

Hillslope Hydrology and Stability

Landslides occur when hillslopes become mechanically unstable, because of meteorological and geologic processes, and pose a serious threat to human environments in their proximity. The mechanical balance within hillslopes is governed by two coupled physical processes: hydrologic or subsurface flow and stress. The stabilizing strength of hillslope materials depends on effective stress, which is diminished by rainfall, increasing the risk of gravity destabilizing the balance and causing a landslide.

This book presents a cutting-edge quantitative approach to understanding hydro-mechanical processes in hillslopes, and to the study and prediction of rainfall-induced landslides. Combining geomorphology, hydrology, and geomechanics, it provides an interdisciplinary analysis that integrates the mechanical and hydrologic processes governing landslide occurrences, across variably saturated hillslope environments. Topics covered include a historic synthesis of hillslope geomorphology and hydrology, total and effective stress distributions, critical reviews of shear strength of hillslope materials, and different bases for stability analysis. Exercises and homework problems are provided for students to engage with the theory in practice.

This is an invaluable resource for graduate students and researchers in hydrology, geomorphology, engineering geology, geotechnical engineering, and geomechanics, and also for professionals in the fields of civil and environmental engineering, and natural hazard analysis.

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To
Connie, Vivian, and Shemin
and
Neva and Laura

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Foreword

Even a cursory inspection of *Hillslope Hydrology and Stability* by Lu and Godt will impress most professionals interested in processes at the interface between geotechnical engineering and hydrology. This unique textbook represents an attempt to systematically unify concepts from vadose zone hydrology and geotechnical engineering into a new hydro-geo-mechanical approach with special emphasis on quantifying natural mechanisms for the onset of hydrologically induced landslides. Professionals will particularly appreciate the comprehensive coverage of concepts ranging from fundamentals of geomechanics and soil properties to the state-of-the-art concepts of hillslope hydrology, with explicit treatment of soil heterogeneity, layering, and vegetation mechanical and hydrologic functions. The authors have been able to weave a coherent picture based on the cutting-edge state of knowledge regarding landslides as natural geomorphological processes and as ubiquitous natural hazards in mountainous regions.

Students will appreciate the lucid coverage of topics offering a systematic introduction to key ingredients essential for understanding the occurrence of landslides in their broader natural context (often missing in technical textbooks). Students are guided through aspects of precipitation with its instantaneous to inter-annual patterns, as well as aspects of soil types and the geomorphological context of landslides. This provides a solid foundation for introduction of more specific technical aspects of infiltration, hillslope hydrology, and hydro-mechanical properties, and assembles the roles of these factors on a hillslope mechanical state. Students will find clear explanations of fundamental concepts inspired by numerical examples to help them develop appreciation for the orders of magnitude for the quantities involved. Numerous motivating homework problems further promote self-study.

Hillslope Hydrology and Stability helps chart the boundaries of the emerging interdisciplinary field of soil hydromechanics. The authors offer a rigorous link between hydrology and soil mechanics by providing a unified treatment of effective stress (suction stress) under variably saturated conditions (Chapter 6). The authors also provide a fresh look at well-established concepts found in textbooks from hydrology and geotechnical engineering fused together using new crucial aspects typically glossed over in standard texts, thereby providing a unique new perspective. For example, the interplay between hillslope subsurface flows and soil layering (forming hydrologic barriers), a critical mechanism for abrupt landslide triggering, has rarely been previously discussed in a quantitative hillslope hydro-mechanical context as done in Chapter 3. The quantitative treatment of root reinforcement and the role of plants in the mechanical picture of natural hillslopes (Chapter 7) is another example of the conceptual integration in the basis of the book. The wealth of information on numerical values of key parameters and the instructive use of case studies described in

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Chapters 9 and 10 make *Hillslope Hydrology and Stability* an outstanding resource for students, researchers, and practitioners alike. No doubt the test of time would add refinement to this labor of love that contains numerous new concepts – I hope students and researchers would be challenged and inspired by the breadth and depth offered in this unique treatise on hydro-mechanical hillslope processes.

