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Historical perspective

The founding fathers of quantum mechanics had already perceived the essence of the difficulties of quantum mechanics; today, after almost a century, the discussions are still lively and, if some very interesting new aspects have emerged, at a deeper level the questions have not changed so much. What is more recent, nevertheless, is a general change of attitude among physicists: until about 1970 or 1980, most physicists thought that the essential questions had been settled, and that "Bohr was right and proved his opponents to be wrong". This was probably a consequence of the famous discussions between Bohr, Einstein, Schrödinger, Heisenberg, Pauli, de Broglie, and others (in particular at the Solvay meetings [1–3], where Bohr's point of view had successfully resisted Einstein's extremely clever attacks). The majority of physicists did not know the details of the arguments. They nevertheless thought that the standard "Copenhagen interpretation" had clearly emerged from the infancy of quantum mechanics as the only sensible attitude for good scientists. This interpretation includes the idea that modern physics must contain indeterminacy as an essential ingredient: it is fundamentally impossible to predict the outcome of single microscopical events; it is impossible to go beyond the formalism of the wave function (or its generalization, the state vector $|\Psi\rangle$) and complete it. For some physicists, the Copenhagen interpretation also includes the difficult notion of "complementarity" – even if it is true that, depending on the context, complementarity comes in many varieties and has been interpreted in many different ways! By and large, the impression of the vast majority was that Bohr had eventually won the debate against Einstein, so that discussing again the foundations of quantum mechanics after these giants was pretentious, passé, and maybe even bad taste.

Nowadays, the attitude of physicists is more open concerning these matters. One first reason is probably that the non-relevance of the "impossibility theorems" put forward by the defenders of the standard interpretation, in particular by Von Neumann [4] , has now been better realized by the scientific community – see [5–7] and [8], as well as the discussion given in [9]). Another reason is, of course, the great impact of the discoveries and ideas of J.S. Bell [6] in 1964. At the beginning of a new century, it is probably fair to say that we are no longer sure that the Copenhagen interpretation is the only possible consistent attitude for physicists – see for instance the doubts expressed by Shimony in [10]. Alternative points of view are considered with interest: theories including additional variables (or "hidden variables"[1]) [11, 12]; modified dynamics of the state vector [7, 13–15] (non-linear and/or stochastic evolution); at the other extreme, we have points of view such as the so-called "many worlds interpretation" (or "many minds interpretation", or "multibranched universe") [16]; more recently, other interpretations such as that of "decoherent histories" [17] have been put forward (this list is non-exhaustive). These interpretations and several others will be discussed in Chapter 10. For a recent review containing many references, see [18], which emphasizes additional variables, but which is also characteristic of the variety of positions among contemporary scientists[2]. See also an older but very interesting debate published in *Physics Today* [19]; another very useful source of older references is the 1971 *American Journal of Physics* "Resource Letter" [20]. But this variety of possible alternative interpretations should not be the source of misunderstandings! It should also be emphasized very clearly that, until now, no new fact whatsoever (or new reasoning) has appeared that has made the Copenhagen interpretation obsolete in any sense.

1.1 Three periods

Three successive periods may be distinguished in the history of the elaboration of the fundamental quantum concepts; they have resulted in the point of view that is called "the Copenhagen interpretation", or "orthodox", or "standard" interpretation. Actually, these terms may group different variants of the general interpretation, as we see in more detail below (in particular in Chapter 10). Here we give only a brief historical summary; we refer the reader who would like to know more about the history of the conceptual development of quantum mechanics to the book of Jammer [21] – see also [22] and [23]. For detailed discussions of fundamental problems in quantum mechanics, one could also read [10, 24, 25] as well as the references contained, or those given in [20].

