

Quantum Concepts in Physics

Written for advanced undergraduates, physicists, and historians and philosophers of physics, this book tells the story of the development of our understanding of quantum phenomena through the extraordinary years of the first three decades of the twentieth century.

Rather than following the standard axiomatic approach, this book adopts a historical perspective, explaining clearly and authoritatively how pioneers such as Heisenberg, Schrödinger, Pauli and Dirac developed the fundamentals of quantum mechanics and merged them into a coherent theory, and why the mathematical infrastructure of quantum mechanics has to be as complex as it is. The author creates a compelling narrative, providing a remarkable example of how physics and mathematics work in practice. The book encourages an enhanced appreciation of the interactions between mathematics, theory and experiment, helping the reader gain a deeper understanding of the development and content of quantum mechanics than with any other text at this level.

Malcolm Longair is Emeritus Jacksonian Professor of Natural Philosophy and Director of Development at the Cavendish Laboratory, University of Cambridge. He has held many highly respected positions within physics and astronomy, and he has served on and chaired many international committees, boards and panels, working with both NASA and the European Space Agency. He has received much recognition for his work, including the Pilkington Prize of the University of Cambridge for Excellence in Teaching and a CBE in the millennium honours list for his services to astronomy and cosmology. His previous well-received books for Cambridge University Press include *Theoretical Concepts in Physics* (2003), *The Cosmic Century: A History of Astrophysics and Cosmology* (2005) and *High Energy Astrophysics* (2011).

Cambridge University Press
978-1-107-01709-2 - Quantum Concepts in Physics: An Alternative Approach to the Understanding of Quantum Mechanics
Malcolm Longair
Frontmatter
[More information](#)

Cambridge University Press

978-1-107-01709-2 - Quantum Concepts in Physics: An Alternative Approach to the Understanding of Quantum Mechanics

Malcolm Longair

Frontmatter

[More information](#)

Quantum Concepts in Physics

An Alternative Approach to the Understanding of Quantum Mechanics

MALCOLM LONGAIR

Cavendish Laboratory, University of Cambridge



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press

978-1-107-01709-2 - Quantum Concepts in Physics: An Alternative Approach to the Understanding of Quantum Mechanics

Malcolm Longair

Frontmatter

[More information](#)

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107017092

© M. Longair 2013

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2013

Reprinted 2014

Printed in the United Kingdom by TJ International Ltd, Padstow, Cornwall

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-01709-2 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Cambridge University Press
978-1-107-01709-2 - Quantum Concepts in Physics: An Alternative Approach to the Understanding of Quantum Mechanics
Malcolm Longair
Frontmatter
[More information](#)

For Deborah

Cambridge University Press
978-1-107-01709-2 - Quantum Concepts in Physics: An Alternative Approach to the Understanding of Quantum Mechanics
Malcolm Longair
Frontmatter
[More information](#)

Contents

<i>Preface</i>	<i>page</i> xii
<i>Acknowledgements</i>	xvi

Part I The Discovery of Quanta

1 Physics and theoretical physics in 1895	3
1.1 The triumph of nineteenth century physics	3
1.2 Atoms and molecules in the nineteenth century	4
1.3 The kinetic theory of gases and Boltzmann’s statistical mechanics	6
1.4 Maxwell’s equations for the electromagnetic field	11
1.5 The Michelson–Morley experiment and the theory of relativity	13
1.6 The origin of spectral lines	15
1.7 The spectrum of black-body radiation	19
1.8 The gathering storm	23
2 Planck and black-body radiation	24
2.1 The key role of experimental technique	24
2.2 1895–1900: The changing landscape of experimental physics	25
2.3 Planck and the spectrum of black-body radiation	29
2.4 Towards the spectrum of black-body radiation	36
2.5 Comparison of the laws for black-body radiation with experiment	41
2.6 Planck’s theory of black-body radiation	42
2.7 Planck and ‘natural units’	45
2.8 Planck and the physical significance of h	46
3 Einstein and quanta 1900–1911	48
3.1 Einstein in 1905	48
3.2 Einstein on Brownian motion	49
3.3 <i>On a heuristic viewpoint concerning the production and transformation of light</i> (Einstein 1905a)	50
3.4 The quantum theory of solids	55
3.5 Debye’s theory of specific heats	58
3.6 Fluctuations of particles and waves – Einstein (1909)	60
3.7 The First Solvay Conference	64
3.8 The end of the beginning	67

