CAMBRIDGE TRACTS IN MATHEMATICS

General Editors

B. BOLLOBÁS, W. FULTON, A. KATOK,
F. KIRWAN, P. SARNAK, B. SIMON, B. TOTARO

---

195 A Universal Construction for Groups Acting Freely on Real Trees

CAMBRIDGE TRACTS IN MATHEMATICS

A complete list of books in the series can be found at www.cambridge.org/mathematics.
Recent titles include the following:

166. The Lévy Laplacian. By M. N. Feller
167. Poincaré Duality Algebras, Macaulay's Dual Systems, and Steenrod Operations. By D. Meyer and L. Smith
168. The Cube-A Window to Convex and Discrete Geometry. By C. Zong
169. Quantum Stochastic Processes and Noncommutative Geometry. By K. B. Sinha and D. Goswami
170. Polynomials and Vanishing Cycles. By M. Tibăr
171. Orbifolds and Stringy Topology. By A. Adem, J. Leida, and Y. Ruan
172. Rigid Cohomology. By B. Le Stum
173. Enumeration of Finite Groups. By S. R. Blackburn, P. M. Neumann, and G. Venkataraman
174. Forcing Idealized. By J. Zapletal
175. The Large Sieve and its Applications. By E. Kowalski
176. The Monster Group and Majorana Involutions. By A. A. Ivanov
177. A Higher-Dimensional Sieve Method. By H. G. Diamond, H. Halberstam, and W. F. Galway
178. Analysis in Positive Characteristic. By A. N. Kochubei
179. Dynamics of Linear Operators. By F. Bayart and É. Matheron
180. Synthetic Geometry of Manifolds. By A. Kock
181. Totally Positive Matrices. By A. Pinkus
182. Nonlinear Markov Processes and Kinetic Equations. By V. N. Kolokoltsov
183. Period Domains over Finite and p-adic Fields. By J.-F. Dat, S. Orlik, and M. Rapoport
184. Algebraic Theories. By J. Adámek, J. Rosický, and E. M. Vitale
185. Rigidity in Higher Rank Abelian Group Actions I: Introduction and Cocycle Problem. By A. Katok and V. Niţică
186. Dimensions, Embeddings, and Attractors. By J. C. Robinson
187. Convexity: An Analytic Viewpoint. By B. Simon
188. Modern Approaches to the Invariant Subspace Problem. By I. Chalendar and J. R. Partington
189. Nonlinear Perron–Frobenius Theory. By B. Lemmens and R. Nussbaum
190. Jordan Structures in Geometry and Analysis. By C.-H. Chu
191. Malliavin Calculus for Lévy Processes and Infinite-Dimensional Brownian Motion. By H. Osswald
192. Normal Approximations with Malliavin Calculus. By I. Nourdin and G. Peccati
193. Distribution Modulo One and Diophantine Approximation. By Y. Bugeaud
194. Mathematics of Two-Dimensional Turbulence. By S. Kuksin and A. Shirikyan
195. A Universal Construction for R-free Groups. By I. Chiswell and T. Müller
196. The Theory of Hardy's Z-Function. By A. Ivić
197. Induced Representations of Locally Compact Groups. By E. Kaniuth and K. F. Taylor
198. Topics in Critical Point Theory. By K. Perera and M. Schechter
199. Combinatorics of Minuscule Representations. By R. M. Green
200. Singularities of the Minimal Model Program. By J. Kollár

A Universal Construction for Groups Acting Freely on Real Trees

IAN CHISWELL
Queen Mary, University of London

THOMAS MÜLLER
Queen Mary, University of London

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment,
a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of
education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107024816

First published 2012

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-02481-6 Hardback

Dedicated
to the memory of
KARL W. GRUENBERG
1928–2007

Contents

Preface

In summer 2004, V. N. Remeslennikov, during a visit to Queen Mary and Westfield College, gave a series of three talks in which he outlined the construction of a class of groups $\mathscr{R}\mathscr{F}(G)$, starting from the collection of (set-theoretic) functions $f : [0,\alpha] \to G$, where α is any non-negative real number and G is a given (discrete) group. Apparently, his main motivation was to imitate the construction of free groups in a continuous setting. He indicated that these new groups would have natural $\mathbb{R}$-tree actions associated with them, and he pointed out that it might be possible to study the centraliser of a hyperbolic element f in terms of (suitably defined) periods of f. However, no proofs were given.

Nevertheless, the picture emerging was felt to be interesting; the authors set out to try to fill in missing proofs, at first with the modest aim of establishing that the construction really produced groups. This task alone turned out to be rather difficult, leading to the development of a substantial body of cancellation theory (as given in Chapter 2 of the present book) before the actual proof that 'reduced multiplication' was associative could be given. By the time this task was accomplished (more than half a year later), the authors were already absorbed in what turned out to be a difficult but ultimately rewarding theory.

Now, several years further on, we present the fruits of our labour. To mention just a few highlights: the bounded subgroups of $\mathscr{R}\mathscr{F}(G)$ are determined; it is shown that $\mathscr{R}\mathscr{F}(G)$ (if non-trivial) is not generated by its elliptic elements and that the quotient of $\mathscr{R}\mathscr{F}(G)$ by the span $E(G)$ of the elliptic elements has an isomorphism type depending at most on two cardinal numbers, the number of involutions in G as well as the cardinality of its complement. Moreover, the conjugacy relation for hyperbolic elements is characterised, thereby yielding a continuous analogue of the classical conjugacy theorem for free groups; cf. Theorem 1.3 in Magnus, Karrass, and Solitar [31].

Also, Remeslennikov's prediction concerning the centralisers of hyperbolic elements ultimately turns out to be substantially true, with some modification, but to prove this involves a considerable amount of work. Further, the last section of Chapter 10 contains the beginnings of a structure theory for $\mathscr{R}\mathscr{F}(G)$ and its quotient $\mathscr{R}\mathscr{F}(G)/E(G)$, while Chapter 4 explains our recent finding that $\mathscr{R}\mathscr{F}$-groups and their associated $\mathbb{R}$-trees are universal (with respect to inclusion) for free $\mathbb{R}$-tree actions.

Something has been accomplished, yet much remains to be done. Nevertheless, as far as the case $\Lambda = \mathbb{R}$ is concerned the theory is beginning to shape nicely, despite the fact that there are still a large number of open problems (see Appendix B for a sample); thus it seemed a good idea, and the right time, to present our findings obtained so far in the hope of stimulating further research in what the authors feel is an exciting new area.