1.1.1 Prehistory

Planck's name is obviously the first that comes to mind when one thinks about the birth of quantum mechanics: in 1900, he was the one who introduced the famous

[1] As we discuss in more detail in §10.6, we prefer to use the words "additional variables" since they are not hidden, but actually appear directly in the results of measurements.
[2] For instance, the contrast between the titles of [10] and [18] is interesting.

constant h, which now bears his name. His method was phenomenological, and his motivation was actually to explain the properties of the radiation in thermal equilibrium (blackbody radiation) by introducing the notion of finite grains of energy in the calculation of the entropy [26]. Later he interpreted them as resulting from discontinuous exchange between radiation and matter. It is Einstein who, still later (in 1905), took the idea more seriously and really introduced the notion of quantum of light (which would be named "photon" only much later, in 1926 [27]) in order to explain the wavelength dependence of the photoelectric effect – for a general discussion of the many contributions of Einstein to quantum theory, see [28].

One should nevertheless realize that the most important and urgent question at the time was not so much to explain the fine details of the properties of interactions between radiation and matter, or the peculiarities of the blackbody radiation. It was more general: to understand the origin of the stability of atoms, that is of all matter which surrounds us and of which we are made! According to the laws of classical electromagnetism, negatively charged electrons orbiting around a positively charged nucleus should constantly radiate energy, and therefore rapidly fall onto the nucleus. Despite several attempts, explaining why atoms do not collapse but keep fixed sizes was still a complete challenge for physics[3]. One had to wait a little bit longer, until Bohr introduced his celebrated atomic model (1913), to see the appearance of the first ideas allowing the question to be tackled. He proposed the notion of "quantized permitted orbits" for electrons, as well as of "quantum jumps" to describe how they would go from one orbit to another, for instance during radiation emission processes. To be fair, we must concede that these notions have now almost disappeared from modern physics, at least in their initial forms; quantum jumps are replaced by a much more precise and powerful theory of spontaneous emission in quantum electrodynamics. But, on the other hand, one may also see a resurgence of the old quantum jumps in the modern use of the postulate of the wave packet (or state vector) reduction (§1.2.2.a). After Bohr, came Heisenberg, who, in 1925, introduced the theory that is now known as "matrix mechanics"[4], an abstract intellectual construction with a strong philosophical component, sometimes close to positivism; the classical physical quantities are replaced by "observables", corresponding mathematically to matrices, defined by suitable postulates without much help of intuition. Nevertheless, matrix mechanics contained many elements which turned out to be essential building blocks of modern quantum mechanics!

In retrospect, one can be struck by the very abstract and somewhat mysterious character of atomic theory at this period of history; why should electrons obey

[3] For a review of the problem in the context of contemporary quantum mechanics, see [29].
[4] The names of Born and Jordan are also associated with the introduction of this theory, since they immediately made the connexion between Heisenberg's rules of calculation and those of matrices in mathematics.

such rules, which forbid them to leave a restricted class of orbits, as if they were miraculously guided on simple trajectories? What was the origin of these quantum jumps, which were supposed to have no duration at all, so that it would make no sense to ask what were the intermediate states of the electrons during such a jump? Why should matrices appear in physics in such an abstract way, with no apparent relation with the classical description of the motion of a particle? One can guess how relieved physicists probably felt when another point of view emerged, a point of view which looked at the same time much simpler and more in the tradition of the physics of the nineteenth century: the undulatory (or wave) theory.

1.1.2 The undulatory period

The idea of associating a wave with every material particle was first introduced by de Broglie in his thesis (1924) [30]. A few years later (1927), the idea was confirmed experimentally by Davisson and Germer in their famous electron diffraction experiment [31]. For some reason, at that time de Broglie did not proceed much further in the mathematical study of this wave, so that only part of the veil of mystery was raised by him (see for instance the discussion in [32]). It is sometimes said that Debye was the first, after hearing about de Broglie's ideas, to remark that in physics a wave generally has a wave equation: the next step would then be to try and propose an equation for this new wave. The story adds that the remark was made in the presence of Schrödinger, who soon started to work on this program; he successfully and rapidly completed it by proposing the equation which now bears his name, one of the most basic equations of all physics. Amusingly, Debye himself does not seem to have remembered the event. The anecdote may be inaccurate; in fact, different reports about the discovery of this equation have been given and we will probably never know exactly what happened. What remains clear is that the introduction in 1926 of the Schrödinger equation for the wave function[5] [33] is one of the essential milestones in the history of physics. Initially, it allowed one to understand the energy spectrum of the hydrogen atom, but it was soon extended and gave successful predictions for other atoms, then molecules and ions, solids (the theory of bands for instance), etc. It is presently the major basic tool of many branches of modern physics and chemistry.

