Fundamentals of Electro-Optic Systems Design

Communications, Lidar, and Imaging

Using fundamentals of communication theory, thermodynamics, information theory and propagation theory, this book explains the universal principles underlying a diverse range of electro-optical systems. From visible / infra-red imaging, to free space optical communications and laser remote sensing, the authors relate key concepts in science and device engineering to practical systems issues.

A broad spectrum of coherent and incoherent electro-optical systems is considered, accompanied by many real-world examples. The authors also present new insights into the fundamental limitations of these systems when operating through deleterious channels.

Accompanied by online examples of processed images and ideas, this uniquely tailored guide to the fundamental principles underlying modern electro-optical systems is an essential reference for all practicing engineers, graduate students, and academic researchers in optical engineering.

Sherman Karp received his Ph.D. from the University of Southern California, and has gone on to work with NASA, and as Principal Scientist at DARPA. The author of several books, he has also been awarded the SECDEF Medal for Meritorious Civilian Service, and NOSC "Scientist of the Year." He is a Fellow of the IEEE.

Larry B. Stotts received his Ph.D. from the University of California at San Diego, and is a former Deputy Office Director for the Strategic Technology Office, DARPA. He has been awarded two DARPA Technical Achievement Awards, two SECDEF Medals for Meritorious Civilian Service, and the NOSC Technical Director's Award. He is a Fellow of the IEEE and SPIE.

> "With the completion of *Fundamentals of Electro-Optic Systems Design*, Sherman Karp and Larry B. Stotts have created a single comprehensive book for anyone having anything to do with the vast field of electro-optics. The detailed systems design principles, examples, charts, graphs, and methods bring together under one cover the information to handle the applications given by the subtitle *Communications, Lidar, and Imaging*. The basic theories and their relationship to real-world hardware constraints such as noise and scattering are covered in full detail with necessary citations to decades of electro-optics research. From a systems design point-of-view, Karp and Stotts blend Lidar, laser communications, and imaging into a logical path to analyze, design, and test complex electro-optics. The communication chapters covering modulation, coding, and propagation in various media are not found anywhere else unless one wades through thousands of research papers and reports. If you are a scientist or engineer who has to manipulate photons, *Fundamentals of Electro-Optic Systems Design* belongs on your bookshelf – near the front."

> > Robert K. Tyson, The University of North Carolina at Charlotte

"This book uniquely treats electro-optical system design from an engineering viewpoint emphasizing real world applications and where theory works and does not work. These perspectives make this book a must-have reference for the scientist or engineer involved with electro-optical system design."

Tony Tether, Former DARPA Director 2001 to 2009

"Fundamentals of Electro-Optic Systems Design is a comprehensive and authoritative treatment of free-space optical communications and Lidar. Topics range from diffraction, photoelectric detection, effects of scattering and optical turbulence, and even signal coding, modulation and error correction."

Joseph W. Goodman, Stanford University

"The book is written by very knowledgeable and very experienced individuals in the field of electro-optical systems. Their writing and explanations make the material very accessible. It is clear and well presented."

Ronald Phillips, University of Central Florida

Fundamentals of Electro-Optic Systems Design

Communications, Lidar, and Imaging

SHERMAN KARP LARRY B. STOTTS



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In Memorandum

The authors dedicate this book to our esteemed friend and colleague, Professor Robert M. Gagliardi, who recently passed away. We have known Bob for almost fifty years, and we will miss him. He was truly a great and gentle man.

