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978-1-107-01777-1 - Normal Approximations with Malliavin Calculus: From Stein's Method to Universality

Ivan Nourdin and Giovanni Peccati

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