

Definitions

Definition 1 The quantity of matter is a measure of the same that arises from its density and its volume together.

If air is doubled in density, and the volume is also doubled, this quantity is multiplied by four; if the volume is trebled it is multiplied by six. The same applies to snow or dust, if it is condensed by compression or melting. And the same is true of all bodies that are condensed in any way whatever. However, I do not take into account here the medium, if such exists, that freely pervades the interstices between the parts of a body. The above-mentioned quantity will henceforth be called ‘body’ or ‘mass’. We become aware of the mass of any body through its weight: For I have found that the mass is proportional to the weight by very accurate experiments with pendulums, as will be explained later.

When this was written the fact that the force of gravity on a body, in other words its weight, varies slightly from place to place was becoming known; see the note to Definition 7. So Newton realised that there is something fundamental that does not change, namely the quantity of matter. And so the concept of ‘mass’ was born, with a preliminary name of ‘quantity of matter’.

Newton’s concern with the possible existence and properties of the ether runs throughout *The Principia*.

Newman & Searle (1957) begin their influential textbook with the following observation: ‘The mass of a body, usually described as the quantity of matter in it, is one of the fundamental entities which are more easily understood than adequately defined.’ It is impossible to produce an adequate definition of mass, within the Newtonian framework, without reference to the laws of motion. Newton is not trying to produce definitions here that would be regarded as adequate by modern standards. He is trying to make the reader feel at home with these concepts. In the same way Euclid’s *Elements* starts with definitions of ‘point’, ‘line’, and ‘superficies’ that have no relation to anything that follows. It is striking that Newton regards density as more fundamental than mass; a case can be made out for this point of view; the density of water (at standard temperature and pressure) is a fundamental constant. But, like Euclid, Newton makes no further reference to this definition. In Corollary 4 to Proposition 6, Book 3, Newton turns this definition round, as follows: ‘When I say that particles are of the same density I mean that their masses are proportional to their volumes.’ Here *vis inertiae* has been translated, perhaps inappropriately, as ‘mass’; see Definition 3.

In order to define mass, it is sufficient to explain how the masses of two bodies are to be compared. One possibility (for small bodies) is

to compare their weights, when they are weighed in the same place. An essentially equivalent definition is to define the gravitational attraction of two particles, of masses m_1 and m_2 , at a distance r , to be Gm_1m_2/r^2 , where G is a constant (the universal gravitational constant). This also allows the comparison of the masses of celestial bodies by comparing their gravitational fields, as measured by the periods of their planets, or satellites. It is not so easy to compare the masses of celestial bodies with the masses of small terrestrial bodies in this way, or to estimate the masses of celestial bodies without planets, or satellites. But this is not a theoretical difficulty. An alternative way to compare the masses of two objects is to allow them to collide, and then apply the conservation of momentum. Thus one may define a gravitational mass and an inertial mass. In Newtonian mechanics it is a matter of experimental evidence that these two definitions are equivalent; in general relativity the two definitions cohere, or, as Einstein put it, they are *wesensgleich*. If one takes *vis inertiae* to mean ‘inertial mass’, as is inevitable, then Newton defines density in Book 3 in terms of inertial mass, and hence, in Definition 1, he is defining the mass of a body to be its inertial mass.

Newton’s decision to use the terms ‘body’ (*corpus*) and ‘mass’ (*massa*) as synonyms is rather unfortunate. The word *corpus* occurs throughout *The Principia*, with various meanings, and *massa* is very rarely used.

Definition 2 The quantity of motion is a measure of the same that arises from the speed and quantity of matter together. Quoted in the proof of Prop. 69, §11.

The motion of the whole is the sum of the motions of each of its parts; and so, in a body that is twice as great, and with the same speed, the motion is doubled; and if the speed is also doubled then the motion is quadrupled.

Much has been made of the fact that these two definitions use ‘arising from’ (*orta ex*), where *proportionalis*, as in Definitions 7 and 8, was an alternative. Any attempt to assess the significance of this variation must be measured against a detailed study of Newton’s use of elegant variation, which is a central component of his style of writing. Thus he uses words and phrases that, out of context, have slightly different shades of meaning, but within the context must bear precisely the same meaning. We take the view that any nuance between *orta ex* and *proportionalis* is far below the threshold of Newton’s variations, and is unintended.