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Preface

The authors have been active participants in the area of electro-optic systems for over four decades, covering the introduction of laser systems and low loss optical fibers and the institutionalizing of photonic systems into everyday life. Yet for all the literature that exists, and all the work that has been accomplished, we felt that no single book existed that integrated the entire field of electro-optics, reaching back to all the fundamental building blocks and providing enough examples to be useful to practicing engineers. After much discussion and a slow start, we decided first to reference as much material as possible, bringing forth only the highlights necessary to guide researchers in the field. Then we decided to minimize mathematical developments by relegating them, as much as possible, to explanatory examples. What has evolved in our development is a clear statement of the duality of time and space in electro-optic systems. This had been touched upon in our earlier work, but has been brought forth clearly in this book in the duality of modulation index in time, and contrast in space. In doing so, and in other areas, we feel that this book contains new material with regard to the processing of spatial images which have propagated through deleterious channels. We feel that this book contains much new material in the areas of communications and imaging through deleterious channels.

In Chapter 1, we reach back to the true foundations of modern physics, the establishment of the first two laws of thermodynamics. While taken for granted, it is the first law that explains why we can see stars at the edge of the universe, and governs the radiant properties of propagating systems. The second law and the insight of Claude Shannon have created the modern field of Information Theory. Using his fundamental definitions of channel capacity we are able to establish the duality of time and space in electro-optics. This requires one basic mathematical development that is included in Appendix A, and is developed in Chapters 3 and 4.

In Chapter 2, we present the development of Maxwell's equations, which we correlate with radiant properties of propagating fields. We bring this development to the point where we can establish the conditions under which duality applies. In the process we demonstrate how the radiant properties can be used to make engineering calculations in both temporal and spatial systems, passing from the transmitting optics through to the detector.

In Chapter 3, we focus on the behavior of the photo-detection system. Using Appendix A, we introduce the duality between temporal systems and spatial systems. We identify the information-bearing portion of the received signal (as defined by Shannon) and the modulation index of incoherent communication systems.

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In Chapter 4, we delve more deeply into spatial systems and the effects on imaging. Again using Appendix A, we show the duality between the modulation index in temporal systems and contrast in spatial systems. This is related to the classic definition and merely extends it based upon the newer understanding of the behavior of electro-optic systems. We then proceed further in digital communication systems and show how we develop optimum systems under a few different constraints.

In Chapter 5, we investigate in detail the concepts of contrast and visibility and their relationship to signal-to-noise ratio. We then discuss the duality of modulation index in communication theory and contrast in imaging. We also introduce the concept of sub-visibility imaging, which allows a direct trade-off between improved contrast and deteriorated signal-to-noise ratio. Imagery demonstrating this trade-off is presented, and applications are discussed.

In Chapter 6, we review various optical modulation schemes used in optical communications. This will include a discussion of spectral efficiency and the energy per bit per unilateral noise density, parameters used extensively in RF communications, but just coming into use in optical communications. These latter aspects help us determine the optimal means for optical communications, especially when erbium-doped fiber amplifiers are involved.

In Chapter 7, we provide the basics on block forward error correction (FEC) encodings, and highlight the arguably most popular FEC, Reed-Solomon codes. We then will note other important types of FEC schemes that can be employed, based on the authors' bias. This material is not meant to be an exhaustive survey of FEC coding, but rather included in this book to give the reader a basic background and knowledge of FEC to illustrate their utility in optical communications today. Application of FEC to optical systems will be discussed in a subsequent chapter.

In Chapter 8, we discuss some of the key aspects of the signal modulation and coding schemes used in fiber optic and free-space optical communications systems today. Most notably, we will review the use of return-to-zero and non-return-to-zero in coding the information streams and see their effect on systems performance, as well as receiver sensitivity.

In Chapter 9, we lay out the fundamentals of lidar and address a variety of applications. Although each of these applications has a huge area of research behind it, there are nevertheless some common threads which we discuss in the context of our development. We also reference some relevant papers in these areas for those interested in further investigation.

In Chapter 10, we discuss the performance of incoherent and coherent communications systems when operating in the optical turbulence channel. We will discuss a new statistical link budget approach for characterizing incoherent FSOC link performance, and compare experimental results with statistical predictions.

