

Contents

	<i>Preface</i>	<i>page ix</i>
1	First and second variational formulas for area	1
2	Volume comparison theorem	10
3	Bochner–Weitzenböck formulas	19
4	Laplacian comparison theorem	32
5	Poincaré inequality and the first eigenvalue	40
6	Gradient estimate and Harnack inequality	57
7	Mean value inequality	68
8	Reilly’s formula and applications	77
9	Isoperimetric inequalities and Sobolev inequalities	86
10	The heat equation	96
11	Properties and estimates of the heat kernel	109
12	Gradient estimate and Harnack inequality for the heat equation	122
13	Upper and lower bounds for the heat kernel	134
14	Sobolev inequality, Poincaré inequality and parabolic mean value inequality	149

viii	<i>Contents</i>	
15	Uniqueness and the maximum principle for the heat equation	169
16	Large time behavior of the heat kernel	177
17	Green's function	189
18	Measured Neumann Poincaré inequality and measured Sobolev inequality	203
19	Parabolic Harnack inequality and regularity theory	216
20	Parabolicity	241
21	Harmonic functions and ends	256
22	Manifolds with positive spectrum	267
23	Manifolds with Ricci curvature bounded from below	284
24	Manifolds with finite volume	299
25	Stability of minimal hypersurfaces in a 3-manifold	306
26	Stability of minimal hypersurfaces in a higher dimensional manifold	315
27	Linear growth harmonic functions	326
28	Polynomial growth harmonic functions	340
29	L^q harmonic functions	349
30	Mean value constant, Liouville property, and minimal submanifolds	361
31	Massive sets	370
32	The structure of harmonic maps into a Cartan–Hadamard manifold	381
<i>Appendix A</i>	Computation of warped product metrics	392
<i>Appendix B</i>	Polynomial growth harmonic functions on Euclidean space	395
	<i>References</i>	399
	<i>Index</i>	404