# 1 Introduction

The economy is a complex system with nonlinear interactions and feedback loops. Early traces of this view date back, for example, to Schumpeter and Hayek, and to Simon. The complexity modeling paradigm has been strongly advocated since the 1980s by economists and multidisciplinary scientists from various fields, such as physics, computer science and biology, linked to the Santa Fe Institute.[1] More recently the complexity view has also drawn the attention of policy makers, who are faced with complex phenomena, irregular fluctuations and sudden, unpredictable market transitions. For example, the chairman of the FED, Ben Bernanke, noted that the 1000-point collapse of the Dow Jones Industrial Average on the afternoon of May 6, 2010, reflected the complexity of financial-market systems:

> The brief market plunge was just a small indicator of how complex and chaotic, in the formal sense, these systems have become. Our financial system is so complicated and so interactive – so many different markets in different countries and so many sets of rules. What happened in the stock market is just a little example of how things can cascade or how technology can interact with market panic.
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> (interview Ben Bernanke, IHT, May 17, 2010).

The recent financial-economic crisis is a dramatic example of large movements, similar to critical transitions that are so characteristic for complex evolving systems. These large changes of global financial markets can hardly be viewed as a rational response to news about economic fundamentals and cannot be explained by traditional representative rational agent macro-finance models. A more compelling and intuitive explanation is that these extreme large movements have been triggered by bad economic news, and subsequently strongly amplified by an "irrational" overreaction of a heterogeneous population of boundedly rational, interacting agents. In a well-known speech the former president of the ECB, Jean-Claude Trichet, called for a new approach for policy makers to managing crises:

> First, we have to think about how to characterise the homo economicus at the heart of any model. The atomistic, optimising agents underlying existing models do not capture behaviour during a

[1] See, e.g., the early collections of papers in the Santa Fe conference proceedings Anderson et al. (1988) and Arthur et al. (1997a).

> crisis period. We need to deal better with heterogeneity across agents and the interaction among those heterogeneous agents. We need to entertain alternative motivations for economic choices. Behavioural economics draws on psychology to explain decisions made in crisis circumstances. Agent-based modelling dispenses with the optimisation assumption and allows for more complex interactions between agents. Such approaches are worthy of our attention.
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> Second, we may need to consider a richer characterisation of expectation formation. Rational expectations theory has brought macroeconomic analysis a long way over the past four decades. But there is a clear need to re-examine this assumption. Very encouraging work is under way on new concepts, such as learning and rational inattention.
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> (Speech by Jean-Claude Trichet, ECB Central Banking Conference, Frankfurt, November 18, 2010)

This book presents some simple, stylized complexity models in economics. Our main focus will be an underlying *behavioral theory of heterogeneous expectations* of boundedly rational individual agents in a complex, adaptive economic environment. We will also discuss empirical validation, both at the micro and at the macro level, of a behavioral theory of heterogeneous expectations through financial time series data and laboratory experiments with human subjects. The need for an empirically grounded behavioral theory of expectations for economic dynamics has already been stressed by Herb Simon (1984, p. 54):

> A very natural next step for economics is to maintain expectations in the strategic position they have come to occupy, but to build an empirically validated theory of how attention is in fact directed within a social system, and how expectations are, in fact, formed. Taking that next step requires that empirical work in economics take a new direction, the direction of micro-level investigation proposed by Behavioralism.

## 1.1 Economic dynamics, nonlinearity and complexity

Economic dynamics is concerned with modeling fluctuations in economic and financial variables, such as commodity prices, output growth, unemployment, interest rates, exchange rates and stock prices. Broadly speaking, there are two contrasting views concerning the main sources of economic fluctuations. According to the first, business cycles are mainly driven by "news" about economic fundamentals, that is, by random *exogenous* shocks to preferences, endowments, technology, firms' future earnings or dividends, etc. These random shocks typically act on an inherently stable (linearized) economic system. This view dates back to the 1930s, to Frisch, Slutsky and Tinbergen, who showed that a stable linear system subject to an irregular sequence of external, random shocks may produce fluctuations very similar to those observed in real business cycles.

