

### The Physics of Dilute Magnetic Alloys

Available for the first time in English, this classic text by Jun Kondo describes the Kondo effect thoroughly and intuitively. Its clear and concise treatment makes this book of interest to graduate students and researchers in condensed matter physics.

The first half of the book describes the rudiments of the theory of metals at a level that is accessible for undergraduate students. The second half discusses key developments in the Kondo problem, covering topics including magnetic impurities in metals, the resistance minimum phenomenon, infrared divergence in metals and scaling theory, including Wilson's renormalization group treatment and the exact solution by the Bethe ansatz. A new chapter has been added covering advances made since the Japanese edition was published, such as the quantum dot and heavy fermion systems.

JUN KONDO is Emeritus Professor of Toho University and Special Advisor at the National Institute for Advanced Industrial Science and Technology (AIST), Japan, and a Member of the Japan Academy. He is well known for solving the problem of the resistance minimum phenomenon, now known as the Kondo effect. Theoretical and experimental studies in this field continue today, and new applications for the theory are still being found.





# The Physics of Dilute Magnetic Alloys

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CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9781107024182

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First published 2012

Printed in the United Kingdom at the University Press, Cambridge

A catalog record for this publication is available from the British Library

Library of Congress Cataloging in Publication data Kondo, Jun, 1930–

The physics of dilute magnetic alloys / Jun Kondo ; translated by Shigeru Koikegami ... [et al.].

p. cm.

Includes bibliographical references and index. ISBN 978-1-107-02418-2 (hardback)

Dilute alloys.
 Magnetic alloys.
 Kondo effect.
 Free electron theory of metals.
 Title.
 QC176.8.M3K66
 2012

QC176.8.M3K66 2012 538'.4-dc23 2012014479

ISBN 978-1-107-02418-2 Hardback

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### **Preface**

I wrote this book as an introduction to the theory of electrons in metals. There are a good many texts on this topic, and the emphasis of this present book is in discussing the physics of dilute magnetic alloys. The first half of this book is devoted to discussion of the topics that are necessary for the discussions in the later chapters. Recent activity in the theory of dilute magnetic alloys has made the field highly complex, so I have tried to describe this at a level that is suitable for those who are new to the subject area.

While metals are characterized by the presence of electrons that move about freely in them, it is most important that, unlike in semi-conductors, there is huge electron density. As a result, quantum effects become predominant and, because electrons are fermions, the phenomenon of degeneracy takes place. We may even go so far as to say that almost all of the characteristic behavior of metals is due to this phenomenon of degeneracy.

Concerning the quantum theory of electrons in metals, there have been five major developments. The first is the Sommerfeld theory, which introduced the concept of degeneracy to explain the behavior of the electronic specific heat. The second is the Bohm–Pines theory, which discusses the effect of the inter electronic Coulomb interaction, together with the many-body treatment of the problem which emerged from this theory. Next, the sensitivity of the magnetic resistance and the de Haas–van Alphen effect to the shape of the Fermi surface became the motivation for the development that occurred in the field of fermiology. As a result, the Fermi surface came to be called the 'face' of the metal.

The most important development in the field of the theory of electrons in metals has been the Bardeen–Cooper–Schrieffer (BCS) theory of superconductivity. We may say that this meant the discovery of a new state of matter, and furthermore it considerably influenced other areas of physics.



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Fifth, and finally, we have the problem of magnetic alloys. This problem began with the old discovery, in the 1930s, of the electric resistance minimum effect. When metals are cooled down to liquid helium temperatures, at first, the electric resistance goes down. However, below  $\sim \! \! 10 \, \mathrm{K}$ , an increase in the resistivity was often observed. It was established that this was due to the mixing of a small number of magnetic atoms. However, the relationship between the magnetism due to the impurity atoms and the increase in the electric resistivity was unclear.