Here ‘motion’ translates *motus*, a word used by Newton with many meanings. Following Euclid, Newton is unhappy about multiplying quantities, so he writes ‘the speed and the quantity of matter together’ (*velocitate et quantitate materiae conjunctim*). This is just a linguistic convention; he multiplies quantities when the convention would otherwise be too cumbersome; but it also assorts well with Euclidean practice by comparing quantities, rather than measuring them against some absolute scale.

There are two substantive issues with Definition 2. Is Newton defining the quantity of motion of a particle (that is, of a point mass), or of a more general body; and should ‘speed’ (a scalar) be replaced by ‘velocity’ (a vector)?

If Newton is considering a body that is not a particle, then he must define its speed, or velocity, to be that of its centre of mass.

It is easy to deduce the general case from the case of a particle. As to the question of whether he is making a weaker statement about scalars, or a stronger statement about vectors, the linguistic evidence is probably in favour of scalars; consider his comment that follows the definition. But Newton was keenly aware of momentum and velocity as vectors, even if he did not have a word that corresponds unambiguously to either vector, and he was equally aware of dynamical issues of passing between particles and general bodies. It is possible that he was happy for the definition to be interpreted in any reasonable sense.

* We refer to Appendix G for a comment on *vis insita*, or ‘intrinsic force’.

Definition 3 The *vis insita* of matter is its ability to resist, by which any body, as far as it is able, preserves its state by remaining at rest, or by moving uniformly in a straight line.*

The *vis insita* of a body is always proportional to its mass [*corpus*], and only differs from its inertia of mass [*inertia massae*] in the way it is thought of. It is the inertia of mass of a body that determines the difficulty of disturbing it from being at rest, or from moving at a uniform speed. And so the meaning of *vis insita* is well indicated by calling it the *vis Inertiae*. In fact a body only exercises this force when another force is impressed to change its state; and it comes into play in two ways, by resisting and by imparting momentum. Resistance in that the body struggles against the impressed force to maintain its current state; and imparting momentum insofar as the same body, when giving way, with difficulty, to the resistance of an obstacle, tries to impart momentum to that obstacle. It is common to ascribe resistance to objects at rest, and momentum to moving objects. But motion and rest, as commonly understood, are purely relative terms; and bodies are not always at rest when they are commonly considered to be so.

The fact that ‘bodies are not always at rest when they are commonly considered to be so’ is discussed in very great detail in the first scholium, where Newton states his belief in the concept of absolute rest. Newton nearly added a comment to the effect that Kepler, who introduced the term *Trägheit*, or ‘inertia’, in Kepler (1609), believed that bodies naturally come to rest. See Koryé & Cohen (1972, p. 40).

Definition 4 An impressed force is an action exercised on a body to change its state of rest, or of uniform speed in a fixed direction.

The force consists only of the action, and does not remain in the body after the action. For the body’s *vis inertiae* alone will keep it moving at every new speed. Moreover, an impressed force can arise from various causes; for example, from a blow, from pressure, or from a centripetal force.

Definition 5 A centripetal force is one by which bodies are drawn, impelled, or tend in any way and from all directions towards some point, as towards a centre.