The *linear, stable view* was criticized in the 1940s and 1950s, mainly because it did not offer an *economic* explanation of observed fluctuations, but rather attributed those fluctuations to external, non-economic forces. As an alternative, Goodwin, Hicks and Kaldor developed nonlinear, *endogenous* business cycle models, with the savings-investment mechanism as the main economic force generating business fluctuations. According to this *nonlinear view*, the economy may be intrinsically unstable and, even in the absence of external shocks, fluctuations in economic variables can arise. These early Keynesian nonlinear business cycle models, however, were criticized for at least three reasons. Firstly, the limit cycles generated by these models were much too regular to explain the sometimes highly irregular movements in economic and financial time series data. Secondly, the "laws of motion" were considered to be "ad hoc," since they had not been derived from micro foundations, i.e., from utility and profit maximization principles. A third important critique was that agents' behavior was considered as *irrational*, since their *expectations were systematically wrong* along the regular business cycles. Smart, rational traders would learn from experience to anticipate these cyclic movements and revise their expectations accordingly, and, so the story goes, this would cause the cycles to disappear.

These shortcomings triggered the rational expectations revolution in the 1960s and 1970s, inspired by the seminal papers of Muth (1961) and Lucas (1972a and b). New classical economists developed an alternative within the exogenous approach, the stochastic real business cycle (RBC) models, pioneered by Kydland and Prescott (1982). RBC models fit into the general equilibrium framework, characterized by utility-maximizing consumers, profit-maximizing firms, market clearing for all goods at all dates and all traders having rational expectations. More recently, New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models have moved to the forefront of macroeconomic modeling and policy analysis (Clarida et al., 1999; Woodford 2003). Typically these DSGE models are log linearized and assume a representative rational agent framework. A representative, perfectly rational agent nicely fits into a linear view of a globally stable, and hence predictable, economy. By the late 1970s and early 1980s, the debate concerning the main source of business cycles seemed to have been settled in favor of the exogenous shock hypothesis, culminating in the currently dominating DSGE macro models for policy analysis.

### *1.1.1 The discovery of chaos*

In mathematics and physics the view on modeling dynamic phenomena changed dramatically in the 1960s and 1970s due to the discovery of *deterministic chaos*. One of its pioneers, the MIT meteorologist Edward Lorenz (1963), discovered by computer simulations that a simple nonlinear system of three differential equations can generate highly irregular and seemingly unpredictable time series patterns.[2] Moreover, his stylized model of weather prediction was characterized by *sensitive dependence on initial conditions* (the "butterfly effect"): a small perturbation of the initial state leads to a completely different time path prediction in the medium or long run. In the 1970s, Ruelle and Takens (1971) presented a mathematical proof that a simple nonlinear system of three or four differential equations, without any external random disturbances, can indeed exhibit complicated, irregular long run dynamical behavior. They introduced

[2] See, e.g., Gleick (1987) for a stimulating historical overview of "chaos theory." It is interesting to note that one of the traditional Keynesian business cycle models from the 1950s, Hicks' classical nonlinear trade cycle model with ceilings and floors, can in fact generate irregular, chaotic time series. In particular, figures 9 and 10 in Hicks (1950, pp. 76–79), computed by hand at the time, are similar to the computer simulated chaotic series in Hommes (1995), so that in some sense Hicks was close to discovering chaos in his trade cycle model.

the notion of a *strange attractor* to describe irregular long run behavior in a nonlinear deterministic dynamical system. The discovery of deterministic chaos and strange attractors shattered the Laplacian deterministic view of perfect predictability and made scientists realize that, because initial states can only be measured with finite precision, long run prediction may be fundamentally impossible, even when the laws of motion are perfectly known.

In the 1970s, there was yet another important mathematical article with the illuminating title "Period three implies chaos" (Li and Yorke, 1975), which played a stimulating role and was particularly important for applications. Li and Yorke showed that for a large class of simple nonlinear difference equations in one single state variable, a simple sufficient "period three" condition already implies complicated, chaotic dynamical behavior. The best-known example concerns logistic population growth in biology, as described by May (1976). These and other simple mathematical examples together with the rapidly increasing availability of computers for numerical simulations led to an explosion of interest in nonlinear dynamics in mathematics, physics and other applied sciences.