Professor Dani Or
ETH Zurich

Preface

We strive to provide a thorough description on the cutting edge of the spatial and temporal occurrence of rainfall-induced landslides by quantifying the hydro-mechanical processes in hillslopes. Landslides are a pervasive natural phenomenon that constantly shapes the morphology of the earth's surface. Over geologic time, landslides are the result of two episodic, and broadly occurring geologic processes; tectonics and erosion. At human scale, the former operates at a uniform rate barely sensed by humans except during earthquakes. However, the latter is entirely sensible and is driven largely by rainfall. The results of these dynamic geologic processes are the infinite variety of landforms that vary remarkably in geometry; from flat plains to rolling hills, to vertical or even overhanging cliffs, and to shapes that test human's imagination.

Understanding of how landslides occur is vital to the well being of human society and our environment and has been a research focus for many disciplines such as geomorphology, hydrology, geography, meteorology, soil science, and civil and environmental engineering. While each of these disciplines tackles landslide problems from quite different perspectives, a common thread is the mechanics of landsliding. From the vantage of mechanics, no matter how complicated the morphology of the land surface, it is the mechanical balance within hillslopes that determines if they are stable or not. Two coupled physical processes govern the mechanical balance; hydrological or subsurface flow process and stress equilibrium process.

Understanding and quantifying the hydro-mechanical processes provide the key link to the knowledge gained from different disciplines and pathways for predicting the spatial and temporal occurrence of landslides. In each hillslope, driving and resisting forces dictate the state of stability. The driving or destabilizing forces are mainly provided by gravity and the resisting or stabilizing forces are mainly provided by the strength of hillslope materials. This mechanical balance is mediated by the presence of water, which varies dramatically over climatic, seasonal, and shorter time scales and has both a stabilizing and destabilizing effect. The effect of water on the stability of hillslopes is quantified using the concept of effective stress, which provides a connection between subsurface hydrologic and mechanical processes under variably saturated conditions.

In this volume, we present quantitative treatments of rainfall infiltration, effective stress, their coupling, and roles in hillslope stability. An overall introduction to landslide phenomena, their classification, and socio-economic impacts is provided in Chapter 1. The settings where landslides occur are described in Chapter 2: slope geomorphology. Sub-surface hydrologic process under variably saturated conditions is systematically described in the forms of steady infiltration (Chapter 3) and transient infiltration (Chapter 4). The background stress or total stress fields driven by gravity in hillslopes are quantified under

the theory of linear elastostatics in Chapter 5. A unified effective stress framework linking soil suction to effective stress is provided in Chapter 6. The pertinent material properties, both the strength of soil and vegetation roots, and hydrologic constitutive laws, are provided in Chapters 7 and 8. Integration of slope geomorphology, hydrology, and soil mechanics leads to a rigorous treatment of slope stability analysis that is described in Chapters 9 and 10. Chapter 9 provides an in-depth introduction to the classical or conventional slope stability methodologies as well as expansions to include environments under variably saturated conditions by the unified effective stress principle. Chapter 10 presents a framework departing from the conventional slope stability paradigm by employing scalar fields of suction stress and factor of safety, which has potential to reveal spatial and temporal occurrence of rainfall-induced landslides in variably saturated hillslopes. The effectiveness of the proposed hydro-mechanical framework is examined through two case studies in these chapters. The first case study is an analysis of a shallow landslide induced by rainfall and is based on a multi-year field-monitoring program where the reduction of a few kPa of suction stress eventually led to slope failure. The second case study applies the hydro-mechanical framework to analyze a deep-seated landslide that moves each year in response to melting snow.

