Wave Theory of Information

Understand the relationship between information theory and the physics of wave propagation with this expert guide. Balancing fundamental theory with engineering applications, it describes the mechanism and limits for the representation and communication of information using electromagnetic waves. Information-theoretic laws relating functional approximation and quantum uncertainty principles to entropy, capacity, mutual information, rate–distortion, and degrees of freedom of bandlimited radiation are derived and explained. Both stochastic and deterministic approaches are explored, and applications for remote sensing and signal reconstruction, wireless communication, and networks of multiple transmitters and receivers are reviewed. With end-of-chapter exercises and suggestions for further reading enabling in-depth understanding of key concepts, it is the ideal resource for researchers and graduate students in electrical engineering, physics, and applied mathematics looking for a fresh perspective on information theory.

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"This is an excellent textbook that ties together information theory and wave theory in a very insightful and understandable way. It is of great value and highly recommended for students, researchers and practitioners. Professor Franceschetti brings a highly valuable textbook based on many years of teaching and research."
Charles Elachi, ***California Institute of Technology and Director Emeritus of the Jet Propulsion Laboratory (NASA)***

"This book is about the physics of information and communication. It could be considered to be an exposition of Shannon information theory, where information is transmitted via electromagnetic waves. Surely Shannon would approve of it."
Sanjoy K. Mitter, ***Massachusetts Institute of Technology***

"Communication and information are inherently physical. Most of the literature, however, abstracts out the physics, treating them as mathematical or engineering disciplines. Although abstractions are necessary in the design of systems, much is lost in understanding the fundamental limits and how these disciplines fit together with the underlying physics. Franceschetti breaks the disciplinary boundaries, presenting communication and information as physical phenomena in a coherent, mathematically sophisticated, and lucid manner."
Abbas El Gamal, ***Stanford University***

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About the Cover

The picture represents the electromagnetic emission from stellar dust pervading our galaxy, measured by the Planck satellite of the European Space Agency. The colors represent the intensity, while the texture reflects the orientation of the field. The intensity of radiation peaks along the galactic plane, at the center of the image, where the field is aligned along almost parallel lines following the spiral structure of the Milky Way. Cloud formations are visible immediately above and below the plane, where the field's structure becomes less regular. The emission carries information regarding the evolution of our galaxy, as the turbulent structure of the field is related to the processes taking place when stars are born.

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To my wife Isabella,
opera in my head.

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Preface

Claude Elwood Shannon, the giant who ignited the digital revolution, is the father of information theory and a hero for many engineers and scientists. There are many excellent textbooks describing the many facets of his work, so why add another one? The ambitious goal is to provide a completely different perspective. The writing reflects my desire to abhor duplication and to attempt to break through the compartmentalized walls of several disciplines. Rather than copying a Picasso, I have tried to frame it and place it in a broader context.

The motivation also came from my experience as a teacher. The Electrical and Computer Engineering Department of the University of California at San Diego, in the spotlight of its annual workshop on information theory and applications, attracting several hundred participants from around the world, may be considered a holy destination for graduate students in information theory. Many gifted young minds join our department every year with the ultimate goal of earning a PhD in this venerable subject. Here, thanks to the work of many esteemed colleagues, they can become experts in coding and communication theories, point-to-point and network information theories, and wired and wireless information systems. Over my years of teaching, however, I have noticed that sometimes students are missing the master plan for how these topics are tied together and how are they related to the fundamental sciences. Some questions that may catch them off guard are: How much information can be radiated by a waveform at the most fundamental level? How is the physical entropy related to the information-theoretic limits of communication? How does the energy and the quantum nature of radiation limit information? How is information theory related to other branches of mathematics besides probability theory, like functional analysis and approximation theory? On top of these, there is the overarching question, of paramount importance for the engineer, of how communication technologies are influenced by fundamental limits in a practical setting. To fill these gaps, this book focuses on information theory from the point of view of wave theory, and describes connections with different branches of physics and mathematics.