Part II The Old Quantum Theory

4 The Bohr model of the hydrogen atom	71
4.1 The Zeeman effect: Lorentz and Larmor's interpretations	71
4.2 The problems of building models of atoms	74
4.3 Thomson and Rutherford	75
4.4 Haas's and Nicholson's models of atoms	79
4.5 The Bohr model of the hydrogen atom	80
4.6 Moseley and the X-ray spectra of the chemical elements	83
4.7 The Franck–Hertz experiment	88
4.8 The reception of Bohr's theory of the atom	89
5 Sommerfeld and Ehrenfest – generalising the Bohr model	90
5.1 Introduction	90
5.2 Sommerfeld's extension of the Bohr model to elliptical orbits	91
5.3 Sommerfeld and the fine-structure constant	96
5.4 A mathematical interlude – from Newton to Hamilton–Jacobi	99
5.5 Sommerfeld's model of the atom in three dimensions	109
5.6 Ehrenfest and the adiabatic principle	113
5.7 The developing infrastructure of quantum theory	118
6 Einstein coefficients, Bohr's correspondence principle and the first selection rules	119
6.1 The problem of transitions between stationary states	119
6.2 <i>On the quantum theory of radiation</i> (Einstein 1916)	120
6.3 Bohr's correspondence principle	123
6.4 The first selection rules	127
6.5 The polarisation of quantised radiation and selection rules	129
6.6 The Rydberg series and the quantum defect	132
6.7 Towards a more complete quantum theory of atoms	135
7 Understanding atomic spectra – additional quantum numbers	137
7.1 Optical spectroscopy, multiplets and the splitting of spectral lines	137
7.2 The Stark effect	138
7.3 The Zeeman effect	142
7.4 The anomalous Zeeman effect	145
7.5 The Barnett, Einstein–de Haas and Stern–Gerlach experiments	151
8 Bohr's model of the periodic table and the origin of spin	155
8.1 Bohr's first model of the periodic table	155
8.2 The Wolfskehl lectures and Bohr's second theory of the periodic table	157
8.3 X-ray levels and Stoner's revised periodic table	164
8.4 Pauli's exclusion principle	168
8.5 The spin of the electron	169

9 The wave–particle duality	172
9.1 The Compton effect	172
9.2 Bose–Einstein statistics	174
9.3 De Broglie waves	177
9.4 Electron diffraction	181
9.5 What had been achieved by the end of 1924	183

Part III The Discovery of Quantum Mechanics

10 The collapse of the old quantum theory and the seeds of its regeneration	189
10.1 Ladenburg, Kramers and the theory of dispersion	189
10.2 Slater and the Bohr–Kramers–Slater theory	194
10.3 Born and ‘quantum mechanics’	197
10.4 Mathematics and physics in Göttingen	200
11 The Heisenberg breakthrough	203
11.1 Heisenberg in Göttingen, Copenhagen and Helgoland	203
11.2 <i>Quantum-theoretical re-interpretation of kinematic and mechanical relations</i> (Heisenberg, 1925)	205
11.3 The radiation problem and the translation from classical to quantum physics	207
11.4 The new dynamics	212
11.5 The nonlinear oscillator	214
11.6 The simple rotator	221
11.7 Reflections	222
12 Matrix mechanics	224
12.1 Born’s reaction	224
12.2 Born and Jordan’s matrix mechanics	226
12.3 Born, Heisenberg and Jordan (1926) – the Three-Man Paper	232
12.4 Pauli’s theory of the hydrogen atom	241
12.5 The triumph of matrix mechanics and its incompleteness	245
13 Dirac’s quantum mechanics	247
13.1 Dirac’s approach to quantum mechanics	247
13.2 Dirac and <i>The fundamental equations of quantum mechanics</i> (1925)	248
13.3 Quantum algebra, q - and c -numbers and the hydrogen atom	255
13.4 Multi-electron atoms, <i>On quantum algebra</i> and a PhD dissertation	259
14 Schrödinger and wave mechanics	261
14.1 Schrödinger’s background in physics and mathematics	261
14.2 Einstein, De Broglie and Schrödinger	263
14.3 The relativistic Schrödinger wave equation	267
14.4 <i>Quantisation as an Eigenvalue Problem (Part 1)</i>	269

