FEYNMAN DIAGRAM TECHNIQUES IN CONDENSED MATTER PHYSICS

A concise introduction to Feynman diagram techniques, this book shows how they can be applied to the analysis of complex many-particle systems, and offers a review of the essential elements of quantum mechanics, solid-state physics, and statistical mechanics.

Alongside a detailed account of the method of second quantization, the book covers topics such as Green's and correlation functions, diagrammatic techniques, superconductivity, and contains several case studies. Some background knowledge in quantum mechanics, solid-state physics, and mathematical methods of physics is assumed.

Detailed derivations of formulas and in-depth examples and chapter exercises from various areas of condensed matter physics make this a valuable resource for both researchers and advanced undergraduate students in condensed-matter theory, many-body physics, and electrical engineering. Solutions to the exercises are made available online.

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To the memory of
my parents

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Preface

In both theory and practice, condensed matter physics is concerned with the physical properties of materials that are comprised of complex many-particle systems. Modeling the systems' behavior is essential to achieving a better understanding of the properties of these systems and their practical use in technology and industry.

Maximal knowledge about a many-particle system is gained by solving the Schrödinger equation. However, an exact solution of the Schrödinger equation is not possible, so resort is made to approximation schemes based on perturbation theory. It is generally true that, in order to properly describe the properties of an interacting many-particle system, perturbation theory must be carried out to infinite order. The best approach we have for doing so involves the use of Green's function and Feynman diagrams. Furthermore, much of our knowledge about a given complex system is obtained by measuring its response to an external probe, such as an electromagnetic field, a beam of electrons, or some other form of perturbation; its response to this perturbation is best described in terms of Green's function.

Two years ago, I set out to put together a guide that would allow advanced undergraduate and beginning graduate students in physics and electrical engineering to understand how Green's functions and Feynman diagrams are used to more accurately model complicated interactions in condensed matter physics. As time went by and the book was taking form, it became clear that it had turned into a reference manual that would be useful to professionals and educators as well as students. It is a self-contained place to learn or review how Feynman diagrams are used to solve problems in condensed matter physics. Great care has been taken to show how to create them, use them, and solve problems with them, one step at a time. It has been a labor of love. My reward is the thought that it will help others to understand the subject.

The book begins with a brief review of quantum mechanics, followed by a short chapter on single-particle states. Taken together with the accompanying exercises,

these two chapters provide a decent review of quantum mechanics and solid state physics. The method of second quantization, being of crucial importance, is discussed at length in Chapter 3, and applied to the jellium model in Chapter 4. Since Green's functions at finite temperature are defined in terms of thermal averages, a review of the basic elements of statistical mechanics is presented in Chapter 5, which, I hope, will be accessible to readers without extensive knowledge of the subject.

Real-time Green's functions are discussed in Chapter 6, and some applications of these functions are presented in Chapter 7. Imaginary-time functions and Feynman diagram techniques are dealt with in Chapters 8 and 9. Every effort has been made to provide a step-by-step derivation of all the formulas, in as much detail as is necessary. Rules for the creation of the diagrams and their translation into algebraic expressions are clearly delineated. Feynman diagram techniques are then applied to the interacting electron gas in Chapter 10, to electron–phonon and electron–photon interactions in Chapter 11, and to superconductivity in Chapter 12. These techniques are then extended to systems that are not in equilibrium in Chapter 13.

Many exercises are given at the end of each chapter. For the more difficult problems, some guidance is given to allow the reader to arrive at the solution. Solutions to many of the exercises, as well as additional material, will be provided on my website (www.calstatela.edu/faculty/rjishi).

Over the course of the two years that it took me to finish this book, I received help in various ways from many people. In particular, I would like to thank David Guzman for extensive help in preparing this manuscript, and Hamad Alyahyaei for reading the first five chapters. I am indebted to Linda Alviti, who read the whole book and made valuable comments. I am grateful to Professor I. E. Dzyaloshinski for reading Chapter 9 and for his encouraging words. I also want to thank Dr. John Fowler, Dr. Simon Capelin, Antoaneta Ouzounova, Fiona Saunders, Kirsten Bot, and Claire Poole from Cambridge University Press for their help, guidance, and patience. I would also like to express my gratitude to my wife and children for their encouragement and support. Permission to use the quote from Russell's *The Scientific Outlook* (2001) was provided by Taylor and Francis (Routledge). Copyright is owned by Taylor and Francis and The Bertrand Russell Foundation Ltd. Permission to use Gould's quote from *Ever Since Darwin* (1977) was provided by W.W. Norton & Company.

This book is dedicated to the memory of my parents, who, despite adverse conditions, did all they could to provide me with a decent education.

Los Angeles, California

R. A. J.
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