The book is truly the journal of our joint endeavor to advance the understanding of occurrence of landslides. The materials covered here have been grown out of a course, *Hillslope Hydrology and Stability*, taught at Colorado School of Mines, USA, EPFL-Lausanne, Switzerland, and University of Perugia, Italy over the past 6 years. From teaching, we gained much from our interactions with students and professionals. The major part of NL's contribution to the book was written while he was on sabbatical as the Shimizu Visiting Professor at Stanford University and a visiting scientist at the U.S. Geological Survey campus in Menlo Park, California office in 2010–2011. His hosts, Ronaldo Borja at Stanford and Brian Collins at the USGS provided an intellectually stimulating and productive environment. The authors benefitted greatly from contributions from the following colleagues who provide insightful, critical, and thorough reviews of parts of the manuscript: Rex Baum, Brian Collins, Richard Healy, Richard Iverson, and Mark Reid of the U.S. Geological Survey, Dalia Kirschbaum of NASA Goddard Space Flight Center, Giovanni Crosta of the University of Milano-Bicocca, William Likos of the University of Wisconsin-Madison, John McCartney of the University of Colorado-Boulder, Dani Or of ETH Zurich, Ricardo Rigon of the University of Trento, Diana Salciarini of the University of Perugia, Alexandra Wayllace of the Colorado School of Mines, and Raymond Torres of the University of South Carolina-Columbia. We extend special thanks to Rex Baum for looking at the entire proof of the book. Nonetheless, all errors and bias remain ours. Başak Şener-Kaya prepared the figures and tables for the total stress distributions in hillslope in Chapter 5. Finally, the authors would like to express our gratitude to Peter Birkeland who acts as *Pe(te)casso* for illustrating the essentials of our thoughts in art form at the beginning of each part.

Symbols

Symbol	Description	Units
A	Skempton pore pressure parameter for isotropic loading	–
A_L	landslide area	m^2
A	area; cross sectional area	m^2
a_1	root tensile strength parameter	MPa m^{-a_2}
a_2	root tensile strength parameter	–
B	Skempton pore pressure parameter for deviator loading	–
$\mathbf{b}$	body force vector	N/m^3
b	parameter for inter-grain friction angle	–
b_1	root shear strength parameter	MPa
b_2	root shear strength parameter	$\text{MPa m}^3/\text{kg}$
b_n	width of the n th slice in a method of slices	m
b_o	parameter for cumulative rate of root mass with depth	–
b_i	body force components	N/m^3
$C(\psi)$	specific moisture capacity as function of suction	$1/\text{kPa}$
$C(h)$	specific moisture capacity as function of head	$1/\text{m}$
c	cohesion	kPa
c	solute concentration	mol m^3
c_c	cohesion mobilized by cementation bonds	kPa
c_d	mobilized or developed cohesion along failure surface	kPa
c_o	cohesion due to grain inter-locking	kPa
c_s	cohesion mobilized by suction stress	kPa
c_u	undrained shear strength	kPa
c'	cohesion in terms of effective stress	kPa
D	diffusivity	m^2/s
D_o	free vapor diffusivity in air	m^2/s
D_v	free vapor diffusivity in porous media	m^2/s
D_r	maximum depth of landslide body	m
D_r	relative density	–
D_v	diffusion coefficient for water vapor	m^2/s
D_{10}	10% finer particle diameter	m
D_{50}	50% finer particle diameter	m
d	diameter of capillary tube	m
d	root diameter	mm

Symbol	Description	Units
d	shear strength parameter defined by cohesion and friction angle	kPa
d_1, d_2, d_3	root shear strength growth parameters	kPa
d_4	root shear strength growth parameter	y^{-1}
d_5	root shear strength decay parameter	y^{-d_5}
d_6	root shear strength decay parameter	y^{-d_6}
E	Young's modulus	kPa
E	inter-slice normal forces in method of slices	kN
e	void ratio	—
e_{max}	void ratio in loosest state	—
e_{min}	void ratio in densest state	—
e_s	saturation vapor pressure	hPa
FS	factor of safety for a hillslope	—
FS _s	shear strength based factor of safety	—
f	infiltration capacity	cm/hr
$f(u_a - u_w), f(S)$	suction stress characteristic function	kPa
f_c	minimum steady constant infiltration capacity	cm/hr
f_0	initial infiltration capacity	cm/hr
F_{ij}	force components	N
G	elasticity modulus	kPa
G_s	specific gravity of soil solids	—
g	acceleration due to gravity	m/s ²
$\mathbf{g}$	acceleration vector due to gravity	m/s ²
H	Kirchhoff integral transformation	m ² /s
H_{max}	maximum slope height of a finite slope	m
H_{ss}	depth of sliding surface from ground surface	m
H_{wt}	depth of water table from ground surface	m
h	height of capillary rise; head	m
h_a	air-entry head	m
h_c	maximum height of capillary rise	m
h_d	applied increment in matric suction head	m
h_g	total gravitational head	m
h_i	initial suction head in a soil column	m
h_m	matric suction head	m
h_n	height of the water table from the failure surface for slice n	m
h_o	suction head at wetting front	m
h_o	osmotic suction head	m
h_t	total head	m
h_{vap}	potential head of water vapor	m
h_v	kinetic or velocity head	m

