

Microhydrodynamics, Brownian Motion, and Complex Fluids

This text is an introduction to the dynamics of fluids at small scales, the physical and mathematical underpinnings of Brownian motion, and the application of these subjects to the dynamics and flow of complex fluids such as colloidal suspensions and polymer solutions. It brings together continuum mechanics, statistical mechanics, polymer and colloid science, and various branches of applied mathematics, in a self-contained and integrated treatment that provides a foundation for understanding complex fluids, with a strong emphasis on fluid dynamics.

Students and researchers will find that this book is extensively cross-referenced to illustrate connections between different aspects of the field. Its focus on fundamental principles and theoretical approaches provides the necessary groundwork for research in the dynamics of flowing complex fluids.

MICHAEL D. GRAHAM is the Vilas Distinguished Achievement Professor and Harvey D. Spangler Professor of Chemical and Biological Engineering at the University of Wisconsin–Madison. His research focuses on theoretical and computational studies of the fluid dynamics of complex fluids. Among his recognitions are a CAREER Award from the National Science Foundation (NSF), the François Frenkiel and Stanley Corrsin Awards from the American Physical Society Division of Fluid Dynamics, and the Kellett Mid-Career Award at UW–Madison. He has served as associate editor of the *Journal of Fluid Mechanics* and editor-in-chief of the *Journal of Non-Newtonian Fluid Mechanics*. He is coauthor of the textbook *Modeling and Analysis Principles for Chemical and Biological Engineers*.

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978-1-107-02464-9 — Microhydrodynamics, Brownian Motion, and Complex Fluids

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MICHAEL D. GRAHAM

University of Wisconsin–Madison



CAMBRIDGE
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University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India
79 Anson Road, #06–04/06, Singapore 079906

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www.cambridge.org

Information on this title: www.cambridge.org/9781107024649

DOI: 10.1017/9781139175876

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First published 2018

Printed and bound in Great Britain by Clays Ltd, Elcograf S.p.A.

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-02464-9 Hardback

ISBN 978-1-107-69593-1 Paperback

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To Mary Ellen, Nathaniel and Jacob

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