Robust Statistics for Signal Processing

Understand the benefits of robust statistics for signal processing with this authoritative yet accessible text. The first ever book on the subject, it provides a comprehensive overview of the field, moving from fundamental theory through to important new results and recent advances. Topics covered include advanced robust methods for complex-valued data, robust covariance estimation, penalized regression models, dependent data, robust bootstrap, and tensors. Robustness issues are illustrated throughout using real-world examples and key algorithms are included in a MATLAB Robust Signal Processing Toolbox accompanying the book online, allowing the methods discussed to be easily applied and adapted to multiple practical situations. This unique resource provides a powerful tool for researchers and practitioners working in the field of signal processing.

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Preface

With the rapid advance in signal processing driven by technological advances toward a more intelligent networked world, there is an ever-increasing need for reliable and robust information extraction and processing, and this is the domain of robust statistical signal processing. There has been a proliferation of research, applications, and results in this field and an in-depth book that provides a timely overview and a systematic account, along with foundational theory, is of importance. Collectively, the authors have significant expertise in the robust statistical signal processing field, developed over many years, and this expertise underpins the selection of material and algorithms, along with the foundational theory, that comprises the book.

Robust statistical signal processing is part of statistical signal processing that, broadly, involves making inference based on observations of signals that have been distorted or corrupted in some unknown manner (Zoubir, 2014). Often, the term *robust* is loosely used in signal processing. The focus in this book is on statistical robustness in the context of statistical uncertainty that arises in state-of-the-art statistical signal processing applications or in deviations from distributional assumptions. The emphasis is on robust statistical methods to solve problems encountered in current engineering practice. While the focus is practical, the book also presents advances in robust statistics – an established discipline of statistics which has its origins in the middle of the last century. Appropriate introductory material is provided to form a coherent development of robust statistical signal processing theory.

Classical statistical signal processing relies strongly on the normal (Gaussian) distribution, which provides, in many situations, a reasonable model for the data at hand. It also allows for closed-form derivations of optimal procedures. However, there have been deviations from Gaussianity reported in numerous measurement campaigns. An overview that covers a broad range of applications where these deviations occur is given in Zoubir et al. (2012), and in later chapters of this book. Robust statistical methods account for the fact that the postulated models for the data are fulfilled only approximately and not exactly. In contrast to classical parametric procedures, robust methods are not significantly affected by small changes in the data, such as outliers or small model departures. They also provide near-optimal performance when the assumptions hold exactly. While optimality is clearly desirable, robustness is the engineer's choice.

In Figure 0.1 the transition from classical parametric statistical signal processing to robust signal processing is illustrated. This tightrope walker metaphor, which goes back to Hampel et al. (2011), illustrates the danger of walking on a single rope, symbolizing