Symbol	Description	Units
h_w	applied decrement in matric suction head	m
$I_{1\sigma}$	first stress invariant	kPa
i	hydraulic gradient	–
i	initial root orientation with respect to failure plane	deg
i, j, m, s	series indices	–
K	bulk elastic modulus	kPa
K	hydraulic conductivity	m/s
$\mathbf{K}$	hydraulic conductivity tensor	m/s
K^*	dimensionless hydraulic conductivity in Laplace space	–
K_f	permeability-dependent constant for infiltration capacity	hr ^{–1}
K_o	hydraulic conductivity at wetting front	m/s
K_o	horizontal to vertical stress ratio under no horizontal displacement condition	–
K_{eq}	equivalent hydraulic conductivity of soil-HAE ceramic stone system	m/s
K_s	saturated hydraulic conductivity	m/s
K_{sat}	saturated hydraulic conductivity	m/s
K_s^d	saturated hydraulic conductivity for drying state	m/s
K_s^w	saturated hydraulic conductivity for wetting state	m/s
K_s^c	saturated hydraulic conductivity of HAE ceramic stone	m/s
K_x, K_y, K_z	hydraulic conductivity in the x , y , and z directions	m/s
L	diversion width for capillary barrier	m
L	soil layer thickness	m
L	length of soil body in infinite-slope model	m
L	depth of the water table from ground surface	m
L_r	length of the surface of rupture of a landslide body	m
l	sample height plus thickness of HAE ceramic stone	m
l_1, l_s	sample height	m
l_2, l_c	thickness of HAE ceramic stone	m
l_n	length of the base of slice n	m
M	shear strength parameter defined by internal friction angle	–
M_r	cumulative mass fraction in depth z	–
m	total number of slices in a method of slices	–
m	slope stability number for assessing stability of finite slope	–
m_r	root mass per unit volume of the reinforced soil	kg/m ³
m_s	mass of solid	kg
N	index variable	–
N	normal force	N
N_n	normal reacting force	N
n	Corey’s 1954 hydraulic conductivity model parameter	–
n	porosity	–

Symbol	Description	Units
n	SWCC modeling constant	—
n^d	SWCC modeling constant for drying state	—
n^w	SWCC modeling constant for wetting state	—
n	series index	—
$\mathbf{n}$	unit directional vector on boundary	—
n_x, n_y, n_z	components of unit directional vector on boundary	—
n_a	air-filled porosity	%
n_p	porosity	—
P	annual precipitation	mm
PET	annual potential evaporation	mm
p	landslide probability density	m^{-2}
Q	dimensionless flow variable	—
Q	diversion capacity for capillary barrier	m^2/s
Q	total cumulative infiltration	m
q	fluid flow velocity	m/s
$\hat{q}_d(l, t)$	simulated outflow rate during drying	m/s
$\hat{q}_d^{\text{exp}}(l, t)$	experimental outflow rate during drying	m/s
$\hat{q}_w(l, t)$	simulated inflow rate during wetting	m/s
$\hat{q}_w^{\text{exp}}(l, t)$	experimental inflow rate during wetting	m/s
q_{in}	total inflow rate of water into a unit cell	kg/s
q_{out}	total outflow rate of water out of a unit cell	kg/s
q_v	vapor flow velocity	m/s
$\mathbf{q}$	fluid velocity vector	m/s
R	universal gas constant	J/mol K
R	radius of Mohr circle	kPa
R	resultant force	N
RDD	relative dry density	—
R_{max}	maximum resultant force	N
R_r	root shear strength conversion factor	—
REV	representative elementary volume	m^3
r	radius of circular failure surface	m
r	equivalent or mean pore radius	μm
r_u	pore-water pressure parameter in infinite-slope model	—
S	degree of saturation	%
S	cross section	m^2
S_{xy}	cross section perpendicular to z axis	m^2
S_{xz}	cross section perpendicular to y axis	m^2
S_{yz}	cross section perpendicular to x axis	m^2
S	shear force	N
S_{max}	maximum shear force	N
S_e	effective degree of saturation	%
S_n	mobilized shear resistance along the base of the n th slice	N

