

Contents

	<i>Preface</i>	<i>page xi</i>
1	Knots and their relatives	1
	1.1 Definitions and examples	1
	1.2 Plane knot diagrams	5
	1.3 Inverses and mirror images	7
	1.4 Knot tables	9
	1.5 Algebra of knots	10
	1.6 Tangles, string links and braids	12
	1.7 Variations	17
	Exercises	21
2	Knot invariants	26
	2.1 Definition and first examples	26
	2.2 Linking number	27
	2.3 The Conway polynomial	30
	2.4 The Jones polynomial	32
	2.5 Algebra of knot invariants	35
	2.6 Quantum invariants	36
	2.7 Two-variable link polynomials	43
	Exercises	49
3	Finite type invariants	57
	3.1 Definition of Vassiliev invariants	57
	3.2 Algebra of Vassiliev invariants	60
	3.3 Vassiliev invariants of degrees 0, 1 and 2	64
	3.4 Chord diagrams	66
	3.5 Invariants of framed knots	68
	3.6 Classical knot polynomials as Vassiliev invariants	70
	3.7 Actuality tables	77

viii	<i>Contents</i>	
	3.8 Vassiliev invariants of tangles	79
	Exercises	81
4	Chord diagrams	84
	4.1 Four- and one-term relations	84
	4.2 The Fundamental Theorem	87
	4.3 Bialgebras of knots and of Vassiliev knot invariants	89
	4.4 Bialgebra of chord diagrams	92
	4.5 Bialgebra of weight systems	98
	4.6 Primitive elements in \mathcal{A}	101
	4.7 Linear chord diagrams	103
	4.8 Intersection graphs	104
	Exercises	112
5	Jacobi diagrams	115
	5.1 Closed Jacobi diagrams	115
	5.2 IHX and AS relations	118
	5.3 Isomorphism $\mathcal{A} \simeq \mathcal{C}$	123
	5.4 Product and coproduct in \mathcal{C}	125
	5.5 Primitive subspace of \mathcal{C}	126
	5.6 Open Jacobi diagrams	129
	5.7 Linear isomorphism $\mathcal{B} \simeq \mathcal{C}$	134
	5.8 More on the relation between \mathcal{B} and \mathcal{C}	140
	5.9 The three algebras in small degrees	142
	5.10 Jacobi diagrams for tangles	143
	5.11 Horizontal chord diagrams	150
	Exercises	152
6	Lie algebra weight systems	157
	6.1 Lie algebra weight systems for the algebra \mathcal{A}	157
	6.2 Lie algebra weight systems for the algebra \mathcal{C}	169
	6.3 Lie algebra weight systems for the algebra \mathcal{B}	181
	6.4 Lie superalgebra weight systems	187
	Exercises	190
7	Algebra of 3-graphs	195
	7.1 The space of 3-graphs	195
	7.2 Edge multiplication	196
	7.3 Vertex multiplication	201
	7.4 Action of Γ on the primitive space \mathcal{P}	204
	7.5 Lie algebra weight systems for the algebra Γ	206
	7.6 Vogel's algebra Λ	210
	Exercises	214

8	The Kontsevich integral	216
8.1	First examples	216
8.2	The construction	219
8.3	Example of calculation	223
8.4	The Kontsevich integral for tangles	225
8.5	Convergence of the integral	227
8.6	Invariance of the integral	229
8.7	Changing the number of critical points	234
8.8	The universal Vassiliev invariant	236
8.9	Symmetries and the group-like property of $Z(K)$	238
8.10	Towards the combinatorial Kontsevich integral	242
	Exercises	244
9	Framed knots and cabling operations	249
9.1	Framed version of the Kontsevich integral	249
9.2	Cabling operations	253
9.3	Cabling operations and the Kontsevich integral	258
9.4	Cablings of the Lie algebra weight systems	262
	Exercises	263
10	The Drinfeld associator	265
10.1	The KZ equation and iterated integrals	265
10.2	Calculation of the KZ Drinfeld associator	275
10.3	Combinatorial construction of the Kontsevich integral	291
10.4	General associators	302
	Exercises	308
11	The Kontsevich integral: advanced features	310
11.1	Mutation	310
11.2	Canonical Vassiliev invariants	313
11.3	Wheeling	317
11.4	The unknot and the Hopf link	330
11.5	Rozansky's rationality conjecture	334
	Exercises	336
12	Braids and string links	340
12.1	Basics of the theory of nilpotent groups	340
12.2	Vassiliev invariants for free groups	349
12.3	Vassiliev invariants of pure braids	352
12.4	String links as closures of pure braids	357
12.5	Goussarov groups of knots	362
12.6	Goussarov groups of string links	366

x	<i>Contents</i>	
	12.7 Braid invariants as string link invariants	370
	Exercises	373
13	Gauss diagrams	375
	13.1 The Goussarov theorem	375
	13.2 Canonical actuality tables	386
	13.3 The Polyak algebra for virtual knots	387
	13.4 Examples of Gauss diagram formulae	391
	13.5 The Jones polynomial via Gauss diagrams	399
	Exercises	401
14	Miscellany	402
	14.1 The Melvin–Morton Conjecture	402
	14.2 The Goussarov–Habiro theory revisited	410
	14.3 Willerton’s fish and bounds for c_2 and j_3	419
	14.4 Bialgebra of graphs	420
	14.5 Estimates for the number of Vassiliev knot invariants	424
	Exercises	432
15	The space of all knots	434
	15.1 The space of all knots	435
	15.2 Complements of discriminants	437
	15.3 The space of singular knots and Vassiliev invariants	443
	15.4 Topology of the diagram complex	448
	15.5 Homology of the space of knots and Poisson algebras	454
	<i>Appendix</i>	456
	A.1 Lie algebras and their representations	456
	A.2 Bialgebras and Hopf algebras	465
	A.3 Free algebras and free Lie algebras	480
	<i>References</i>	483
	<i>Notations</i>	496
	<i>Index</i>	499