Conceptually, at the time of its introduction, the undulatory theory was welcomed as an enormous simplification of the new mechanics. This is particularly true because Schrödinger and others (Dirac, Heisenberg) promptly showed how it could be used to recover the predictions of matrix mechanics from more intuitive considerations, using the properties of the newly introduced "wave function" – the solution of the Schrödinger equation. The natural hope was then to extend this

[5] See footnote 11 for the relation between the state vector and the wave function.

success, and to simplify all problems raised by the mechanics of atomic particles: one would replace it by a mechanics of waves, which would be analogous to electromagnetic or sound waves. For instance, Schrödinger initially thought that all particles in the universe looked to us like point particles just because we observe them at a scale which is too large; in fact, they are tiny "wave packets" which remain localized in small regions of space. He had even shown that these wave packets remain small (they do not spread in space) when the system under study is a harmonic oscillator – alas, we now know that this is a very special case; in general, the wave packets constantly spread in space!

1.1.3 Emergence of the Copenhagen interpretation

It did not take long before it became clear that the completely undulatory theory of matter also suffered from very serious difficulties, actually so serious that physicists were soon led to abandon it. A first example of difficulty is provided by a collision between particles, where the Schrödinger wave spreads in all directions, like a circular wave in water stirred by a stone thrown into it; but, in all collision experiments, particles are observed to follow well-defined trajectories and remain localized, going in some precise direction. For instance, every photograph taken in the collision chamber of a particle accelerator shows very clearly that particles never get "diluted" in all space! This stimulated the introduction, by Born in 1926, of the probabilistic interpretation of the wave function [34]: quantum processes are fundamentally non-deterministic; the only thing that can be calculated is probabilities, given by the square of the modulus of the wave function.

Another difficulty arises as soon as one considers systems made of more than one single particle: then, the Schrödinger wave is no longer an ordinary wave since, instead of propagating in normal space, it propagates in the so-called "configuration space" of the system, a space which has $3N$ dimensions for a system made of N particles! For instance, already for the simplest of all atoms, the hydrogen atom, the wave propagates in six dimensions[6]. For a collection of atoms, the dimension grows rapidly, and becomes an astronomical number for the ensemble of atoms contained in a macroscopic sample. Clearly, the new wave was not at all similar to classical waves, which propagate in ordinary space; this deep difference will be a sort of Leitmotiv in this text[7], reappearing under various aspects here and there[8].

[6] This is true if spins are ignored; if they are taken into account, four such waves propagate in six dimensions.

[7] For instance, the non-locality effects occurring with two correlated particles can be seen as a consequence of the fact that the wave function propagates locally, but in a six-dimensional space, while the usual definition of locality refers to ordinary space which has three dimensions.

[8] Quantum mechanics can also be formulated in a way that does not involve the configuration space, but just the ordinary space: the formalism of field operators (sometimes called second quantization, for historical reasons). One can write these operators in a form that is similar to a wave function. Nevertheless, since they are quantum operators, their analogy with a classical field is even less valid.