Of this type is weight, by which bodies tend towards the centre of the

earth; magnetic force, by which iron is attracted towards a magnet; and that force, whatever it may be, by which planets are continually pulled aside from linear motion and are forced to revolve in curves. A stone, swung around in a sling, attempts to escape from the hand that swings it; and extends the sling more strongly by this *conatus* as it is swung faster; and as soon as it is released, away it flies. The force that acts against this *conatus*, by which the sling constantly pulls back the stone towards the hand, and keeps it in its orbit, I describe as ‘centripetal’, as it is directed towards the hand as to the centre of the orbit. And the same applies to all bodies that move in a circle. They all try to move away from the centre of the orbit; and unless some force is present to oppose this *conatus*, by which the bodies are restrained and kept in orbit, and which I therefore call a centripetal force, they will fly off in uniform motion along straight lines. In the absence of gravity, a projectile would not be deflected earthwards, but would set off to the heavens, with a uniform speed in a straight line, provided the air resistance was removed. It is pulled back from a straight path by its gravitational force, and is continuously deflected towards the earth, and that to a greater or lesser extent according to its gravitational force and speed of motion. The less its gravitational force in proportion to its mass, or the greater the speed with which it is projected, the less it will be deflected from a straight path, and the further it will go. If a leaden ball were fired horizontally at a given speed from the summit of some mountain by a cannon it might travel two miles in a curved path before falling to earth. It would go about twice as far if fired at twice the speed, and some ten times as far if fired at ten times the speed, provided that the resistance of the air were removed. And by increasing the speed it would be possible to fire the ball to any desired distance; and describing a less and less curved orbit, it might come to earth a distance of ten or thirty or ninety degrees from the starting point; or even go round the whole earth, or finally set off into the heavens, and disappear into infinity. And in the same way that a projectile could be deflected by the force of gravity into an orbit, and go round the whole earth, the moon is also deflected continuously from a straight path towards the earth and turned into its orbit, either by the force of gravity, if it is indeed a question of weight, or by some other force by which it is impelled earthwards; and without such a force the moon would not keep to its orbit. If this force were less than appropriate it would not deflect the moon sufficiently from a straight path, and if greater it would deflect it too much, and it would be deflected from its orbit towards the earth. The force has indeed to be of exactly the right strength; and it is up to Mathematicians to compute the force by which a body could be held precisely in any given orbit with a given speed; and conversely to compute the orbit of a body starting from a given point with a given velocity acted on by a given force. Now this centripetal force comes in three ways: as an absolute, accelerative, or momental force.

For *conatus*, or ‘endeavour’, see Appendix G.

The reader may wonder what Latin expression Newton can have used for his cannon, that, pre-figuring the imaginative flights of Jules Verne, was to fire the leaden ball to infinity. A more literal translation of *vi pulveris tormentarii* would have been ‘by the force of a powder operated catapult’.

The notion that the same force that causes earthly gravity might also be the force that kept the moon in orbit was one of the great inspired thoughts, whether assisted by the fall of an apple or not. It was an insight whose verification required a good estimate of the distance from the moon to the baricentre (or centre of mass) of the earth–moon system; but that is another tale, see Kollerstrom (1991). The idea that the moon was prevented from falling into the earth by a centripetal force similar to that which acts on a stone in a sling goes back at least to Plutarch’s *Moralia*, written at about 80AD; see Plutarch (1957) and Szabó (1987, p. 4).

The circle mentioned above, *gyrus* in Latin, only denotes ‘circle’ in a rather vague sense, and is used in Proposition 1, Section 2, for an arbitrary orbit. Similarly, the centre to which the centripetal force attracts is not intended to be the precise geometrical centre of the circle. In Corollary 1 of Proposition 7, Section 2, Newton considers a case where the orbit is precisely circular and the centre of the centripetal force is on the circumference of the circle, and again when it is a long way outside the circle.

Newton regarded calculating the force, given the orbit and centre of attraction, as the direct problem, and computing the orbit given the force and the initial conditions as the inverse problem. Modern usage would regard computing the orbit as the direct problem. But Newton was given the orbits of the planets, and he had to compute the force.

The three measures of a centripetal force described in Definitions 6, 7, and 8 below are rather obscure. They are respectively called the ‘absolute’, the ‘accelerative’, and the ‘momental’ measures. The word ‘momental’ has been coined here as an adjective derived from ‘momentum’. The accelerative and momental measures of a centripetal force give the change in velocity and the change of momentum that they generate, and hence are closely related. A gravitational field is an accelerative field, in that the effect of a gravitational field is to induce a given acceleration in any particle at a given point, regardless of the mass of the particle. As Newton puts it, the accelerative measure of a (gravitational) field may be referred to the point where it is measured; that is to say, gravity defines a vector field whose units are the units of acceleration. The momental measure of a centripetal force measures the rate of change of momentum that it generates, depending on its position, and thus, as Newton puts it, must be referred to the body on which it acts. The absolute measure of a centripetal force is a physical rather than a mathematical concept, and is a single parameter that determines its strength. Thus the absolute strength of a magnet may be determined by examining the magnet; as Newton puts it, the absolute measure of a centripetal force is referred to the centre of attraction. As an example of this usage, in Proposition 52 of Section 10, a centripetal force acts with a strength proportional to the distance from the centre. Newton then takes the absolute force to be V , so that the accelerative force at O is $CO \times V$, where C is the centre of attraction.