14.5	<i>Quantisation as an eigenvalue problem (Part 2)</i>	274
14.6	Wave-packets	281
14.7	<i>Quantisation as an eigenvalue problem (Part 3)</i>	284
14.8	<i>Quantisation as an eigenvalue problem (Part 4)</i>	289
14.9	Reflections	291
15	Reconciling matrix and wave mechanics	292
15.1	Schrödinger (1926d)	293
15.2	Lanczos (1926)	298
15.3	Born and Wiener's operator formalism	299
15.4	Pauli's letter to Jordan	302
15.5	Eckart and the operator calculus	304
15.6	Reconciling quantum mechanics and Bohr's quantisation of angular momentum – the WKB approximation	307
15.7	Reflections	310
16	Spin and quantum statistics	312
16.1	Spin and the Landé g -factor	312
16.2	Heisenberg and the helium atom	315
16.3	Fermi–Dirac statistics – the Fermi approach	318
16.4	Fermi–Dirac statistics – the Dirac approach	320
16.5	Building spin into quantum mechanics – Pauli spin matrices	324
16.6	The Dirac equation and the theory of the electron	327
16.7	The discovery of the positron	338
17	The interpretation of quantum mechanics	343
17.1	Schrödinger's interpretation (1926)	343
17.2	Born's probabilistic interpretation of the wavefunction ψ (1926)	345
17.3	Dirac–Jordan transformation theory	349
17.4	The mathematical completion of quantum mechanics	355
17.5	Heisenberg's uncertainty principle	357
17.6	Ehrenfest's theorem	360
17.7	The Copenhagen interpretation of quantum mechanics	362
18	The aftermath	368
18.1	The development of theory	368
18.2	The theory of quantum tunnelling	372
18.3	The splitting of the atom and the Cockcroft and Walton experiment	375
18.4	Discovery of the neutron	377
18.5	Discovery of nuclear fission	378
18.6	Pauli, the neutrino and Fermi's theory of weak interactions	379
18.7	Cosmic rays and the discovery of elementary particles	381
18.8	Astrophysical applications	383

Cambridge University Press
978-1-107-01709-2 - Quantum Concepts in Physics: An Alternative Approach to the Understanding of Quantum Mechanics
Malcolm Longair
Frontmatter
[More information](#)

<i>Epilogue</i>	388
<i>Notes</i>	389
<i>References</i>	405
<i>Name index</i>	432
<i>Subject index</i>	436

Preface

How this book came about

This book is the outcome of a long cherished ambition to write a follow-up to my book *Theoretical Concepts in Physics (TCP2)* (Longair, 2003). In that book, I took the story of the development of theoretical concepts in physics up to the discovery of quanta and the acceptance by the physics community that quanta and quantisation are essential features of the new physics of the early twentieth century. There was neither space nor scope to take that story further – it was just too complicated and would have required more advanced mathematics than I wished to include in that volume.

This book is my attempt to do for quantum mechanics what I did for classical physics and relativity in *TCP2*. The objective is to try to reconstruct as closely as possible the way in which quantum mechanics was created out of a mass of diverse experimental data and mathematical analyses through the period from about 1900 to 1930. In my view, quantisation and quanta are the greatest discoveries in the physics of the twentieth century. The phenomena of quantum mechanics have no direct impact upon our consciousness which to all intents and purposes is a world dominated by classical physics. But quantum mechanics underlies all the phenomena of matter and radiation and is the basis of essentially all aspects of civilisation in the twenty-first century.

There is no lack of excellent books on quantum mechanics which is one of the staples of all courses in undergraduate physics. Most of the successful texts adopt an axiomatic approach in which quantum mechanics is derived from a set of basic axioms, the consequences of which are elucidated in the subsequent mathematical elaboration. The first complete exposition of this approach was Dirac’s classic book *Principles of Quantum Mechanics* of 1930 which may be thought of as the ultimate goal of this book (Dirac, 1930a). But how did it all come about? Can we understand why the theory has to be as complex as it is and how did the interpretation of the formalism come about?

