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978-1-107-01887-7 - Encyclopedia of Mathematics and its Applications: Bitangential Direct and Inverse Problems for Systems of Integral and Differential Equations

Damir Z. Arov and Harry Dym

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