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This largely self-contained treatment surveys, unites and extends some 20 years of research on direct and inverse problems for canonical systems of integral and differential equations and related systems. Five basic inverse problems are studied in which the main part of the given data is either a monodromy matrix; an input scattering matrix; an input impedance matrix; a matrix-valued spectral function; or an asymptotic scattering matrix. The corresponding direct problems are also treated.

The book incorporates introductions to the theory of matrix-valued entire functions, reproducing kernel Hilbert spaces of vector-valued entire functions (with special attention to two important spaces introduced by L. de Branges), the theory of J-inner matrix-valued functions and their application to bitangential interpolation and extension problems, which can be used independently for courses and seminars in analysis or for self-study. A number of examples are presented to illustrate the theory.

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ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

Bitangential Direct and Inverse Problems for Systems of Integral and Differential Equations

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Dedicated to our wives Natasha and Irene,
for their continued support and encouragement, and
for being ideal companions on the path of life.

CONTENTS

PREFACE

This book is devoted to direct and inverse problems for canonical integral and differential systems. Five basic problems are considered: those in which an essential part of the data is either (1) a monodromy matrix; or (2) an input scattering matrix; or (3) an input impedance matrix; or (4) a spectral function; or (5) an asymptotic scattering matrix.

There is a rich literature on direct and inverse problems for canonical integral and differential systems and for first- and second-order differential equations that can be reduced to such systems. However, the intersection between most of this work and this book is relatively small. The approach used here combines and extends ideas that originate in the fundamental work of M.G. Krein, V.P. Potapov and L. de Branges:

M.G. Krein studied direct and inverse problems for Dirac systems (and differential equations that may be reduced to Dirac systems) by identifying the matrizant of the system with a family of resolvent matrices for assorted classes of extension problems that are continuous analogs of the classical Schur and Carathéodory extension problems.

In this monograph we present bitangential generalizations of the Krein method that is based on identifying the matrizants of canonical systems of equations as resolvent matrices of an ordered family of bitangential generalized interpolation/extension problems that were studied earlier by the authors and are also reviewed in reasonable detail in the text.

The exposition rests heavily on the theory of J-inner mvf's (matrix-valued functions) that was developed and applied to a number of problems in analysis (including the inverse monodromy problem for canonical differential systems) by V.P. Potapov in his study of J-contractive mvf's. The parts of this theory that are needed here are taken mainly from our earlier monograph *J-Contractive Matrix Valued Functions and Related Topics*. Nevertheless, in order to keep this monograph reasonably self-contained, material that is needed for the exposition

is repeated as needed, though usually without proof, and often in simpler form if that is adequate for the application at hand.

Extensive use is made of reproducing kernel Hilbert spaces of vector-valued entire functions of the kind introduced by L. de Branges. This theory is developed further and used to provide alternate characterizations of certain classes of entire J-inner mvf's and some useful results on the isometric inclusion of some nested families of reproducing kernel Hilbert spaces.

We have tried to give a reasonable sample of the literature that we felt was most relevant to the topics developed in the monograph. But for every item listed, there are tens if not hundreds of articles that are somewhat connected. To keep the length of the list of references reasonable, we have not referenced articles that deal with canonical systems on the full line, or non-Hermitian Hamiltonians and Hamiltonians with negative eigenvalues.

The authors gratefully acknowledge and thank the administration of South-Ukrainian National Pedagogical University for authorizing extended leaves of absence to enable the first author to visit the second and finally, and most importantly, the Minerva Foundation, the Israel Science Foundation, the Arthur and Rochelle Belfer Institute of Mathematics and Computer Science, and the Visiting Professorship program at the Weizmann Institute for the financial support that made these visits possible and enabled the authors to work together under ideal conditions.