Just as the core of *TCP2* was inspired by the essays of Martin J. Klein (1967), so this book was inspired long ago by the book *Sources of Quantum Mechanics* edited by B. L. van der Waerden (1967). I had an ambition to use van der Waerden’s book as the basis of the equivalent of *TCP2* for the development of quantum mechanics. This was reinforced by the appearance of the massive six-volume series *The Historical Development of Quantum Theory* by Jagdish Mehra and Helmut Rechenberg which provides a very thorough, authoritative survey of the history of quantum mechanics and which were published between 1982 and 2001 (Mehra and Rechenberg, 1982a,b,c,d, 1987, 2000, 2001). Equally inspiring

was *The Conceptual Development of Quantum Mechanics* by Max Jammer which covers similar ground in a single volume (Jammer, 1989). Another inspiration was the book *Inward Bound* by Abraham Pais (1985) which sets the development of quantum mechanics and quantum phenomena in a much longer time-frame. In my view, these truly excellent books are quite hard work and can only be readily appreciated by those who already have a strong foundation in classical and quantum physics. They are quite a challenge for those seeking more readily accessible enlightenment.

The historical approach and level of presentation

The experience of teaching and writing a number of books convinced me of the value of rethinking the foundations of physics from a somewhat historical perspective, at the same time making as few assumptions as is reasonable about the mathematical sophistication of the reader. As in *TCP2*, I assume some fluency in physics and mathematics, but nothing that would be beyond the first couple of years of the typical course in physics. It is useful to restate some of the objectives of *TCP2* which apply equally to the approach adopted in this book, in contrast to the standard way in which the subject is tackled.

The origin of *TCP2* can be traced to discussions in the Cavendish Laboratory in the mid-1970s among those of us who were involved in teaching theoretically biased undergraduate courses. There was a feeling that the syllabuses lacked coherence from the theoretical perspective and that the students were not quite clear about the scope of *physics* as opposed to *theoretical physics*. As our ideas evolved, it became apparent that a discussion of these ideas would be of value for all final-year students. The course entitled *Theoretical Concepts in Physics* was therefore designed to be given in the summer term in July and August to undergraduates entering their final year. It was to be strictly non-examinable and entirely optional. Students obtained no credit from having attended the course beyond an increased appreciation of physics and theoretical physics. I was invited to give this course of lectures for the first time, with the considerable challenge of attracting students to 9.00 am lectures on Mondays, Wednesdays and Fridays during the most glorious summer months in Cambridge.

The course was designed to contain the following elements:

- (a) *The interaction between experiment and theory.* Particular stress would be laid upon the importance of experiment and, in particular, the role of advanced technology in leading to theoretical insights.
- (b) The importance of having available the *appropriate mathematical tools for tackling theoretical problems.*
- (c) *The theoretical background to the basic concepts of modern physics,* emphasising underlying themes such as *symmetry, conservation, invariance,* and so on.
- (d) *The role of approximations and models in physics.*
- (e) *The analysis of real scientific papers in theoretical physics,* providing insight into how professional physicists tackle real problems.

- (f) *The consolidation and revision* of many of the basic physical concepts which all final-year undergraduates can reasonably be expected to have at their fingertips.
- (g) Finally, *to convey my own personal enthusiasm for physics and theoretical physics*. My own research has been in high energy astrophysics and astrophysical cosmology, but I remain a physicist at heart. My own view is that astronomy, astrophysics and cosmology are no more than subsets of physics, but applied to the Universe on the large scale. I am one of the very lucky generation who began research in astrophysics in the early 1960s and who have witnessed the astonishing revolutions which have taken place in our understanding of all aspects of the physics of the Universe. But, the same can be said of all areas of physics. The subject is not a dead, pedagogic discipline, the only object of which is to provide examination questions for students. It is an active, extensive subject in a robust state of good health.

My objective in writing *Quantum Concepts in Physics* has been to adopt the same user-friendly approach as in *TCP2* but now applied to the discovery of quantum mechanics. I should emphasise that this is a *personal approach* to the understanding of quantum mechanics, but it has the great virtue of forcing the writer and reader to think hard about the issues at stake at each stage in the development through one of the most dramatic periods in the evolution of our understanding of fundamental processes in physics. One of the differences as compared with *TCP2* is that somewhat more advanced mathematical tools have to be introduced to appreciate the full essence of the story. I have tried to lay out the necessary mathematics in as simple a form as I could devise, without sacrificing rigour. In my view, final-year undergraduates and their teachers should have little trouble in coping with these requirements.

Let me also emphasise that this book is *not* a textbook on quantum mechanics. It is certainly *not* a substitute for the systematic development of these topics through the standard axiomatic approach to the discipline. You should regard this book as a supplement to the standard courses, but one which I hope will enhance your understanding, appreciation and enjoyment of the physics. Certainly, I have learned a huge amount about quantum mechanics through studying the works of genius of the pioneers of the subject.