Symbol	Description	Units
S_r	residual degree of saturation	%
S_s	specific storage	1/m
s	sorptivity	m/s ^{1/2}
T	absolute temperature	K
T	dimensionless time	–
T_s	surface tension	N/m
t	time	s
t_x	traction or stress component in the x direction at boundary	Pa
t_y	traction or stress component in the y direction at boundary	Pa
t_z	traction or stress component in the z direction at boundary	Pa
u	pore-water pressure	kPa
u_x, u_y, u_z	displacement components	m
$\bar{u}_x, \bar{u}_y, \bar{u}_z$	displacement components at boundary	m
u_a	pore air pressure; air pressure	kPa
u_b	air-entry (bubbling) pressure	kPa
u_c	pore pressure due to isotropic stress loading	kPa
u_d	pore pressure due to deviatoric stress loading	kPa
u_{ij}	displacement components	m
u_{sat}	saturated vapor pressure	kPa
u_{v0}	saturated vapor pressure	kPa
u_w	pore-water pressure; water pressure	kPa
$(u_a - u_w)$	matric suction	kPa
V	volume of landslide body	m ³
V_t	volume of soil specimen	m ³
v	discharge velocity	m/s
v_v	volume of void in REV	m ³
v_s	volume of solid in REV	m ³
v_w	volume of water in REV	m ³
v_w	molar volume of water	m ³ /kmol
W	virtual work due to effective stress	J
W	weight of soil body	N
W_n	weight of slice n	N
W_σ^s	virtual work due to suction stress stress	J
W_v	weight of soil column per unit cross section area	N/m ²
X	maximum displacement of failure zone along failure surface	m
X	inter-slice shear forces in method of slices	kN
x, y, z	Cartesian coordinate directions	m
x_*, z_*	Cartesian coordinate aligned with sloping direction	m
Z	dimensionless distance	–

Symbol	Description	Units
Z	wetting from position	m
Z	thickness of failure zone	m
z_w	depth of loose or weathering zone	m
α	rotational angle on Mohr circle	deg
α	local topographic slope	–
α	pore size distribution index; SWCC modeling constant	1/kPa
α^d	pore size distribution index for drying state	1/kPa
α^w	pore size distribution index for wetting state	1/kPa
α_n	angle of slice n	N
α_s	bulk compressibility of soil	m ² /N
β	rotational angle on Mohr circle	deg
β	angle of failure surface with respect to horizontal direction	deg
β	pore size distribution index; SWCC modeling constant	1/m
β_w	compressibility of water	m ² /N
χ	coefficient of matric suction	–
$\chi(u_a - u_w)$	suction stress (capillary stress)	kPa
ε	strain	%
$\varepsilon_x, \varepsilon_{xy}$	strain components	%
ϕ	angle of internal friction	deg
ϕ	angle of dip for capillary barrier	deg
ϕ_c	inter-grain friction angle	deg
ϕ_d	angle of internal friction at dry state	deg
ϕ_0, ϕ_{100}	angle of internal friction at 0 and 100% relative dry density	deg
ϕ'	effective angle of internal friction	deg
ϕ'_d	developed or mobilized effective friction angle	deg
ϕ_{CU}	friction angle under consolidation undrained condition	deg
ϕ'_{NC}	effective friction angle under normal consolidation condition	deg
ϕ'_{OC}	effective friction angle under overly consolidation condition	deg
γ	bulk (total) unit weight	kN/m ³
$\gamma_{dmax}, \gamma_{dmin}$	maximum or minimum dry unit weight	kN/m ³
γ	slope angle	deg
γ_{xy}	strain components for angle of distortion	radian
γ_w	unit weight of water	kN/m ³
Λ_n	n th positive root of pseudoperiodic characteristic equation for K^*	–
λ	latent heat of vaporization	J/kg
λ	slope stability number for assessing soil unit weight	–