The "chaos revolution" in the 1970s had its roots, however, much earlier, at the end of the nineteenth century in the famous French mathematician Henri Poincaré. In 1887 king Oskar II of Sweden promised a prize to the best essay concerning the question "Is our solar system stable?" In his prize-winning essay, Poincaré (1890) showed that the motion in a simple three-body system, a system of sun, earth and moon, need not be periodic, but may become highly irregular and unpredictable. In modern terminology he showed that chaotic motion is possible in a three-body system. Poincaré introduced the notion of a so-called *homoclinic point*, an intersection point between the stable and the unstable manifolds of an equilibrium steady state. His notion of homoclinic orbits turned out to be a key feature of complicated motion and strange attractors and may be seen as an early signature of chaos.

### *1.1.2 Economic applications of chaos*

In the 1980s, inspired by "chaos theory" and within the tradition of endogenous business cycle modeling, economic theorists started looking for nonlinear, deterministic models generating erratic time series similar to the patterns observed in real business cycles. This search led to new, simple nonlinear business cycle models, within the Arrow–Debreu general equilibrium paradigm of optimizing behavior, perfectly competitive markets and rational expectations, generating chaotic business fluctuations (e.g., Benhabib and Day, 1982 and Grandmont, 1985; see, e.g., Lorenz, 1993 for an overview of nonlinear business cycle models and chaos). These model examples show that irregular, chaotic fluctuations can arise under the New Classical Economics paradigm in a perfectly rational representative agent framework. It turned out to be more difficult, however, to calibrate or estimate such chaotic business cycle models to real economic data.

Simultaneously, the search for nonlinearity and chaos in economics was undertaken from an empirical perspective. In physics and mathematics nonlinear methods to distinguish between truly random and deterministic chaotic time series had been developed. For example, correlation dimension tests and Lyapunov exponent tests had been developed by Takens (1981) and Grassberger and Procaccia (1983). When the correlation dimension of a time series is low, this suggests evidence for low-dimensional chaos. In economics, for example, Brock and Sayers (1988) found a correlation dimension of about 3 for macroeconomic data (postwar quarterly US unemployment rates), and Scheinkman and LeBaron (1989) a correlation dimension of about 6 for stock market data (weekly stock returns). A problem for applying these empirical methods, particularly relevant for economic data, is that they require very long time series and that they are extremely sensitive to noise. Furthermore, it turned out that time series generated by fitted stochastic alternative models, such as linear, near unit root autoregressive models for macro data or GARCH-models for stock returns, also generate low correlation dimensions of comparable size. Hence, from these empirical findings, one *cannot* conclude that there is evidence for low-dimensional, purely deterministic chaos in economic and financial data. Brock, Dechert, Scheinkman and LeBaron (1996) have developed a general test (the BDS test), based upon the notion of correlation dimension, to test for *nonlinearity* in a given time series; see Brock et al., (1991) for the basic theory, references and applications. The BDS test has become widely used, in economics but also in physics, and has high power against many nonlinear alternatives. From an empirical viewpoint, evidence for low-dimensional, purely deterministic chaos in economic and financial data is weak, but there is strong evidence for nonlinear dependence. At the same time, it seems fair to add that, because of the sensitivity to noise of these methods, the hypothesis of chaos buffeted with (small) dynamic noise has *not* been rejected either.[3] Nor has higher-dimensional chaos been rejected by these time series methods.

Empirical difficulties, both in calibrating new classical nonlinear endogenous business cycle models to economic data and in finding evidence for low-dimensional chaos in economic and financial time series, thus prevented a full embracement and appreciation of nonlinear dynamics in economics in the 1980s and early 1990s.

### *1.1.3 Expectations*

The most important difference between economics and the natural sciences is perhaps the fact that decisions of economic agents today depend upon their *expectations* or *beliefs* about the future. To illustrate this difference, weather forecasts for tomorrow will not affect today's weather, but investors' predictions about future stock prices may affect financial market movements today. A classic example is the Dutch "tulip mania" in the seventeenth century, as described in Kindleberger (1996). The dreams and hopes of Dutch investors for excessive high returns on their investments in tulip bulbs may have exaggerated the explosion of the price of tulip bulbs by a factor of more than 20 at the beginning of 1636, and its crash back to its original level by the end of that year. Another more recent example is the "dot-com bubble," the rapid run up of stock

[3] See Hommes and Manzan (2006) for a brief recent discussion.