David Hilbert, studying functional representations in terms of orthogonal basis sets in early twentieth-century Germany, contributed to underpinning the mathematical concept of information associated with a waveform. After Shannon's breakthrough work, his approach was later followed by the Soviet mathematician Andrey Kolmogorov, who developed information-theoretic concepts in a purely deterministic setting, and by Shannon's colleagues at Bell Laboratories: Henry Landau, Henry Pollack, and David

Slepian. Their mathematical works are the basis of the wave theory of information presented in this book. We expand upon them, and place them in the context of communication with electromagnetic waves.

From the physics perspective, much has been written on the relationship between thermodynamics and information theory. Parallels between the statistical mechanics of Boltzmann and Gibbs and Shannon's definition of entropy led to many important advancements in the analysis and design of complex engineering systems. Once again, *repetita iuvant, sed continuata secant*. We briefly touch upon these topics, but focus on the less beaten path, uncovering the relationship between information theory and the physics of Heisenberg, Maxwell, and Planck. We describe how information physically propagates using waves, and what the limitations are for this process. What was first addressed in the pioneering works of Dennis Gabor and Giuliano Toraldo di Francia is revisited here in the rigorous setting of the theory of functional approximation. Using these tools we provide, for the first time in a book, a complete derivation of the information-theoretic notion of degrees of freedom of a wave starting from the Maxwell equations, and relate it to the concept of entropy, and to the principles of quantized radiation and of quantum indeterminacy. We also provide analogous derivations for stochastic processes and discuss communication technologies from the point of view of functional representations, which turns out to be very useful to uncover the core architectural ideas fundamental to communication systems. When these are viewed in the context of physical limits, one realizes that there still is "plenty of room at the bottom." Engineers are far from reaching the limits that nature imposes on communication: our students have a bright future in front of them!

Now, a word on style and organization. Although the treatment requires a great deal of mathematics and assumes that the reader has some familiarity with probability theory, stochastic processes, and real analysis, this is not a mathematics book. From the outset, I have made the decision to avoid writing a text as a sequence of theorems and proofs. Instead, I focus on describing the ideas that are behind the results, the relevant mathematical techniques, and the philosophy behind their arguments. When not given, rigorous proofs should follow easily once these basic concepts are grasped. This approach is also reflected in the small set of exercises provided at the end of each chapter. They are designed to complement the text, and to provide a more in-depth understanding of the material. When given, solutions are often sketchy, emphasize intuition over rigor, and encourage the reader to fill in the details. Pointers to research papers for further reading are also provided at the end of each chapter.

A grouping into an introductory sequence of topics (Chapters 1–6), central results (Chapters 8 and 9), and an in-depth sequence (Chapters 10–13) is the most natural for using the book to teach a two-quarter graduate course. The demarcation line between these topics can be somewhat shifted, based on the taste of the instructor. The book could also be used in a one-semester course with a selection of the in-depth topics, and limiting the exposition of some of the details of the central results. Within this organization, Chapter 7 is an *intermezzo*, focusing on wireless communication technologies, and on how they exploit information-theoretic representations. Of course, this can only scratch the surface of a large field of study, and the interested reader should refer to the wide range of literature for a more in-depth account. A *tour d'horizon* of the book's content is provided at the end of the first chapter.

The material has been tested over the course of seven years in the annual graduate course I have taught at the University of California at San Diego. I wish to thank the many students who attended the course and provided feedback on the lecture notes, which were early incarnations of this book, especially the students in my research group, Taehyung Jay Lim and Hamed Omidvar, who read many sections in detail and provided invaluable comments. I also enjoyed interactions with my colleague Young-Han Kim, who read parts of the manuscript and provided detailed feedback. The presentation of blind sensing and compressed sensing in Chapter 3 has been enriched by conversations with my colleague Rayan Saab. Recurrent visits to the group led by Bernard Fleury at Aalborg University, Denmark, influenced the presentation of the material on stochastic models and their relationship to communication systems presented in Chapters 6 and 10. Sergio Verdú of Princeton University kindly offered some stylistic suggestions to improve the presentation of the material in Chapters 1 and 12. Many exchanges with Edward Lee of the University of California at Berkeley and with my colleague George Papen on the physical meaning of information helped to shape the presentation in Chapter 13. Interactions with Sanjoy Mitter of the Massachusetts Institute of Technology also stimulated many of the physical questions addressed in the book. My editors Phil Meyler and Julie Lancashire at Cambridge University Press were very patient with my eternal postponement of manuscript delivery, and provided excellent professional advice throughout.