Symbol	Description	Units
λ	Boltzmann transformation variable	—
λ	Lamé elastic constant	kPa
δ_{ij}	identity tensor	—
ν	Poisson’s ratio	—
μ_t	total chemical potential	J/kg
μ_0	chemical potential of reference state	J/kg
μ_v	chemical potential of water vapor	J/kg
π	osmotic pressure	kPa
Θ	effective water content (effective degree of saturation)	%
θ	volumetric water content	%
θ	mobilized friction angle	deg
θ	angle of shear distortion with respect to initial root orientation	deg
θ	maximum mobilized friction angle	deg
θ	angle of potential failure surface with respect to horizontal direction	deg
θ_1	angle of distortion for element Δx	radian
θ_2	angle of distortion for element Δz	radian
θ_{cr}	critical angle of potential failure surface	deg
θ_r	residual volumetric water content	%
θ_r^d	residual volumetric water content for drying state	%
θ_r^w	residual volumetric water content for wetting state	%
θ_s	saturated volumetric water content	%
θ_s^d	saturated volumetric water content for drying state	%
θ_s^w	saturated volumetric water content for wetting state	%
θ_i	initial volumetric water content	%
θ_o	volumetric water content at wetting front	%
ρ_v	density of water vapor (absolute humidity)	kg/m ³
ρ_w	density of water	kg/m ³
σ	total normal stress	kPa
σ_o	normal stress	kPa
σ_{cap}	suction stress component due to capillarity	kPa
σ_{pc}	suction stress component due to physico-chemical forces	kPa
σ_C	counterbalance stress to suction stress due to Born repulsion	kPa
σ_c	cementation bonding stress	kPa
σ'	effective stress	kPa
σ_r	root tensile strength	kPa
σ_{ri}	root i ’s tensile strength	kPa
σ^s	suction stress (capillary cohesion)	kPa
σ_{xi}	stress components	kPa
σ_1	major principal stress	kPa

Symbol	Description	Units
σ_2	intermediate principal stress	kPa
σ_3	minor principal stress	kPa
σ_n	total normal stress	kPa
σ'_n	effective normal stress	kPa
σ_{tia}	isotropic tensile strength	kPa
σ_{tua}	uniaxial tensile strength	kPa
$(\sigma - u_a)$	net normal stress	kPa
$(\sigma_f - u_a)_f$	net normal stress on failure plane at failure	kPa
τ	shear stress	kPa
$\tau_{xy}, \tau_{xz}, \tau_{zy}$	shear stress components	kPa
τ_{max}	maximum shear stress at a point	kPa
τ_d	mobilized or developed shear stress along failure surface	kPa
τ_f	shear stress at failure	kPa
τ_{rs}	root shear strength mobilized by root tensile strength	kPa
ω_w	molecular mass of water	kg/mol
ω_v	molecular mass of water vapor	kg/mol
ω	capillary barrier efficiency	—
ω	parameter for defining van Genuchten’s SWRC model	—
ψ	composite contact or dilation angle	deg
ψ	suction	kPa
ψ	rupture root orientation with respect to failure plane	deg
ψ_m	matric suction	kPa
ψ_o	osmotic suction	kPa
ψ_o	matric suction beyond wetting front	kPa
ψ_t	total suction	kPa
RH	relative humidity	%
HCF	hydraulic conductivity function	
SSCC	suction stress characteristic curve	
SWCC	soil water characteristic curve	
SWRC	soil water retention curve (also called SWCC)	