prices in financial markets worldwide in the late 1990s, and the subsequent crash. This rise in stock prices was triggered by good news about economic fundamentals, a new communication technology, the internet. An overoptimistic estimate of future growth of ICT industries seems to have contributed to and strongly reinforced the excessively rapid growth of stock prices in 1995–2000, leading to extreme overvaluation of stock markets worldwide, and their subsequent fall in 2000–2003. A more recent example is the 2008–2012 financial-economic crisis. It is hard to believe that the decline of worldwide financial markets in 2008 of more than 50% was completely driven by changes in economic fundamentals. Rather it seems that the large decline was strongly amplified by pessimistic expectations and market psychology. A similar observation applies to the 2011–2012 EU debt crisis. While the budget deficits of EU countries are partly caused by economic fundamentals, the sharp rise in the spread of, e.g., Italian and German bonds in 2011 seems to have been exaggerated by investors' pessimistic expectations. The predictions, expectations or beliefs of consumers, firms and investors about the future state of the economy are part of the "law of motion." The economy is a highly nonlinear *expectations feedback* system, and therefore a *theory of expectations* is a crucial part of any dynamic economic model or theory.

Since the introduction of rational expectations by Muth (1961) and its popularization in macroeconomics by Lucas (1972a and b) and others, the *rational expectations hypothesis* (REH) became the dominating expectations formation paradigm in economics. According to the REH all agents are rational and take as their subjective expectation of future variables the objective prediction by economic theory. In economic modeling practice, expectations are given as the mathematical conditional expectation given all available information. Rational agents do not make "systematic mistakes" and their expectations are, on average, correct. The REH provides an elegant "fixed-point" solution to an economic expectations feedback system by imposing that, on average, expectations and realizations coincide. In the absence of exogenous shocks, rational expectations implies that agents have perfect foresight and make no mistakes at all. This shortcut solution excludes all irrationality and market psychology from economic analysis, and instead postulates that expectations are in equilibrium and perfectly self-fulfilling.

The rational expectations revolution in economics took place *before* the discovery of chaos, at least before the time that the irregular behavior and complexity of nonlinear dynamics were widely known among economists. The fact that chaos can arise in simple nonlinear systems and its implications for limited predictability, however, shed important new light on the expectations hypothesis. In a simple (linear) stable economy with a unique steady state, predictability prevails and it seems natural that agents may have rational expectations, at least in the long run. A representative, perfectly rational agent model nicely fits into a linear view of a globally stable and predictable economy. But how can agents have rational expectations or perfect foresight in a *complex, nonlinear world*, when the true law of motion is unknown and prices and quantities move irregularly on a strange attractor exhibiting sensitivity to initial conditions? A boundedly rational world view with agents using simple forecasting strategies, which may not be perfect but are at least approximately right, seems more appropriate for a complex nonlinear environment. Indeed, already around 1900 Poincaré, one of the founding fathers of nonlinear dynamics, expressed his concerns about the implications of limited predictability in nonlinear systems for economics in a letter to Walras, one of the founders of mathematical economics:[4]

> You regard men as infinitely selfish and infinitely farsighted. The first hypothesis may perhaps be admitted in a first approximation, the second may call for some reservations.

### *1.1.4 Bounded rationality and adaptive learning*

In economics in the 1950s, Herbert Simon emphasized that rationality requires extreme assumptions concerning agents' information gathering and computing abilities. Firstly, rational agents are typically assumed to have perfect information about economic fundamentals and perfect knowledge about underlying market equilibrium equations. This assumption seems unrealistically strong, especially since the "law of motion" of the economy depends on the expectations of *all other* agents. Secondly, even if such information and knowledge were available, typically in a nonlinear market equilibrium model it would be very hard, or even impossible, to derive the rational expectations forecast analytically, and it would require quite an effort to do it computationally. As an alternative, Simon strongly argued for *bounded rationality*, with limited computing capabilities and agents using simple rules of thumb instead of perfectly optimal decision rules, as a more accurate and more realistic description of human behavior. Simon's reasoning lost against the rational expectations revolution in the 1970s, but in the last two decades similar reasoning has caused an explosion of interest in bounded rationality. Modeling a world with boundedly rational agents, who adapt their behavior and learn from past experiences over time, leads to a complex and highly nonlinear dynamic system.