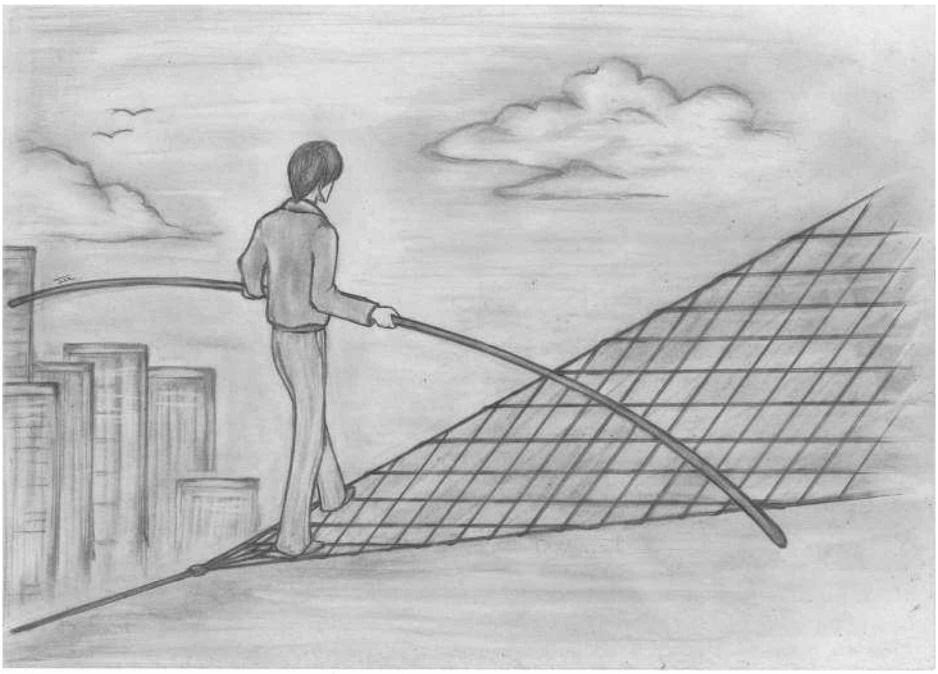

Figure 0.1 Tightrope walker metaphor. Artwork by Sahar Khawatmi.

a single distribution or a parametric model in a real-world situation. For some of today's problems, many robust methods and coherent results exist, as symbolized by the tight mesh that a researcher may use. However, the increasing complexity of the data found in applications necessitates new robust advanced methods. These form a broader mesh that also includes advanced robust signal processing techniques that are in demand more than ever before in today's engineering problems as a pathway to designing systems and data analysis tools.

Aim and Intended Audience

The primary aim of the book is to make robust methods accessible for everyday signal processing practice. The book has been written from a signal processing practitioner's perspective and focuses on data models and applications that we have frequently encountered, and algorithms that we have found useful. Throughout the book, examples taken from practical applications, that is, real-life examples, are used to motivate the use of robust methods and to illustrate the underlying concepts. Our objective is to give a tutorial-style treatment of fundamental concepts, as well as to provide an accessible overview of lesser-known aspects and more recent trends in robust statistics for signal processing.

The intended audience is broad and includes applied statisticians and scientists in areas such as biomedicine, data analytics, electrical and mechanical engineering, communications, and others. Some sections of the book present fundamental concepts, whereas others highlight recent developments. Therefore, beginners, for example, graduate-level students or practitioners with a basic knowledge in probability theory, linear algebra, and statistics, can use the book as a starting point to familiarize themselves with the concepts of robustness. Further, experts and more theoretically oriented readers are likely to benefit from proofs and recent results.

Organization of the Book

Chapter 1 provides the foundation for robust estimation theory that is introduced in a signal processing context and in a signal processing framework. It includes a brief historical account and provides the basic outlier and heavy-tailed distribution models. The M-estimator of location and scale parameters is explained by using the example of estimating the direct current value in independently and identically distributed noise. Important measures of robustness such as the influence function and the breakdown point are then introduced.

The important topic of parameter estimation in linear regression models is considered in Chapter 2. Linear regression models are used in many practical problems including those where the interference is impulsive or the noise has a heavy-tailed distribution. As an example, we discuss the geolocation of a user equipment in mixed line-of-sight/non-line-of-sight observations in wireless communications. Because of its practical importance, the chapter begins with a brief introduction to complex derivatives and optimization. Chapter 2 then discusses least squares estimators, least absolute deviation estimators, rank-least absolute deviation estimators, maximum likelihood estimators, M-estimators, and positive breakdown point estimators. Measures of robustness are defined for the linear regression model and simulation examples are provided to compare the estimators.

The major focus of Chapter 3 is robust and sparse estimation in linear models. Regularized robust estimators are considered and an example from the area of image denoising is given. Regularized robust estimators are also important for compressive sensing, which is a signal processing technique for efficiently acquiring and reconstructing a signal by finding solutions to underdetermined linear systems. Sparse regression methods have become increasingly important in modern data analysis due to the frequently occurring case of both the measurement and feature space being high-dimensional. In high-dimensional regression problems, the regression model is often ill-posed, that is, the number of regressors often exceeds the number of measurements. The least absolute shrinkage and selection operator (Lasso) has become a benchmark method for sparse regression, and the purpose of this chapter is to develop and review robust alternatives to the Lasso estimator and its extensions. For illustration, these methods are applied to a benchmark prostate cancer data set.

Robustness for the multichannel data case, as arises, for example, in sensor array processing, is of increasing importance in signal processing practice and is the subject of Chapter 4. The complex-valued case is considered and a brief review of complex elliptically symmetric distributions is provided before the theory of robust estimation of the multivariate location and scatter (covariance) matrix is introduced. This chapter places special attention to M-estimators for the scatter matrix as these estimators have been, by far, the most popular estimators in the signal processing literature. We illustrate the usefulness of robust covariance estimators in a signal detection application by using the normalized matched filter.