A final "thank you" goes to my family, who patiently accepted, with "minimal" complaint, my lack of presence, due to the long retreats in my downstairs hideout.

Massimo Franceschetti

Notation

Asymptotics

$f(x) \sim g(x)$ as $x \to x_0$	$\iff \lim_{x\to x_0} f(x)/g(x) = 1$
$f(x) = o(g(x))$ as $x \to x_0$	$\iff \lim_{x\to x_0} f(x)/g(x) = 0$
$f(x) = O(g(x))$ as $x \to x_0$	$\iff \lim_{x\to x_0} \lvert f(x)/g(x) \rvert < \infty$

Approximations

$f(x) \simeq g(x)$	$g(x)$ is a finite-degree Taylor polynomial of $f(x)$
$f(x) \approx g(x)$	$f(x)$ is approximately equal to $g(x)$ in some numerical sense
$f(x) \gg g(x)$	$f(x)$ is much greater than $g(x)$ in some numerical sense
$f(x) \ll g(x)$	$f(x)$ is much smaller than $g(x)$ in some numerical sense

Domains

$t \in \mathbb{R}$	time
$\omega \in \mathbb{R}$	angular frequency
$\mathbf{r} \in \mathbb{R}^3$	spatial
$\mathbf{k} \in \mathbb{R}^3$	wavenumber
$\phi \in [0, 2\pi]$	angular
$\lambda \in \mathbb{R}^+$	wavelength
$w \in \mathbb{R}^+$	scalar wavenumber
$f(t) \leftrightarrow F(\omega)$	time–angular frequency Fourier transform pairs
$f(\mathbf{r}) \leftrightarrow \widehat{f}(\mathbf{k})$	space–wavenumber Fourier transform pairs
$f(\phi) \leftrightarrow \widehat{f}(w)$	angle–wavenumber Fourier transform pairs

Signals

Ω	angular frequency bandwidth
W	wavenumber bandwidth
$\operatorname{sinc}(t)$	waveform $(\sin t)/t$
$\operatorname{rect}(t/T)$	rectangular waveform of support T and unitary amplitude
$U(t)$	Heaviside's step function: $U(t) = 0$ for $t < 0$, $U(t) = 1$ for $x \geq 0$
$\delta(t)$	Dirac's impulse distribution

Complex Numbers

j	imaginary unit
$f^*(\cdot)$	conjugate of complex signal f
$\Re f(\cdot)$	real part of complex signal f
$\Im f(\cdot)$	imaginary part of complex signal f
$\mathrm{M}^\dagger$	conjugate transpose of matrix M

Functional Spaces

L^2	square-integrable signals
$\mathscr{B}_\Omega$	bandlimited signals of spectral support $[-\Omega, \Omega]$
$\mathscr{T}_T$	timelimited signals of time support $[-T/2, T/2]$
N_0	Nyquist number, $N_0 = \Omega T/\pi$
$\alpha^2(T)$	fraction of a signal's energy in $[-T/2, T/2]$
$\beta^2(\Omega)$	fraction of a signal's energy in $[-\Omega, \Omega]$
$\mathscr{E}(\epsilon_T)$	the set of ϵ_T-concentrated, bandlimited signals
$\|\cdot\|$	norm
$\langle\cdot\rangle$	inner product
$\mathscr{S}_n \subset \mathscr{S}$	an n-dimensional subspace of the space $\mathscr{S}$
$D_{\mathscr{S}_n}(\mathscr{A})$	deviation of the set $\mathscr{A}$ from $\mathscr{S}_n$
$d_n(\mathscr{A}, \mathscr{S})$	Kolmogorov n-width of the set $\mathscr{A}$ in $\mathscr{S}$
$N_\epsilon(\mathscr{A})$	number of degrees of freedom at level ϵ of the set $\mathscr{A}$