In passing, it is interesting to notice that the recent observation of the phenomenon of Bose–Einstein condensation in dilute gases [35] can be seen, in a sense, as a sort of realization of the initial hope of Schrödinger: this condensation provides a case where a many-particle matter wave does propagate in ordinary space. Before condensation takes place, we have the usual situation: the gas has to be described by wave functions defined in a huge configuration space. But, when the atoms are completely condensed into a single-particle wave function, they are restricted to a much simpler many-particle state built with the same ordinary wave function, as for a single particle. The matter wave then becomes similar to a classical field with two components (the real part and the imaginary part of the wave function), resembling an ordinary sound wave for instance. This illustrates why, somewhat paradoxically, the "exciting new states of matter" provided by Bose–Einstein condensates are not an example of an extreme quantum situation; in a sense, they are actually more classical than the gases from which they originate (in terms of quantum description, interparticle correlations, etc.). Conceptually, of course, this remains a very special case and does not solve the general problem associated with a naive view of the Schrödinger waves as real waves.

The purely undulatory description of particles has now disappeared from modern quantum mechanics. In addition to Born and Bohr, Heisenberg [36], Jordan [37, 38], Dirac [39] and others played an essential role in the appearance of a new formulation of quantum mechanics [23], where probabilistic and undulatory notions are incorporated in a single complex logical edifice. The probabilistic component is that, when a system undergoes a measurement, the result is fundamentally random; the theory provides only the probabilities of the different possible outcomes. The wave component is that, when no measurements are performed, the Schrödinger equation is valid. The wave function is no longer considered as a direct physical description of the system itself; it is only a mathematical object that provides the probabilities of the different results[9] – we come back to this point in more detail in §1.2.3.

The first version of the Copenhagen interpretation was completed around 1927, the year of the fifth Solvay conference [3]. Almost immediately, theorists started to extend the range of quantum mechanics from particle to fields. At that time, the interest was focussed only on the electromagnetic field, associated with the photon, but the ideas were later generalized to fields associated with a wide range of particles (electrons, muons, quarks, etc.). Quantum field theory has now enormously expanded and become a fundamental tool in particle physics, within a relativistic formalism (the Schrödinger equation itself does not satisfy Lorentz invariance).

[9] In the literature, one often finds the word "ontological" to describe Schrödinger's initial point of view on the wave function, as opposed to "epistemological" to describe the probabilistic interpretation.

A generalization of the ideas of gauge invariance of electromagnetism has led to various forms of gauge theories; some are at the root of our present understanding of the role in physics of the fundamental interactions (electromagnetic, weak, strong[10]) and led to the successful prediction of new particles. Nevertheless, despite all these remarkable successes, field theory remains, conceptually, on the same fundamental level as the theory of a single non-relativistic particle treated with the Schrödinger equation. Since this text is concerned mostly with conceptual issues, we will therefore not discuss field theory further.

1.2 The state vector

Many discussions concerning the foundations of quantum mechanics are related to the status and physical meaning of the state vector. In §§1.2.1 and 1.2.2, we begin by first recalling its definition and use in quantum mechanics (the reader familiar with the quantum formalism might wish to skip these two sections); then, in §1.2.3, we discuss the status of the state vector in standard quantum mechanics.

1.2.1 Definition, Schrödinger evolution, Born rule

We briefly summarize how the state vector is used in quantum mechanics and its equations; more details are given in §11.1.1 and following.

1.2.1.a Definition

Consider a physical system made of N particles with mass, each propagating in ordinary space with three dimensions; the state vector $|\Psi\rangle$ (or the associated wave function[11]) replaces in quantum mechanics the N positions and N velocities which, in classical mechanics, would be used to describe the state of the system. It is often convenient to group all these positions and velocities within the $6N$ components of a single vector $\mathbf{V}$ belonging to a real vector space with $6N$ dimensions, called "phase space"[12]; formally, one can merely consider that the state vector $|\Psi\rangle$ is the quantum equivalent of this classical vector $\mathbf{V}$. It nevertheless belongs to a space that is completely different from the phase space, a complex vector space called "space of states" (or, sometimes, the "Hilbert space" for historical reasons) with infinite dimension. The calculations in this space are often made with the help of

[10] There is a fourth fundamental interaction in physics, gravitation. The "standard model" of field theory unifies the first three interactions, but leaves gravitation aside. Other theories unify the four fundamental interactions, but for the moment they are not considered standard.