In the end, we came to the understanding of this problem as being due to the magnetic moment which affects the degenerate electrons, but the effort of many was necessary for the complete solution to this problem. A system composed of electrons in metals and magnetic moments may seem simple at first, but the theoretical treatment of this problem is quite involved, and commonplace methods of approximation are not the best way of tackling the core of the problem. Many approaches blossomed in this flowerbed, supported by the stem of the combined intellect of many, nurturing an especially fruitful area of physics which also affected the development of other areas.

The aim and goal of this book is to showcase this theoretical fruit, which will be found in Chapters 5 and beyond. The first four chapters can be thought of as preparatory introductory material. Chapters 1 and 2 discuss the more elementary topics in the description of many-electron systems. Chapters 3 and 4 move on to the elementary topics in the description of metals. We start the discussion of magnetic alloys in Chapter 5. In my opinion, the discussions up to this chapter are fairly simple. Chapters 6 and beyond turn to a more complex content, but I took care to keep the discussion thorough. Thus I believe that those who are new to the subject area may also understand the full content of this book upon careful study. Nonetheless, the author's incapabilities may have encumbered his intentions. I very much hope that readers will be forgiving in this respect.

Finally, my sincere gratitude goes to Professor Yukihito Tanabe of the University of Tokyo, who encouraged me to write this book; Mr. Kyohei Endo and Mr. Saneatsu Makiya of Shokabo; and Ms. Masako Kobayashi who helped me especially in the proofreading.

Jun Kondo October 1983



## Translators' foreword

This present volume is a translation of "kinzoku denshi-ron – jisei-gokin wo chushin to shite" ("Theory of electrons in metals – with emphasis on magnetic alloys"), written by Professor J. Kondo in Japanese and published by Shokabo in 1983. The translation contains an additional chapter which discusses some of the developments that have taken place since the original publication of the book. The title of the book could very well have been "The Kondo effect", had the author not been Professor Kondo himself. The discussion of the Kondo effect takes the prime position in this book, though the author never refers to it as such!

The author and his theory need no introduction. Suffice it to say that his work has been a milestone in condensed matter physics, with far-reaching consequences such as those in the study of many-body problems in general. But not only that. The Kondo effect also marked the beginning of the concept of asymptotic freedom, where the relevant coupling strength increases logarithmically with decreasing energy/temperature scale. This phenomenon is of central importance in the physics of strong interaction, in particle and nuclear physics, which is now believed to be described by quantum chromo-dynamics (QCD). We should add that there has been renewed interest in the study of the Kondo effect in the context of heavy electron systems and quantum dots.

Professor Kondo's famous work was carried out in the 1960s, in what has now become a central block of the National Institute of Advanced Industrial Science and Technology (AIST) in Tsukuba, Japan, and was then called the Electro-technical Laboratory or ETL and situated in Tanashi, Japan. This is the laboratory in which the translators are presently based, and Professor Kondo has been a mentor to all four of us.

In the preparation of the initial manuscript, the workload was split as follows. K. Y. produced the initial manuscript of the first four chapters. S. K. took responsibility for Chapters 5, 7, 8 and the Appendix. T. Y. worked on



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Chapter 6. The new chapter on recent developments was written by Professor Kondo, and this was translated by K. O. The manuscript was then wholly and thoroughly revised by T. Y. and K. O. for the language and physics; on the whole we believe that we have been able to maintain the original style of exposition.

As stated by the author in the Preface, the book is written at a level that is suitable for newcomers to the field, and the Japanese language version of the book indeed exemplifies his lucid style of exposition. Nonetheless, we benefited from numerous discussions with Professor Kondo, who would willingly, but ever unassumingly, respond to our queries concerning the more intricate points.

We would like to thank Tatsuya Ono of Shokabo, Lindsay Barnes, Simon Capelin, Graham Hart, Caroline Mowatt, Mairi Sutherland and Antoaneta Ouzounova of Cambridge University Press, for their help and generosity during the preparation of the manuscript and the publication of the book. We would also like to thank Drs. Izumi Hase, Naoki Shirakawa and all of our other colleagues at AIST for discussions, help and encouragement.

S.K.

K.O.

K.Y.

T. Y.

October 2011