Important aspects of robust covariance estimation for sensor array processing applications are discussed in Chapter 5. First, the basic array processing signal model for ideal arrays and the underlying assumptions used in estimating the angles of arrival is presented. The uncertainties in the signal model, including modeling emitted signals, the array configuration, and the propagation environment, are considered. Some array configurations and direction-of-arrival estimation methods that provide robustness in the face of such uncertainties are briefly discussed.

Chapter 6 is concerned with tensor representations, which are a natural way to approach the modeling of high-dimensional data. First, a brief overview of tensor representations is provided and a standard notation is used. The basic ideas underpinning canonical tensor decomposition and Tucker decomposition are then described. This is followed by a discussion of a statistically robust method for finding a tensor decomposition. The case of finding a statistically robust way of decomposing tensors while promoting sparseness is also discussed.

Robust filtering constitutes an important field of research with a long and rich history, reaching back to even before the mathematical formalization of robust statistics by Huber (1964). Considering the large amount of material on robust filtering, a book on this topic is easily justified. In this context, a brief overview of selected topics is provided in Chapter 7 without any claim for completeness. In particular, we first discuss robust Wiener filtering and then describe some nonparametric nonlinear robust filters, such as the weighted median and weighted myriad filters. Real-world applicability of robust methods is exemplified by analyzing electrocardiographic data. Finally, we consider robust filtering based on state-space models, that is, the robust Kalman filter and the approximate conditional mean filter. Robust filters that can deal with outliers in the state equation and in the measurement equation are discussed. We also provide an example of tracking user equipment in a mixed line-of-sight/non-line-of-sight environment by means of robust extended Kalman filtering.

Correlated data streams are commonly measured in areas such as engineering, data analytics, economics, biomedicine, radar, or speech signal processing, to mention a few. The basic concepts of robustness introduced in the independent data case, however, cannot be straightforwardly extended to the dependent data case. Robust methods for dependent data form the most practical case for signal processing practitioners and are the subject of Chapter 8. This chapter focuses on robust parameter estimation for autoregressive moving-average (ARMA) models associated with random processes, for which the majority of the samples are appropriately modeled by a stationary and invertible ARMA model and a minority consists of outliers with respect to the ARMA model.

Robust spectral estimation is the subject of Chapter 9. Here, we consider nonparametric robust methods, as well as robust parametric spectral estimation. We discuss the robust estimation of the power spectral density of an ARMA process using methods for robust estimation of the model parameters detailed in Chapter 8. Further, we discuss parametric methods such as MUltiple SIgnal Classification for the robust estimation of line spectra by employing the robust eigendecomposition of the covariance matrix, discussed in Chapter 4.

Robust bootstrap methods are introduced in Chapter 10. The bootstrap is a powerful computational tool for statistical inference that allows for the estimation of the distribution of an estimator without distributional assumptions on the underlying data, reliance on asymptotic results, or theoretical derivations. However, the robustness properties of the bootstrap in the presence of outliers are very poor, irrespective of the robustness of the bootstrap estimator. We briefly review recent developments of robust bootstrap methods for estimation and highlight their importance. Again, the example of robust geolocation is used for illustration purposes. The chapter also features a robust bootstrap method that is scalable to very large volume and high-dimensional data sets, that is, big data. Moreover, it is compatible with distributed data storage systems and distributed and parallel computing architectures. Finally, an inference example using real-world data from the Million Song data set is considered.

Chapter 11 is devoted to real-life applications of robust methods. Here, we give several examples of how the theory detailed in preceding chapters can be applied in areas as diverse as short-term load forecasting, diabetes monitoring, heart-rate variability analysis by means of photoplethysmography, inverse atmospherical problems, and indoor localization.

To reproduce the examples that are given in the book and to allow the practitioner to directly apply the methods detailed in the book a MATLAB© toolbox – RobustSP – has been developed, and this is downloadable as ancillary material.