[11] For a system of spinless particles with masses, the state vector $|\Psi\rangle$ is equivalent to a wave function, but for more complicated systems this is not the case. Nevertheless, conceptually they play the same role and are used in the same way in the theory, so that we do not need to make a distinction here.

[12] The phase space therefore has twice as many dimensions as the configuration space mentioned above.

the Dirac notation [39], which actually we will use here, and where the vectors belonging to the space of states are often called "kets".

Because the state vector belongs to a linear space, any combination of two arbitrary state vectors $|\Psi_1\rangle$ and $|\Psi_2\rangle$ belonging to the space of states:

$$|\Psi\rangle = \alpha\,|\Psi_1\rangle + \beta\,|\Psi_2\rangle \tag{1.1}$$

(where α and β are arbitrary complex numbers) is also a possible state for the system. This is called the "superposition principle" of quantum mechanics, and has many consequences.

Moreover, to each physical observable of the system, position(s), momentum(ta), energy, angular momentum, etc., the formalism of quantum mechanics associates a linear operator acting in the space of states, and provides rules for constructing these operators. For historical reasons (§1.1.1), each of these operators is often called "observable"; they belong to the category of mathematical operators called "linear Hermitian operators".

1.2.1.b Schrödinger evolution

The evolution of the state vector $|\Psi(t)\rangle$ between time t_0 and t_1 is given by the Schrödinger equation:

$$i\hbar\frac{d}{dt}\,|\Psi(t)\rangle = H(t)\,|\Psi(t)\rangle \tag{1.2}$$

where $H(t)$ is the Hamiltonian evolution of the system (including the internal interactions of this system as well as the effects of classical external fields applied to it, for instance static or time-dependent magnetic fields). The Schrödinger equation is a linear differential equation, similar to many other such equations in physics. It leads to a progressive evolution of the state vector, without any quantum jump or discontinuity. It is as general as the Newton or Lagrange equations in classical mechanics, and can be applied to all physical situations, provided of course the system is well defined with a known Hamiltonian.

In particular, the Schrödinger equation can also be applied to a situation where the physical system interacts with a measurement apparatus (a spin 1/2 particle entering the magnetic field gradient created by a Stern–Gerlach apparatus for instance); it then does not select precise experimental results, but keeps all of them as potentialities (within a so-called "coherent superposition"). One more ingredient is then introduced into the theory, the Born probability rule.

1.2.1.c Born probability rule

We assume that, at time t_1, when the solution $|\Psi(t)\rangle$ of equation (1.2) takes the value $|\Psi(t_1)\rangle$, the system undergoes a measurement, associated with an operator

M (observable) acting in the space of states. We note $| m_i >$ the eigenvectors of M associated with eigenvalues m_i $(i = 1, 2, \ldots)$; if some eigenvalues are degenerate, several consecutive values in the series of m_i are equal, but associated with different vectors $| m_i >$. Since M is an Hermitian operator, the $| m_i >$ can be chosen as an orthonormal basis of the space of states.

The Born probability rule then states that, in an ideal measurement:

(i) the result of a measurement associated with M can only be one of the m_i; other results are never obtained.

(ii) if a particular eigenvalue m_i is non-degenerate, the probability $\mathcal{P}_i$ of obtaining result m_i is given by the square modulus of the scalar product of $|\Psi(t_1)\rangle$ by $|m_i\rangle$:

$$\mathcal{P}_i = |\langle m_i |\Psi(t_1)\rangle|^2 \tag{1.3}$$

(iii) the probability of measuring a degenerate eigenvalue is the sum of the probabilities (1.3) corresponding to all the orthonormal eigenvectors associated with this eigenvalue[13].

