, 9, *see also* function
- maximum, 5, 16
- maximum metric, 93
- mean value theorem, 59
 - for integrals, 68
 - generalised, 60
- metric, 91
 - equivalent, 93
 - Euclidean, 92
 - induced, 95
 - L^1 , 93
 - L^2 , 93
 - L^∞ , 93
 - maximum, 93
 - p -adic, 93
 - supremum, 94
 - uniform, 94
- metric space, 91
 - compact, 101
 - complete, 97
 - discrete, 92
 - ultrametric, 93
- minimum, 5, 16
- monotone
 - function, 51
 - sequence, 27
- monotone convergence theorem, 27
- multiplication, 2
- multiplicative
 - identity, 2
 - inverse, 2

- natural logarithm, 69
- natural number, 1, 22
- negative number, 4
- neighbourhood, 24, 95
- nested interval property, 19
- nondegenerate interval, 4
- norm, Euclidean, 92
- number
 - algebraic, 22
 - integer, 1
 - natural, 1, 22
 - negative, 4
 - positive, 4
 - rational, 1
 - transcendental, 22

- odd function, 61
- one-to-one function, 10
- onto function, 10
- open
 - ball, 95

- cover, 101
- interval, 4
- set, 39, 98
- or (conjunction), 7
- order limit theorem, 26
- ordered field, 3
- ordered pair, 8
- p -adic metric, 93
- pair, ordered, 8
- partial sum, 28
- partition, 63
- period, 86
- period group, 86
- periodic function, 86
- π , 86
- Picard's theorem, 109
- pointwise convergent
 - sequence, 73
 - series, 76
- polynomial function, 48
- positive number, 4
- power series, 77
- preimage, 9
- product of sets, 8
- product rule, 56
- proper subset, 6
- quotient rule, 56
- radius of convergence, 78
- range, 9
- ratio test, 30
- rational function, 48
- rational number, 1
- rearrangement, 31
- recursion formula, 27
- recursively defined sequence, 27
- refinement, 63
- reflexive relation, 20
- remainder, 81
- Riemann integrable function, 64
- Rolle's theorem, 59
- root, 18, 48, 53, 58
- root test, 31
- rule, 9, 11
- sequence, 23
 - bounded, 25
 - above, 25
 - below, 25
 - Cauchy, 33, 97
 - convergent, 23, 95
 - pointwise, 73
 - uniformly, 74
 - decreasing, 27
 - strictly, 27
 - divergent, 23
 - eventually constant, 95
 - increasing, 27
 - strictly, 27
 - inductively defined, 27
 - monotone, 27
 - strictly, 27
 - recursively defined, 27
- series, 28
 - alternating harmonic, 31
 - convergent, 28, 76
 - absolutely, 30
 - conditionally, 30
 - pointwise, 76
 - uniformly, 76
 - divergent, 28
 - geometric, 29
 - harmonic, 29
 - Maclaurin, 81
 - power, 77
 - Taylor, 81
- set, 6
 - bounded, 15, 101
 - above, 15
 - below, 15
 - closed, 40, 99
 - compact, 41, 101
 - countable, 20
 - countably infinite, 20
 - dense, 18
 - inductive, 22
 - open, 39, 98
 - symmetric, 61
 - uncountable, 20
- sine, 84
 - inverse, 87
- source, 9
- squeeze theorem, 25, 46
- strictly decreasing
 - function, 51
 - sequence, 27
- strictly increasing
 - function, 51
 - sequence, 27
- strictly monotone
 - function, 51
 - sequence, 27
- subcover, 101
- subgroup, 85
- subsequence, 32
- subset, 6
 - proper, 6
- subspace, 95
- substitution, 72
- sum
 - lower, 63
 - of a series, 28

- upper, 63
- supremum, 16
- supremum metric, 94
- surjection, 10
- surjective function, 10
- symmetric relation, 20
- symmetric set, 61

- target, 9
- Taylor series, 81
- term, 23
- test
 - alternating series, 31
 - comparison, 29
 - integral, 72
 - limit comparison, 30
 - ratio, 30
 - root, 31
 - Weierstrass M-, 76
- theorem
 - algebraic limit, 25, 47
 - Banach fixed point, 104
 - Bolzano-Weierstrass, 33
 - Darboux's, 58
 - Dini's, 88
 - extreme value, 49
 - fundamental, of calculus, 67
 - Heine-Borel, 41
 - intermediate value, 50
 - inverse function, 57
 - Lagrange's remainder, 81
 - mean value, 59
 - for integrals, 68
 - generalised, 60
 - monotone convergence, 27
 - order limit, 26
 - Picard's, 109
 - Rolle's, 59
 - squeeze, 25, 46
 - transcendental number, 22
 - transitive relation, 4, 20
 - triangle inequality, 5, 91
 - trigonometric function, 33, 83–87

 - ultrametric, 93
 - uncountable set, 20
 - uniform metric, 94
 - uniformly continuous function, 49
 - uniformly convergent
 - sequence, 74
 - series, 76
 - union, 6, 8
 - upper
 - bound, 15
 - integral, 64
 - sum, 63
 - value, 9
 - Weierstrass M-test, 76
 - well-ordering property, 1, 22