The book does not cover robustness for signal detection and robust hypothesis testing (Sion, 1958; Huber, 1965; Huber and Strassen, 1973; Österreicher, 1978; Kassam and Poor, 1985; Dabak and Johnson, 1993; Poor and Thomas, 1993; Song et al., 2002) or classification, although much progress has been made in this area recently, for example robust detection using f-divergence balls (Levy, 2009; Gül and Zoubir, 2016, 2017; Gül, 2017), the importance of density bands (Kassam, 1981; Fauß and Zoubir, 2016), or robust sequential detection (DeGroot, 1960; Brodsky and Darkhovsky, 2008; Fellouris and Tartakovsky, 2012; Fauß and Zoubir, 2015; Fauß, 2016).

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Abbreviations

ACE	adaptive coherence estimator
ACG	angular central Gaussian
ACM	approximate conditional mean
AIC	Akaike's information criterion
ALS	alternating least squares
AO	additive outlier
AOA	angle-of-arrival
AR	autoregressive
ARE	asymptotic relative efficiency
ARMA	autoregressive moving-average
AWGN	additive white Gaussian noise
BIC	Bayesian information criterion
BIP	bounded influence propagation
BLB	bag of little bootstraps
BLFRB	bag of little, fast, and robust bootstraps
BP	breakdown point
CANDECOMP	Canonical Decomposition
CCD	cyclic coordinatewise descent
CD	coordinate descent
cdf	cumulative distribution function
CES	complex elliptically symmetric
CFAR	constant false alarm rate
CI	confidence interval
CN	complex normal
CRLB	Cramér-Rao lower bound
CS	compressed sensing
CV	cross-validation
DC	direct current
DF	degrees of freedom
DOA	direction-of-arrival
DTFT	discrete-time Fourier transform
ECG	electrocardiogram
ECP	empirical coverage probability

EEG	electroencephalogram
EIF	empirical influence function
EKF	extended Kalman filter
EN	elastic net
ES	elliptically symmetric
ESPRIT	estimation of signal parameters via rotational invariance techniques
ETEX	European Tracer Experiment
EVD	eigenvalue decomposition
FFT	fast Fourier transform
FIR	finite impulse response
fMRI	functional magnetic resonance imaging
FNR	false negative rate
FOBI	fourth-order blind identification
FP	fixed-point
FPR	false positive rate
FRB	fast and robust bootstrap
GES	gross-error-sensitivity
GLRT	generalized likelihood ratio test
GPS	global positioning system
GUT	generalized uncorrelating transform
HOS	higher-order statistics
HQ	Hannan and Quinn
HRV	heart rate variability
IC	information criteria
ICA	independent component analysis
ICP	intracranial pressure
IF	influence function
IFB	influence function bootstrap
i.i.d.	independently and identically distributed
IO	innovations outlier
IRWLS	iterative re-weighted least squares
JADE	joint approximate diagonalization of eigen-matrices
JNR	jammer-to-noise ratio
JSR	jammer-to-signal ratio
KDE	kernel density estimator
LAD	least absolute deviations
LASSO	least absolute shrinkage and selection operator
LCD	liquid crystal display
LMS	least-median of squares
LOS	line-of-sight
LPDM	Lagrangian particle dispersion model
LSE	least squares estimator
LTI	linear time-invariant

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LTS	least trimmed squares
MA	moving-average
MAP	maximum posteriori
MAPE	mean absolute percentage error
MBC	maximum-bias curve
MC	Monte Carlo
MCD	minimum covariance determinant
MDL	minimum description length
MeAD	mean absolute deviation
MHDE	minimum Hellinger distance estimator
MIMO	multiple-input multiple-output
ML	maximum likelihood
MLE	maximum likelihood estimator or estimate
MMSE	minimum mean squared error
MR	median-of-ratios
MRA	minimum redundancy arrays
MS	mean-shift
MSD	matched subspace detector
MSE	mean squared error
MSWF	multistage Wiener filter
MUSIC	MUltiple SIgnal Classification
MVDR	minimum variance distortionless response
MVE	minimum volume ellipsoid
NLOS	non-line-of-sight
NMSE	normalized mean squared error
PARAFAC	parallel factors
PCA	principal component analysis
PCI	peripheral component interconnect
PD	probability of detection
pdf	probability density function
PDH	positive definite Hermitian
PE	prediction error
PFA	probability of false alarm
PPG	photoplethysmogram
PRV	pulse rate variability
PSD	power spectral density
RA	robust autocovariance
RARE	rank-reduction
RCM	rank covariance matrix
REKF	robust extended Kalman filter
RES	real elliptically symmetric
RLAD	rank-least absolute deviations
RM	ratio-of-medians