Rules (ii) and (iii) may be grouped in a simple form, which will be useful in what follows. If the result corresponds to an eigenvalue m that is p times degenerate, the series of p numbers $m_i, m_{i+1}, \ldots, m_{i+p}$ have the same value m. We can then introduce the sum of the projectors (§11.1.3) over the corresponding eigenvectors:

$$P_M(m) = |m_i\rangle\langle m_i| + |m_{i+1}\rangle\langle m_{i+1}| + \ldots + |m_{i+p}\rangle\langle m_{i+p}| \tag{1.4}$$

This operator is also a projector (it is equal to its square), which can be applied to the state vector $|\Psi(t_1)\rangle$ before the measurement:

$$P_M(m)\left|\Psi(t_1)\right\rangle = \left|\Psi'_m\right\rangle \tag{1.5}$$

The probability of obtaining result m in the measurement is then nothing but the square of the norm of $\left|\Psi'\right\rangle$:

$$\mathcal{P}_m = \left\langle \Psi'_m \middle| \Psi'_m \right\rangle = \left\langle \Psi(t_1)\right| P_M(m) \left|\Psi(t_1)\right\rangle \tag{1.6}$$

1.2.2 Measurement processes

The standard interpretation of quantum mechanics contains the progressive, deterministic, evolution of the wave function/state due to the Schrödinger equation. Usually, one also includes in this interpretation a second postulate of evolution;

[13] Similarly, in the classical theory of probabilities, if an event E can be obtained either as event e_1, or e_2, …, or e_i, …, and if all events e_i are exclusive, the probability of E is the sum of the probabilities of the e_i.

this postulate is associated with the process of measurement, and completely different from the Schrödinger evolution since it is discontinuous. It is often called the "wave packet reduction", or "wave function collapse", or again "state vector reduction", and was introduced by Von Neumann in his famous treatise (Chapter VI of [4]). This is the version found in most textbooks. But Bohr himself preferred another point view where state vector reduction is not used[14]; we discuss this point of view afterwards (there exist also other interpretations of quantum mechanics that do not make use of state vector reduction, as discussed in Chapter 10; see for instance §§10.1.2, 10.6, or 10.11).

1.2.2.a Von Neumann, reduction (collapse)

Suppose now that the system we study is prepared at time t_0, evolves freely (without being measured) until time t_1 where it undergoes a first measurement, and then evolves freely again until time t_2 where a second measurement is performed. Just after the first measurement at time t_1, when the corresponding result of measurement is known, it is very natural to consider that both the initial preparation and the first measurement are part of a single preparation process of the system. One then associates to this preparation a state vector that includes the information of the first result; this is precisely what the state vector reduction (or state collapse) postulate does. The new "reduced" state vector can then be used as an initial state to calculate the probabilities of the different results corresponding to the second measurement, at time t_2.

Dirac also takes this point of view when he writes (page 9 of "Quantum mechanics" [39]): "There are, however, two cases when we are in general obliged to consider the disturbance as causing a change in state of the system, namely, when the disturbance is an observation and when it consists in preparing the system so as to be in a given state".

We assume that the measurement is ideal[15] – it preserves the integrity of the system, as opposed to destructive measurements such as the absorption of a photon in a detector. Then, after the measurement associated to M has provided result m_i corresponding to a non-degenerate eigenvalue (and therefore to a single $|m_i\rangle$), the reduced state vector is:

$$\left|\Psi'_{m_i}\right\rangle = \left|m_i\right\rangle \tag{1.7}$$

[14] As stated in [40]: "Most importantly, Bohr's complementarity interpretation makes no mention of wave packet collapse ... or a privileged role for the subjective consciousness of the observer. Bohr was also in no way a positivist. Much of what passes for the Copenhagen interpretation is found in the writings of Werner Heisenberg, but not in Bohr".

[15] We come back in more detail on the Von Neumann model of measurement in §8.1.1 and on the notion of QND (quantum non-demolition) measurement.