Newton sometimes tacitly assumes that the strength of a centripetal force acting on a body depends only on the distance of the body from the centre, as in Proposition 40, Section 8; but not, for example, in Proposition 7, Section 2.

Definition 6 The absolute quantity of a centripetal force is a measure of the same that is greater or less in proportion to the efficacy of the cause in propagating the force from the centre throughout the neighbouring regions.

For example the magnetic field, according to the bulk of the magnet and how strongly it has been magnetised, is greater in one magnet and less in another.

Definition 7 The accelerative quantity of a centripetal force is a measure of the same that is proportional to the speed that it generates in a given time.

Quoted in the proof of Prop. 69, §11.

For example, the strength of the same magnet is greater at a smaller remove, and less at a greater. Similarly the force of gravity is greater in valleys, less at the tops of high mountains, and even less (as will appear later) at greater distances from the sphere of the earth; but everywhere equal at equal distances, because it accelerates equally, at the same distance, all falling bodies (heavy or light, large or small), once the resistance of the air has been removed.

If the earth is assumed to be a homogeneous ball the force of gravity will become less as one moves upwards (at the tops of mountains), but also as one moves downwards from the surface of the earth (in valleys). Newton was very actively aware of this, despite the above. See Proposition 73, Section 12, Book 1, and Proposition 9, Book 3. That a pendulum swings more slowly at the top of a mountain was suggested by Hooke in a letter to Newton, dated 6 January, 1680. In this letter he states that Halley told him that he had observed at St. Helena that a pendulum swung more slowly at the top of a mountain than at the bottom; at the top of Green Mountain it would lose about 27 seconds in 24 hours. But Halley's observation, on his way to St. Helena, that a pendulum swings more slowly towards the equator was better publicised.

Definition 8 The momental quantity of a centripetal force is a measure of the same that is proportional to the momentum that it generates in a given time.

Quoted in the proof of Prop. 69, §11.

For example the force of weight is greater acting on a greater mass and less on a smaller; and in the same body greater nearer the earth, less in the heavens. This quantity is the centripetal force of the whole body, or its propensity to move towards the centre, or (as I might say) its weight; and it may be measured by the equal and opposite force needed to prevent it from falling.

And these measures of forces may, for the sake of brevity, be referred to as momental, accelerative, and absolute forces; and by way of distinction may be referred respectively to the bodies that are being drawn to the centre, to the positions of these bodies, and to the centre of attraction. Of course the momental force could be referred to the body, as being the *conatus* of the whole body towards the centre, which is the sum

of the *conatus* of each of its parts; and the accelerative force could be referred to the position of the body, as being a certain efficacy that spreads out from the centre to all the neighbouring points to move bodies that occupy these points; and the absolute force could be referred to the centre of attraction, since this is provided with some cause without which the momental forces would not be propagated to the neighbouring points, this cause being either intrinsic to a body at the centre of attraction (such as a magnet at the centre of a magnetic field, or the earth at the centre of a gravitational field) or some other cause that is not apparent. Force is considered in this work only from a mathematical point of view: For I am not considering here the causes or physical seats of forces.

The accelerative force thus bears the same relation to the momental force that speed does to momentum. For momentum arises from the speed and mass together, and momental force from the accelerative force and mass together. For the sum of the effects of the accelerative force exerted on the particles of which a body is composed is the momental force exerted on the whole. Thus, at the earth's surface, where the accelerative force of gravity, that is, the gravitational force, is the same for all bodies, the momental force of gravity, that is, the weight, is proportional to the mass; and if the body is taken up into regions where the accelerative force of gravity is less, the weight is equally reduced, and will always be proportional to the mass and the accelerative force of gravity together. Thus in regions where the accelerative force of gravity is half the normal value, if a body is replaced by one whose weight is less than that of the original by a factor of two or three, the lighter body in this region will have a weight that is a quarter or a sixth of the weight of the original body on the earth's surface.

Further I define attractive forces and impulses to be accelerative and momental in the same sense. Moreover I use words for attraction, for impulse, or for any propensity to fall towards a centre, as having the same meaning, and use them interchangeably; considering these forces not physically but only mathematically. So the reader should be careful not to consider that, by the use of such words, I am defining the type or the method of operation, or the cause or physical explanation of anything, or that I am attributing true and physical forces to centres (which, as points, are mathematical abstractions), even if I shall have happened to assert that centres attract, or that forces belong to centres.

