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GEOMETRIC ANALYSIS

The aim of this graduate-level text is to equip the reader with the basic tools and techniques needed for research in various areas of geometric analysis. Throughout, the main theme is to present the interaction of partial differential equations (PDE) and differential geometry. More specifically, emphasis is placed on how the behavior of the solutions of a PDE is affected by the geometry of the underlying manifold, and vice versa. For efficiency, the author mainly restricts himself to the linear theory, and only a rudimentary background in Riemannian geometry and partial differential equations is assumed.

Originating from the author's own lectures, this book is an ideal introduction for graduate students, as well as a useful reference for experts in the field.

Peter Li is Chancellor's Professor at the University of California, Irvine.

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I would like to dedicate this book to my wife, Glenna, for her love and unwavering support.

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Preface

The main goal of this book is to present the basic tools that are necessary for research in geometric analysis. Though the main theme centers around linear theory, i.e., the Laplace equation, the heat equation, and eigenvalues for the Laplacian, the methods of dealing with these problems are quite often useful in the study of nonlinear partial differential equations that arise in geometry.

A small portion of this book originated from a series of lectures given by the author at a Geometry Summer Program in 1990 at the Mathematical Sciences Research Institute in Berkeley. The lecture notes were revised and expanded when the author taught a regular course in geometric analysis. During the author's visit to the Global Analysis Research Institute at Seoul National University, he was encouraged to submit these notes, though still in a rather crude form, for publication in their lecture notes series [L6].

The part of this book that concerns harmonic functions originated from the author's lecture notes for a series of courses he gave on the subject at the University of California, Irvine. A part of this material was also used in a series of lectures the author gave at the XIV Escola de Geometria Diferencial in Brazil during the summer of 2006. These notes [L9] were printed for distribution to the participants of the program.

As well as updating the Korean lecture notes with more recent developments and combining with them the harmonic function notes, the author has also inserted a treatment on the heat equation. The result takes the form of an introduction to the subject of geometric analysis on the one hand, with some application to geometric problems via linear theory on the other. Due to the vast literature in geometric analysis, it is prudent not to make any attempt to try to discuss nonlinear theory. The interested reader is encouraged to consult the excellent book of Schoen and Yau [SY2] in this direction.

The aim of this book is to address entry-level geometric analysts by introducing the basic techniques in the most economical way. The theorems discussed are chosen sometimes for their fundamental usefulness and sometimes for the purpose of demonstrating various techniques. In many cases, they do not represent the best possible or most current results.

The book is roughly divided into three main parts. The first part (Chapters 1–9) contains basic background material, including reviews of various topics that may be found in a standard Riemannian geometry book. It also provides a quick glimpse of a powerful technique, namely the maximal principle method, in obtaining estimates on a manifold.

The second part (Chapters 10–19) gives an outline of the theory of the heat equation that forms a basis for further study of nonlinear geometric flows. It also established various estimates for nonnegative solutions of the heat equation. As a consequence, estimates for the constants that appear in the Sobolev inequality, the Poincaré inequality, and the mean value inequality in terms of geometric quantities are established. Chapters 18 and 19 are quite technical and contain a presentation of Moser's argument for the parabolic Harnack inequality on a manifold. Moreover, the dependency on the background geometry is explicitly stated.

The last part (Chapters 20–32) of the book is primarily on harmonic functions and various applications to other geometric problems, such as minimal surfaces, harmonic maps, and the geometric structure of certain manifolds.

The author would like to express his gratitude to Ovidiu Munteanu, Lei Ni, and Jiaping Wang for their suggestions on how to improve this book. He is particularly in debt to Munteanu for his detailed proof-reading of the draft. Acknowledgement is also due to the author's graduate students Lihan Wang and Fei He, who were extremely helpful in pointing out necessary corrections to the manuscript. The preparation of this manuscript was partially supported by NSF grant DMS-0801988.