In Chapter 11, we discuss a set of approaches for optical communications in diffusive scattering channel, augmenting the results described in Chapter 9. Specifically, we highlight the accepted models for characterizing the statistical channel effects and techniques for providing communications in this channel.

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As with many fields, we stand on the shoulders of great men and women whose contributions have shaped the field of engineering, especially optical system engineering. Of those researchers, the authors would like to acknowledge five pillars that have influenced the authors greatly through their teachings, personal interactions and insightful contributions – Professors Robert S. Kennedy, Irving S. Reed, Robert M. Gagliardi, Adolf Lohmann and Siebert Q. Duntley.

We also would like to thank our many colleagues who helped us over the years succeed in our optical systems projects to gain the insights we discuss in this book. They also have influenced us greatly. In writing this book, we want to acknowledge Drs. Larry Andrews, Juan Juarez, David Fried, Paul Kolodzy, Gary Lee, Ron Phillips, H. Alan Pike, David Young, and Mr. David Buck, Bob James, Todd Martin and Ned Plasson for their help and assistance.

In addition, we would like to acknowledge Dr. Anthony J. Tether, who over the years has encouraged and supported our various efforts in optical systems.

Finally, we recognize the biggest contributors to our book, our families, who have shown their patience during the preparation of this manuscript. Anyone who has imbedded themselves in an all-consuming goal knows the importance of the understanding and patience of loved ones.

Notation

| $E[f(t)],\bar{f}$ | expected value of $f(t)$ |
|----------------------------------|---|
| $m^2(t), \bar{m}^2$ | modulation index; average value |
| $n(\vec{r},t),\bar{n}$ | Poissonrate parameter, average value |
| $2e^2\bar{n}B, (2ei_{dc}B)$ | referred to as shot noise |
| PWE | probability of symbol (word) error |
| (x,y) | coordinates of aperture plane |
| (x_0, y_0) | coordinates of receiver plane |
| 1/2B | pulse width τ |
| a | absorption coefficient radius of the aperture, length of the aperture |
| A | area, optical amplitude |
| AO | adaptive optics |
| b | volume scattering coefficient, length of the aperture |
| В | bandwidth |
| $B(\mathbf{u})$ | source irradiance function |
| BER | bit error rate |
| <i>C</i> , <i>C</i> ₀ | volume extinction coefficient; speed of light in free space |
| $C(\alpha), S(\alpha)$ | Cornu spirals |
| С, Со | information capacity (bits/s), constant, contrast; inherent contrast |
| $C_{\rm corr}$ | correlation length |
| C_n^2 | refractive index structure parameter |
| C_w | curvature of the wave front |
| CNR | carrier-to-noise ratio |
| cx, cy | centroid in x- and y-direction |
| d | linear dimension of the photo-detector, aperture diameter |
| d_T, d_R | transmitter, receiver aperture diameter |
| d_S | beam diameter |
| d_d | photo-detector diameter |
| d_R | converging lense diameter |
| d_D | diffraction-limited focus diameter |
| $D; D_s$ | mode number, dimension, aperture diameter; depth, Secchi depth |
| D_n | refractive index structure function |
| dB | decibel |
| dBmW | decibel milliwatts |
| | |