RMSE	relative mean squared error
RMSSD	root mean square of successive differences
RO	replacement outlier
RSP	robust starting point bootstrap
RSS	residual sum-of-squares
RV	random variable
r.v.	random vector
SARIMA	seasonal integrated autoregressive moving-average
SC	sensitivity curve
SCM	sample covariance matrix
SDNN	standard deviation of N-N intervals
SML	stochastic maximum likelihood
SNR	signal-to-noise power ratio
SOI	signal of interest
SP	signal processing
SSR	sparse signal reconstruction
ST	space-time or soft-thresholding
SUT	strong uncorrelating transform
SVD	singular value decomposition
TCM	Kendall’s Tau covariance matrix
TOA	time-of-arrival
UCA	uniform circular array
ULA	uniform linear array
w.l.o.g.	without loss of generality
WSN	wireless sensor network
WSS	wide-sense stationary

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List of Symbols

$\mathrm{AG}_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	p-dimensional Angular Gaussian (AG) distribution with mean vector $\boldsymbol{\mu}$ and scatter matrix $\boldsymbol{\Sigma}$
arg max	argument of the maximum
arg min	argument of the minimum
$\mathbf{a}_i$	ith column vector of a matrix $\mathbf{A} = (\mathbf{a}_1 \ \cdots \ \mathbf{a}_p)$
$\mathbf{a}_{[i]}$	ith (transposed) row vector of a matrix $\mathbf{A} = (\mathbf{a}_{[1]} \ \cdots \ \mathbf{a}_{[p]})^\top$
$\mathsf{bias}\,(T(F), F)$	asymptotic bias of an estimator $T(F)$ at distribution F
$\mathbb{C}$	field of complex numbers
$\mathsf{col}(\mathbf{X})$	column space of $\mathbf{X}$
$\#\{\cdot\}$	cardinality of a set
$\mathbb{C}\mathcal{E}_p(\boldsymbol{\mu}, \boldsymbol{\Sigma}, g)$	p-dimensional complex elliptically symmetric distribution with mean vector $\boldsymbol{\mu}$, scatter matrix $\boldsymbol{\Sigma}$ and density generator g
$\mathbb{C}\mathcal{N}_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	p-variate complex normal distribution with mean vector $\boldsymbol{\mu}$ and scatter matrix $\boldsymbol{\Sigma}$
$(\cdot)^*$	complex conjugate
$\boldsymbol{\Sigma}$	covariance matrix
$r_{xy}(\cdot)$	cross-second-order moment function of the random processes $x_t(\zeta)$ and $y_t(\zeta)$
$c_{xy}(\cdot)$	central cross-second-order moment function (cross-covariance function) of the random processes $x_t(\zeta),\ y_t(\zeta)$
$x \overset{\mathrm{d}}{=} y$	x has the same distribution as y
$\lvert\mathbf{A}\rvert$	determinant of a matrix $\mathbf{A}$
$\mathsf{diag}(\mathbf{a})$	diagonal matrix with $\mathbf{a} = (a_1, \ldots, a_p)^\top$ as diagonal elements
$\triangleq$	defined as
F_N	empirical distribution function
$\exp(\cdot)$	exponential function
$\mathsf{E}[\cdot]$	expectation operator
$\mathsf{E}[\cdot]_F$	expectation operator at distribution F
$\mathsf{GES}\,(T(F), F)$	gross error sensitivity of estimator $T(F)$ at nominal distribution F
$(\cdot)^{\mathsf{H}}$	Hermitian transpose of a matrix or a vector
$\mathbb{1}_{\mathcal{A}}$	is the indicator of the event $\mathcal{A}$
$\lfloor\cdot\rfloor$	denotes the integer part
$\mathbf{I}$	identity matrix
$\mathbf{I}_p$	$p \times p$ identity matrix