Here '(which, as points, are mathematical abstractions)' would, more literally, be '(which are mathematical points)'. Also 'impulse' translates *impulsus*, and explicitly includes continuous forces. See Appendix G.

The very famous scholium that follows is hard to translate. Newton starts by asserting that he is giving the scientific meaning of four terms, namely *tempus*, *spatium*, *locus*, and *motus*, which he does in four correspondingly numbered paragraphs. These have been consistently translated throughout the scholium as 'time', 'space', 'place', and 'motion'. Two other terms, *dimensio* and *situs*, occur frequently,

and have been translated as ‘extent’ and ‘placement’ respectively. Of these six terms, only the translation of *tempus* as ‘time’ is inevitable. The precise meaning intended for the other words can only be judged by the context of the scholium, on account of Newton’s linguistic inconsistency.

A particular problem arises with *locus*, whose primary meaning is indeed ‘place’. Now *locus* has two plurals in Latin. Classically, the masculine *loci* means ‘places’ with no suggestion that they are related, and the neuter *loca* means ‘places that are related’. Newton almost always uses *loca*, though *loci* occurs in the third sentence of the paragraph numbered 3. We see no evidence that Newton regarded *loca* and *loci* as having different meanings. As a single exception to our translation of *locus* as ‘place’ we have translated *quasi loca* in the third sentence of the paragraph beginning ‘As the order of the parts of time is immutable’ as ‘in an orderly way’.

The issues discussed were not only of concern to Newton’s predecessors and contemporaries, but are relevant to Mach’s principle, relativity, and beyond.

Scholium

So far we have considered it proper to state the sense in which less well-known terms are to be understood in what follows. Time, space, place, and motion are very well known to everyone. But it should be observed that these quantities are commonly conceived only in relation to what the senses can detect. This gives rise to various preconceptions, and in order to remove these it is convenient to distinguish the absolute and relative, the true and apparent, the mathematical and common meanings of these terms.

1. Absolute, true, and mathematical time, of itself and by its nature, without relation to any external cause whatsoever, passes at an even rate; and by another name is known as ‘duration’. Relative, apparent, and common time is any perceptible and external measure of duration by motion (either accurate or uneven), that is commonly used in place of true time, such as hour, day, month, and year.[†]

2. Absolute space, by its nature, without relation to any external cause whatsoever, remains always the same, and at rest. Relative space is some movable measure or extent of absolute space, which is made known to our senses by placement with respect to bodies, and this, in common use, takes the place of unmoving space. Thus, for example, the extent of space that is beneath the surface of the earth, or in the air, or in the heavens above, is determined by placement with respect to the earth. Absolute and relative regions of space are identical in shape and volume, but they do not always remain exactly the same. For if the earth is considered to move, then the space occupied by our atmosphere, which always remains the same relative to and with respect to the earth, will now be one part of the absolute space through which the air passes, and now another; and so, in absolute terms, it will constantly change.[‡]

3. Position is the part of space that a body occupies, and is absolute or relative accordingly as the space is absolute or relative. I say that place is

[†] ‘Time passes at an even rate’ is meaningless; one can only ask whether two measures of time are mutually consistent. ‘Accurate or uneven’ probably refers to the fact that, for example, hours and days, as measured by a sundial, are of unequal length, as measured by a clock.

[‡] Here ‘they do not always remain exactly the same’ translates *non permanent idem semper numero*. The usual meaning of *numero* is ‘exactly’, but it could also mean ‘by number’, which is the interpretation of all translations consulted.

a part of space; it is not the placement of the body, nor is it the ambient surface. For the places occupied by bodies of the same size have the same volumes, but the surface areas of two bodies of the same size are, in general, different, because of their different shapes. Placements, properly speaking, cannot be quantified, and are not so much places as attributes of places. The motion of the whole is the same as the sum of the motions of the parts; that is to say, the translation of the whole from its place is the same as the sum of the translations of the parts from their places; and hence the place of the whole is the same as the sum of the places of the parts; and hence the place of a body is intrinsic to the body as a whole.