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| e, q | electron charge |
|-------------------------------------|--|
| E(z,r) | spatial impulse response point spread function |
| E_d, E_u | downwelling, upwelling radiance at depth |
| E_b | energy per bit |
| erf | error function |
| erfc | complementary error function |
| F | F-number of lens |
| f | frequency (cycles/s) |
| $F(\mathbf{r},t)$ | solution to Fresnel integral (volts/m) |
| f , (f_x, f_y) | spatial frequency, bold |
| f_c | focal length |
| f_g | Greenwood frequency |
| f_{sg} | normalized received power |
| F_w | off-axis irradiance correction |
| FEC | forward error correction |
| FOV | field of view |
| FSL | Fraunhofer spreading loss |
| G | gain |
| g | number of indistinguishable states in ground level of the atom, asymmetry factor |
| h | Planck's constant height |
| Н | scale height |
| $h(\mathbf{r},t)$ | effect of single electron flow |
| $h(x,y;\omega)$ | linear spatial effect at frequency ω |
| H, E(I) | irradiance (intensity) (watts/m ²) |
| HV | Hufnagel-Valley |
| HAP | Hufnagel-Andrews-Phillips |
| $I_e, I(\mathbf{R}); I(\mathbf{v})$ | radiant intensity (watts/sr): image intensity 2D; image evolving in time |
| I_D | dark current |
| $I_0()$ | modified Bessel function of the first kind with order zeo |
| J_0, J_1 | zeroth and first-order Bessel functions |
| k | Boltzmann's constant (J/deg), electron count, diffuse attenuation coefficient |
| k , <i>κ</i> | wavenumber vector |
| Κ | photo-electron counts |
| Κ | degrees Kelvin (degrees) |
| k_D, i_T | detection threshold |
| k_T | integer value of the detection threshold that minimizes the probability of error |
| ℓ ₀ | inner scale of turbulence |
| L_e | radiance (w/m ² sr), propagation path length |
| L(z) | normalized range equation (log of range corrected range equation $\times r^2$) |
| L_0 | outer scale of turbulence |
| L_a, L_c | atmospheric, cloud transmittance |
| $L_{a/s}$ | air/sea interface transmittance |
| т | mass of an atom |
| | |

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|--|---|--|--|
| | | | |
| <i>M_e</i> , <i>M</i> (r) | radiant exitance (radiant emittance) (watts/m ²), point source coherence function, number of slots, number of symbols | | |
| M_{FM} | FM modulation index | | |
| M_L | number of local field modes | | |
| M_R | number of received modes | | |
| $m_{\nu}, m(t), m(t)$ | •) Poisson intensity variable, number of electron counts; time modulation, spatial modulation | | |
| m_0 | total noise counts | | |
| m_1 | total signal plus noise counts | | |
| MTF | modulation transfer function | | |
| п | number of atoms; index of refraction of air | | |
| n_s | average number of photo-electrons per bit for a BER = 10^{-9} | | |
| n_q | average number of photo-electrons per bit at the quantum limit | | |
| N_e | spectral radiance (w/m ² sr λ); number density of air molecules, number of slots, | | |
| | number of Zernike modes | | |
| N_0 | unilateral spectral noise density | | |
| NRZ | non-return-to-zero | | |
| OAGC | optical automatic gain control | | |
| OTF | optical transfer function | | |
| P | probability density | | |
| P_E , PE | probability of bit error | | |
| P_t | probability of t-errors | | |
| P(x,y), P(z) | pupil function; pressure | | |
| $p(\theta)$ | scattering phase function | | |
| $P_e, P, P(z)$ | radiant flux (radiant power) (watts); probability; pressure, power | | |
| PIB | power in the bucket | | |
| PIF | power in the fiber | | |
| POF | power out of the fiber | | |
| PRF | pulse repetition frequency | | |
| Q(a,b) | Marcum Q-function | | |
| Q, Q_e, Q_s | radiant energy (joules); extinction efficiency; scattering efficiency, compensation factor | | |
| R | distance, data rate, water reflectance | | |
| RZ | return to zero | | |
| $r r(R_b)$ | scalar range spectral efficiency, detector radius | | |
| \overrightarrow{r} | radius vector | | |
| r_c, \mathbf{r} | radius, radius of the aperture | | |
| r_0 | Fried parameter, radius of the aperture | | |
| $R(\rho)$ | covariance of homogeneous process | | |
| $R(\tau)$ | covariance of wide sense stationary process | | |
| R_n^m | radial polynomial | | |
| R_e | radius of the Earth | | |
| R_L | load resistor | | |