$\mathsf{IF}(y;T(F),F)$	influence function of estimator $T(F)$ at nominal distribution F
$\overset{i.i.d.}{\sim}$	independently and identically distributed
i,j,k	discrete index
$\mathsf{Im}(\cdot)$	operator extracting the imaginary part of its complex-valued argument
$(\cdot)^{-1}$	inverse of a matrix
$\langle\cdot,\cdot\rangle$	inner product
$\jmath$	imaginary unit
$\mathsf{kurt}[y]$	kurtosis of a random variable y
$L_{\text{ML}}(\mu,\sigma\|\mathbf{y})$	negative log-likelihood function with parameters μ and σ given observations $\mathbf{y}$
$\mathbf{A}^{\dagger}$	Moore–Penrose pseudoinverse of matrix $\mathbf{A}$
$\mathsf{med}(\mathbf{x})$	median of vector $\mathbf{x}$
$\|a\|$	modulus of a complex number a
μ	location parameter (mean)
$\mathsf{MSE}\,(T(F),F)$	mean-squared error of estimator $T(F)$ of the parameter β at distribution F
$\mathcal{M}(N,\mathbf{p})$	multinomial distribution with N trials and outcome probability vector $\mathbf{p}$
$\mathcal{N}_p(\boldsymbol{\mu},\boldsymbol{\Sigma})$	p-variate real normal distribution with mean vector $\boldsymbol{\mu}$ and scatter matrix $\boldsymbol{\Sigma}$
N	number of observations
$\\|\cdot\\|_1$	ℓ_1-norm
$\\|\cdot\\|_2$	Euclidean (ℓ_2-)norm
$\\|\cdot\\|_{p,q}$	mixed $\ell_{p,q}$-norm
$\\|\cdot\\|_{\text{F}}$	Frobenius ($\ell_{2,2}$-) norm
$\mathbf{1}$	column vector of 1's
$\mathbf{A}^{\perp}$	orthogonal complement of $\mathbf{A}$
$\xrightarrow{P}$	convergence in probability
$f(x\|\beta)$	pdf of random variable x given parameter β
$\mathsf{Arg}(\cdot)$	principal argument of a complex number
$\mathsf{Prob}(\cdot)$	probability
p	dimension of model or observations
$\mathbb{S}_{++}^{p\times p}$	set of all positive definite real (or complex) symmetric (or Hermitian) matrices of dimension $p\times p$
$I_{xx}(e^{\jmath\omega})$	periodogram at frequency ω
$S_{xx}(e^{\jmath\omega})$	power spectral density of random process x_t at frequency ω
$\psi(\cdot)$	score function of an M-estimator
$\mathcal{E}_p(\boldsymbol{\mu},\boldsymbol{\Sigma},g)$	p-dimensional real elliptically symmetric distribution with mean vector $\boldsymbol{\mu}$, scatter matrix $\boldsymbol{\Sigma}$ and density generator g
$\mathbb{R}$	field of real numbers
$\mathsf{Re}(\cdot)$	operator extracting the real part of its complex-valued argument
$\mathbb{R}^+$	$\{x\in\mathbb{R}\|x>0\}$
$\mathbb{R}_0^+$	$\{x\in\mathbb{R}\|x\geq 0\}$

$\rho(\cdot)$	objective function of an M-estimator
$\diamond$	replica operator
$\mathsf{SC}(y; T(F_N), F_N)$	sensitivity curve of estimator $T(F)$ at empirical distribution F_N
σ	scale parameter (standard deviation)
$\mathbf{\Sigma}$	scatter matrix
$\mathbb{C}t_{p,\nu}(\boldsymbol{\mu}, \mathbf{\Sigma})$	p-dimensional complex t-distribution with mean vector $\boldsymbol{\mu}$, scatter matrix $\mathbf{\Sigma}$ with $\nu > 0$ degrees of freedom
$(\cdot)^\top$	transpose of a matrix or a vector
$\mathsf{Tr}(\cdot)$	matrix trace
$\mathcal{U}(a, b)$	uniform distribution on the interval a, b
$\sigma_y^2, \mathsf{var}(y)$	variance of a random variable y
ω	radian frequency (normalized)
χ_p^2	chi square distribution with p degrees of freedom
$\mathbb{Z}$	set of integers
$\mathbb{Z}^+$	set of positive integers