4. Absolute motion is the translation of a body from one absolute place to another absolute place; relative motion is the translation from one relative place to another relative place. Thus, in a ship under sail, the relative place of an object is the part of the ship where it is to be found, or that part of the hold that it occupies, which hence moves along with the ship: and to be relatively at rest is to occupy a fixed place in the ship, or a fixed part of the hold. But to be truly at rest is to remain in the same part of that unmoving space in which the ship itself, with its hold and all its contents, is moving. So if the earth is absolutely at rest, a body that is relatively at rest in the ship will have a true and absolute speed which is the speed of the ship relative to the earth. But if the earth also moves, the true and absolute motion of the body will arise in part from the true motion of the earth in unmoving space, and in part from the motion of the ship relative to the land. And if the body also moves relative to the ship, its true motion will arise in part from the true motion of the earth in unmoving space, and in part from the motion of the ship relative to the land, and of the body relative to the ship; and the motion of the body relative to the land arises from these last two relative motions. So if the region of the earth where the ship is to be found is moving eastwards at a speed of 10,010 units, and the wind and the sails bear the ship westwards at a speed of ten units, and a sailor walks eastwards along the deck at a speed of one unit, the sailor's true and absolute speed in unmoving space will be 10,001 units eastwards, and relative to the earth his speed will be nine units westwards.

Absolute time is distinguished from relative time in Astronomy by the correction of common time. For the lengths of natural days are unequal, though they are commonly regarded as equal for the measurement of time. Astronomers correct for this inequality, so that they can measure celestial movements by truer time. Perhaps there is no uniform motion by which time can be accurately measured. All motion can be speeded up or slowed down; but the flow of absolute time can never change. The duration or continuance of things is the same, whether the things are moving rapidly, slowly, or not at all. Hence this duration is rightly distinguished from its visible measures, and is calculated from these

measures by an astronomical correction. That there is a need for this correction in dealing with phenomena is shown both by an experiment with a pendulum clock, and also by the eclipses of the satellites of Jupiter.

The length of a day does not refer to daylight hours but to the time from noon to noon, where noon is the time when the sun is at its highest. It depends, in the first place, on the angular velocity of the earth about its axis, as measured against the fixed stars, and secondarily on the angular velocity of the earth about the sun. The variation in the length of a day has two causes. First of all, the distance of the earth from the sun varies, because of the eccentricity of the earth's orbit, and when the earth is nearer the sun its angular velocity about the sun is greater, by Kepler's second law. This gives a variation with a period of a year. The second cause, with an amplitude similar to that of the first, and a period of six months, lies in the fact that the axis around which the earth rotates is not orthogonal to the ecliptic, and this causes a variation in the way that the rotation of the earth about its axis and that about the sun combine to give a solar day. The discrepancy between clock time and sundial time is known as 'the equation of time'; though in the now obsolete sense of equation: 'the correction of time' would be a better modern expression.

There were two direct ways to measure true time as opposed to solar time. One was to use a clock. The famous clock makers of the time were Huygens, Hooke, and Harrison. Huygens invented the pendulum clock in 1657. The primary objective was to make clocks accurate enough and seaworthy enough to enable mariners to calculate longitude. Clearly a spring balance has a better prospect of being seaworthy, and Hooke and Huygens quarrelled over the patent. By far the greatest of the three, as a clockmaker, was Harrison. He built his first clock in 1713, and his first nautical clock, the famous H1, was completed in 1735. At the time that Newton was writing, clocks were not very accurate. The more reliable measure of absolute time came from observing the satellites of Jupiter, whose eclipses could be observed, and whose periods could be taken as constant. A complication was the fact that the light from Jupiter takes a significant amount of time to reach the earth. As the distance between Jupiter and the earth varies, so does this time. This enabled Rømer, in 1676, to produce the first measurement of the speed of light.

As the order of the parts of time is immutable, so also is the order of the parts of space. Were these parts to be displaced from their places they would be displaced (as I would say) away from themselves. For times and places are related to each other in an orderly way. All things are placed in order; in time with respect to the order of succession, and in space with respect to the order of placement. The essence of time and of space is the essence of place, and for the primary places to be moved would be absurd. Primary places are, therefore, absolute places; and it is only motions with respect to these places that are absolute motions.

This is another difficult passage. Newton is arguing for the concept of absolute rest, as a sequel to his argument in the previous paragraph for absolute time. Here 'order' is meant in a rather general sense, as a geometrical relationship; and the parts of time and of space are the moments in time and the points in space.

Moreover, since these parts of space cannot be seen, and hence cannot