Fields

$\bar{\mathbf{x}}, \bar{\mathbf{y}}, \bar{\mathbf{z}}$	unit vectors along the coordinate axes
$f(x,y,z)$	a scalar field
$\mathbf{f}(x,y,z)$	a vector field: $f_x(x,y,z)\,\bar{\mathbf{x}} + f_y(x,y,z)\,\bar{\mathbf{y}} + f_z(x,y,z)\,\bar{\mathbf{z}}$
∇f	gradient
$\nabla \cdot \mathbf{f}$	divergence
$\nabla \times \mathbf{f}$	curl
$\nabla^2 f$	scalar Laplacian
$\nabla^2 \mathbf{f}$	vector Laplacian
$\underline{\mathbf{g}}(\mathbf{r},t)$	dyadic space–time Green's function
$\underline{\mathbf{G}}(\mathbf{r},\omega)$	dyadic space–frequency Green's function

Physical Constants

ℓ_{p}	Planck's length [m]
$\hbar$	reduced Planck's constant [J s]
k_{B}	Boltzmann's constant [J K^{-1}]
ϵ_0	permittivity of the vacuum [F m^{-1}]
μ_0	permeability of the vacuum [H m^{-1}]
$\epsilon = \epsilon_{\mathrm{r}}\epsilon_0$	permittivity of the medium [F m^{-1}]
$\mu = \mu_{\mathrm{r}}\mu_0$	permeability of the medium [H m^{-1}]
σ	conductivity of the medium [S m^{-1}]
$c = 1/\sqrt{\epsilon\mu}$	propagation speed of electromagnetic wave [m s^{-1}]
$\beta = 2\pi/\lambda = \omega/c$	propagation coefficient [m^{-1}]

Probability

$\mathscr{A}$	a set
$\lvert\mathscr{A}\rvert$	cardinality of the set $\mathscr{A}$
Z	a random variable
$\mathbb{P}(\mathsf{Z} \in \mathscr{A})$	probability that realization of random variable Z is in $\mathscr{A}$
$\mathbb{E}(\mathsf{Z})$	expected value of Z
$f_{\mathsf{Z}}(z)$	probability density function of random variable Z
$\mathsf{Z}(t)$	a random process
$s_{\mathsf{Z}}(t,t')$	autocorrelation of $\mathsf{Z}(t)$
$s_{\mathsf{Z}}(\tau)$	autocorrelation of wide-sense stationary process $\mathsf{Z}(t)$
$S_{\mathsf{Z}}(\omega)$	power spectral density of wide-sense stationary process $\mathsf{Z}(t)$

Entropy and Capacity

H_{C}	Thermodynamic entropy [J K^{-1}]
H_{B}	Boltzmann entropy [J K^{-1}]
H_{G}	Gibbs entropy [J K^{-1}]
H	Shannon entropy [bits]
H_ϵ	Kolmogorov ϵ-entropy [bits]
$\bar{H}_\epsilon$	Kolmogorov ϵ-entropy per unit time [bits s^{-1}]
R_N	Shannon rate distortion function [bits s^{-1}]
C	Shannon capacity [bits s^{-1}]
C_ϵ	Kolmogorov ϵ-capacity [bits]
$\bar{C}_\epsilon$	Kolmogorov ϵ-capacity per unit time [bits s^{-1}]
C_ϵ^δ	ϵ-delta capacity [bits]
$\bar{C}_\epsilon^\delta$	ϵ-delta capacity per unit time [bits s^{-1}]
$h(f)$	differential entropy of probability density function f
$D(p\|\|q)$	relative entropy between probability mass functions p and q
$I(\mathsf{X};\mathsf{Y})$	mutual information between random variables X and Y