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|----------|------|
|----------|------|

| RR | Rayleigh range |
|---|--|
| r, s, | vector distance (arrow, bold) |
| S | entropy (logarithm of degrees of uncertainty - information), spectral efficiency |
| s, s(I, j) | volume scattering coefficient, signal |
| $S(\omega)$ | power spectral density |
| SNR, $\left[\frac{S}{N}\right]$ | signal-to-noise ratio |
| SR | Strehl ratio |
| <i>t</i> , <i>T</i> , T _o | time, temperature |
| T_D | lase dead time |
| U | input potential to Fresnel integral (volts/m) |
| Var[] | variance |
| $V_{\rm HV~5/7}$ | HV 5/7 vertical profile for wind speed |
| VOA | variable optical attentuator |
| W | wind speed, error of focus |
| W_g | ground wind speed |
| Ws | beam slew rate |
| W(x,y) | aberration function |
| x | Mie parameter |
| $x(\mathbf{r},t)$ | shot noise process |
| <i>z</i> , <i>R</i> | range |
| Δ | maximum offset |
| ΔT | slotwidth |
| $\Delta\lambda$ | spectral passband |
| Δv | Laser frequency linewidth |
| $\Phi_m(\mathbf{f},t)$ | spatial power spectral density of $m(\mathbf{r},t)$ |
| Φ_n | refractive index probability density function |
| $\Gamma(\mathbf{r,r'}; t,t');$ | mutual coherence function |
| $M(\rho,z)$ | |
| Λ | likelihood ratio |
| $\Theta()$ | optical transfer function (OTF) |
| Ω | solid angle |
| α | detector sensitivity parameter |
| δ | Stephan-Boltzmann constant $(J-m^2 deg^{-4} s^{-1})$ |
| 3 | emissivity |
| 3 | focus error |
| $\varepsilon, \varepsilon_0$ | permittivity, (free space) |
| φ | scalar potential, angular coordinate; beamwidth |
| $\varphi(\mathbf{\rho}), \varphi(\tau)$ | eigenfunctions |
| Φ_c | laser signal phase angle |
| Φ_o | laser oscillator phase angle |
| φ | optical phase |
| η | quantum efficiency |
| γ_{Tx}, γ_{Rx} | transmitter, receiver transmittance |

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Notation

| Yfiber | fiber transmittance |
|---|---|
| 70 70 | signal-to-noise ratio per symbol per unit bandwidth |
| $\Gamma(n, x)$ | incomplete gamma function |
| λ | wavelength |
| $\lambda_{\tau,I}, \lambda_{s,j}, \lambda(q,l)$ | eigenvalues |
| μ, μ_0 | permeability, (free space) |
| $\mu_{S,B}$ | average number of photo-electrons per bit from laser signal |
| $\mu_{N,B}$ | average number of photo- electrons per bit from background radiation and dark current |
| $\pi_k(heta),\xi_k(heta)$ | spherical harmonics (can be written as Legendre polynomials) |
| $\theta, \alpha, \varphi_s$ | angular coordinate |
| θ_0 | isoplanatic angle |
| ρ | reflection coefficient; bits of resolution |
| $ ho_0$ | plane wave lateral coherence length |
| σ | cross section, standard deviation |
| $\sigma_{fit}^2 \\ \sigma_{\chi}^2 \\ \sigma_{T}^2$ | residual fitting error |
| σ_{χ}^{2} | Rytov number |
| | angle of arrival tilt variance (jitter) |
| $\sigma_{	heta}$ 2 | tilt variance |
| $\sigma_{i_{TH}}{}^2$ | thermal noise variance |
| τ | optical thickness, time between symbol errors |
| $	au_{ m d}$ | diffusion thickness |
| $	au_0$ | Greenwood time period |
| $	au_B$ | bit period |
| ω | frequency (rad/s); single-scatter albedo, radian frequency |
| ψ , ψ_{BB} | Wien's law, black-body equation |