

QUANTUM MEASUREMENT THEORY AND ITS APPLICATIONS

Recent experimental advances in the control of quantum superconducting circuits, nano-mechanical resonators and photonic crystals have meant that quantum measurement theory is now an indispensable part of the modeling and design of experimental technologies.

This book, aimed at graduate students and researchers in physics, gives a thorough introduction to the basic theory of quantum measurement and many of its important modern applications. Measurement and control is explicitly treated in superconducting circuits and optical and optomechanical systems, and methods for deriving the Hamiltonians of superconducting circuits are introduced in detail. Further applications covered include feedback control, metrology, open systems and thermal environments, Maxwell's demon, and the quantum-to-classical transition.

KURT JACOBS is an Associate Professor of Physics at the University of Massachusetts at Boston. He is a leading researcher in quantum measurement theory and feedback control, and applications in nano-electromechanical systems. He is author of the textbook *Stochastic Processes for Physicists: Understanding Noisy Systems* (Cambridge University Press, 2010).

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To my mother, Sandra Jacobs, for many things.
Not least for the Nelson Southern Link Decision, a great triumph unsung.

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Preface

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While I have endeavored to cite a fairly comprehensive and representative set of research papers on the topics I have covered in this text, it is likely that I have omitted some that

deserve to be included. If you discover that your important paper on topic X has been missed, please send me the reference and I will be glad to correct this omission in any further edition.

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