A common assumption underlying models of bounded rationality is that agents do *not* know the actual "law of motion" of the economy, but instead base their forecasts upon time series *observations*. They behave like economic statisticians, forming expectations based upon time series observations, using a simple statistical model for their perceived law of motion. *Adaptive learning*, sometimes also referred to as *statistical learning*, means that agents adapt their beliefs over time by updating the parameters of their perceived law of motion according to some learning scheme (e.g., recursive ordinary least squares), as additional observations become available. The adaptive learning approach has been used extensively in macroeconomics. Sargent (1993) gives an early overview of learning in macroeconomics, while Evans and Honkapohja (2001) contains a more recent extensive and detailed treatment; see also Conlisk (1996) for a stimulating discussion of bounded rationality. An important issue that has received much attention in the literature is the *stability* of rational expectations equilibria under adaptive learning. If adaptive learning enforces convergence to a rational

[4] Front quotation in Grandmont (1998) and Ingrao and Israel (1990), from letter of October 1, 1901 of Henri Poincaré to Léon Walras.

expectations equilibrium, the REH would be more plausible as a (long run) description of the economy, since the underlying informational assumptions could be considerably relaxed. However, many examples have been found where adaptive learning does *not* converge to rational expectations, but rather settles down to some kind of "learning equilibrium" displaying endogenous, sometimes even chaotic, fluctuations and excess volatility (e.g., Bullard, 1994, Grandmont, 1998, Hommes and Sorger, 1998 and Schönhofer, 1999).

### *1.1.5 Heterogeneity in complex adaptive systems*

The representative agent model has played a dominant role in modern economics for quite some time. Most rational expectations models assume a single, *representative agent*, representing average consumer, average firm or average investment behavior. An important motivation for the rational agent model dates back to the 1950s, to Milton Friedman (1953) who argued that non-rational agents will be driven out of the market by rational agents, who will trade against them and earn higher profits. In recent years however, this view has been challenged and heterogeneous agent models are becoming increasingly popular in finance and in macroeconomics. Kirman (1992, 2010), for example, provides an illuminating critique on representative rational agent modeling.

Bounded rationality and learning in a complex environment naturally fit with *heterogeneous expectations*, with the economy viewed as a complex evolving system composed of many different, boundedly rational, interacting agents, using different decision strategies, heuristics and forecasting rules. Heterogeneous strategies compete against each other and an evolutionary selection mechanism, e.g., through genetic algorithm learning, disciplines the class of strategies being used by individual agents. In such a complex system, expectations and realizations coevolve over time. The work at the Santa Fe Institute has played a stimulating role and the collections of papers in Anderson et al. (1988) and Arthur et al. (1997a) of Santa Fe conferences provide early examples of the complexity modeling approach in economics.

The complexity view in economics is naturally linked to *agent-based computational economics (ACE)*, characterized by agent-based computer simulation models with many heterogeneous agents; see, e.g., the recent *Handbook* of Tesfatsion and Judd (2006) for surveys of the state of the art of ACE. An advantage of agent-based models is that one can use a "bottom up" approach and build "realistic" models from micro interactions to simulate and mimic macro phenomena. However, in agent-based models with many interacting agents, the "wilderness of bounded rationality" is enormous, there are infinitely many possibilities for individual decision rules and, for a given model, it is often hard to pin down what exactly causes certain stylized facts at the macro level in agent-based micro simulations.

### *1.1.6 Behavioral rationality and heterogeneous expectations*

A good feature of the rational expectations hypothesis (REH) is that it imposes strong discipline on agents' forecasting rules and minimizes the number of free parameters in dynamic economic models. In contrast, the "wilderness of bounded rationality" in agent-based models leaves many degrees of freedom in economic modeling, and it seems far from clear which rules are the most reasonable out of an infinite class of potential behavioral rules. Stated differently in a popular phrase: "*There is only one way (or perhaps a few ways) you can be right, but there are many ways you can be wrong*." To avoid "ad hoccery," a successful bounded rationality research program needs to discipline the class of expectations and decision rules. The REH assumes *perfect consistency* between beliefs and realizations. For a successful bounded rationality research program a *reasonable* and *plausible* form of consistency between beliefs and realizations is necessary.

This book focusses on "simple" complexity models, where only a few different types of heterogeneous agents interact. Our main focus is on the role of *behavioral rationality* and *heterogeneous expectations* within stylized complexity models. Our consistency story of bounded rationality contains three important elements: (i) agents use simple decision rules, with an intuitive behavioral interpretation; (ii) agents switch between different decision rules based on evolutionary selection and learning; and (iii) the models of bounded rationality are empirically validated, at both the micro and the macro levels.

Behavioral rationality emphasizes the use of simple, intuitive decision rules – *heuristics* – with a plausible behavioral interpretation. These heuristics are not perfect and need not be optimal, but within an environment that is too complex to fully understand individual agents look for simple decision rules that perform reasonably well to a first-order approximation; for a similar approach and extensive discussions, see, e.g., the collection of papers on smart heuristics and the adaptive toolbox in Gigerenzer et al. (1999) and Gigerenzer and Selten (2001).

Two forms of learning further discipline the class of decision heuristics. First, we use the heterogeneous strategy switching framework of Brock and Hommes (1997a, 1998) of *endogenous evolutionary selection* or *reinforcement learning* among heterogeneous decision or forecasting rules. The main idea here is that agents tend to switch to rules that have performed better, according to some suitable economic performance measure such as realized profits or forecasting accuracy, in the recent past. The forecasting rules may be divided into different classes, with different degrees of rationality, ranging from simple behavioral rules such as naive or adaptive expectations, trend extrapolating rules or contrarian rules, to more sophisticated rules, such as statistical learning rules, fundamental market analysis or even rational expectations. These more sophisticated rules may be more costly – due to information-gathering costs – than alternative forecasting heuristics. The second form of learning takes place within each class of forecasting heuristics, with some parameters changing over time following some adaptive learning process. For example, within the class of trend-following heuristics, the trend coefficient or the anchor from which the trend is extrapolated may change over time and depend upon market realizations. This type of learning also has a behavioral interpretation and can be linked to the *anchor and adjustment* heuristics used in psychology (e.g., Tversky and Kahneman, 1974, Kahneman, 2003).

To discipline behavioral models and boundedly rational decision heuristics, *empirical validation both at the micro and at the macro level* is important. Laboratory experiments with human subjects, in particular experimental macroeconomics, plays a key role here, with the experimenter having full control over the type of micro interactions and the macroeconomic fundamentals. Duffy (2006, 2008a and b) provides a stimulating overview of experimental macroeconomics; the learning-to-forecast experiments surveyed in Hommes (2011) can be used to study the interactions of individual heterogeneous expectations and their aggregate effect in the laboratory.

Behavioral rationality and heterogeneous expectations naturally lead to highly *nonlinear* dynamical systems, because the fractions attached to the different rules are changing over time. Evolutionary selection of heterogeneous expectations sometimes enforces convergence to a rational expectations equilibrium. More often, however, the evolutionary system may be unstable and exhibit complicated, perpetual fluctuations, with several simple forecasting heuristics surviving evolutionary selection. In particular, we will see that when some rules act as "far from the steady state stabilizing forces" and other rules act as "close to the steady state destabilizing forces," evolutionary selection of expectations rules may lead to Poincaré's classical notion of a homoclinic orbit and may be seen as a signature of potential instability and chaos in a complex adaptive system with behaviorally rational agents.

An economy with heterogeneous, behaviorally rational agents is a highly nonlinear complex evolving system. The tools of nonlinear dynamics and complex systems are crucial to understand the behavior of markets with heterogeneous boundedly rational agents and to provide the insights to managing complex adaptive systems. This book introduces the most important analytical and computational tools in simple nonlinear complexity models and applies them to study economic dynamics with heterogeneous boundedly rational agents and learning. The remainder of this introduction gives the reader a quick overview of the contents of the book, discussing important concepts such as behavioral rationality and heterogeneous expectations in some simple examples of complex economic systems and briefly discussing their empirical validation with time series data and laboratory experiments with human subjects.

## 1.2 Adaptive expectations in a nonlinear economy

The simplest economic example nicely illustrating the role of expectations feedback is the "hog cycle" or *cobweb model*. Traditionally it has played a prominent role as a didactic benchmark model and has been used, for example, in the seminal article of Muth (1961) introducing rational expectations. Here we focus on the role of simple expectation rules, in particular adaptive expectations, in a *nonlinear* cobweb model.

The model is partial equilibrium and describes an independent competitive market for a non-storable consumption good, such as corn or hogs. Production takes a fixed unit of time, and suppliers therefore have to base their production decision upon their anticipation or expectation $p_t^e$ of the market equilibrium price $p_t$ that will prevail.