The challenge

Let me make it clear at the outset that the amount of material which has to be condensed into a single manageable volume is immense. Some impression of the magnitude of the task can be appreciated from the almost 4500 pages of the magnificent series by Mehra and Rechenberg. In addition, the history of physics literature is vast. As a result, I have had to be selective, and although the course is tortuous, I have had to streamline the story to reach my goal in a finite space. For further enlightenment, which I thoroughly recommend, there is no alternative but to delve into the writings of Mehra, Rechenberg, Jammer, Pais and the many other authors cited in the text.

Cambridge University Press

978-1-107-01709-2 - Quantum Concepts in Physics: An Alternative Approach to the Understanding of Quantum Mechanics

Malcolm Longair

Frontmatter

[More information](#)

I should also confess that, although I have taught numerous courses on quantum physics, I do not regard myself as a ‘black-belt’ quantum physicist. This has the advantage that I am embarking on a voyage of personal intellectual discovery as well. I like very much the splendid remark of Fitzgerald,

‘A Briton wants emotion in his science, something to raise enthusiasm, something with human interest.’ (Fitzgerald, 1902)

I confess to belonging to that school. I hope you will enjoy this adventure as much as I do.

Malcolm Longair
Cambridge and Venice, 2012

Acknowledgements

I have greatly benefitted from interactions with many colleagues in the Cavendish Laboratory over the years while this project has been mulled over in my mind. These began in the 1970s when the hot debates in the Cavendish Teaching Committee, which I chaired for a number of years, gave fascinating insights into what my distinguished colleagues considered to be the central core of physical thinking. The stimulus of giving parallel courses in undergraduate physics with my colleagues undoubtedly influenced my understanding. These colleagues include John Waldram, David Green and Paul Alexander, whose insights I much appreciate. I also benefitted from the lecture notes in quantum mechanics prepared by Michael Payne and Howard Hughes, which I have used as reference points in my thinking to ensure that I was to end up in the right place by the end of the book. Once the writing was completed, I was most grateful to receive the advice of Malcolm Perry and Anna Żytkow, who generously scrutinised parts of the text where I felt deeper mathematical insights would improve the clarity of the exposition – I am most grateful to them for the changes they recommended and which I implemented.

As in my earlier books, I have greatly benefitted from the advice of David Green on the subtleties of *LaTeX* coding. His expert advice has greatly improved the appearance of the text and the mathematics. I am most grateful to the librarians in the Rayleigh Library, Nevenka Huntic and Helen Suddaby, who have been unfailingly helpful in tracking down many of the rarer books and papers referred to in the text. Similar thanks go to Mark Hurn, Librarian at the Institute of Astronomy, for uncovering various little-known treasures in that library.

I would emphasise that *Quantum Concepts in Physics* is a much more personal voyage of discovery than my previous books. I knew what I wanted to achieve at the outset, but the working out and research were carried out in parallel with the writing. As such, the usual disclaimers that I am solely responsible for errors of content and judgement are even more apposite than usual. This is a story which can be told in many different ways, with different emphases according to the inclination of the writer. It is all the more important that the reader should consult the many texts referenced in this book to obtain the fuller picture.

As in all my work, the love, support, encouragement and understanding of my family, Deborah, Mark and Sarah, means more to me than can ever be expressed in words.

Figure credits

Most of the authors of the papers which include figures reproduced in this book are deceased. I am most grateful to the publishers who have been most helpful in giving permission for

the use of the figures from journals, books and other media for which they now hold the copyright. Every effort has been made to track down the copyright owners of all the pictures, but some of them have proved to be beyond what I consider to be reasonable effort. The origins of all the figures are given in the figure captions. In addition, the following list includes the original publishers of the figures and the publishers who now hold the copyright, as well as the specific forms of acknowledgement requested by them.

Annalen der Physik. Reproduced by kind permission of the Annalen der Physik. Figs 3.2 and 14.1. Also, with kind permission from John Wiley and Sons.

Annales de Chimie et de Physique. Reproduced by kind permission of the Annales de Chimie et de Physique. Fig. 3.1.

Bickerstaff, R., Manchester. Reproduced by kind permission of R. Bickerstaff, Manchester. Fig. 1.1.

Cavendish Laboratory. Reproduced by kind permission of the Cavendish Laboratory. Figs 1.3*b*, 2.2, 4.2*b*, 16.3, 18.4, 18.5 and 18.6.

Chapman and Hall, Ltd., London. Reproduced by kind permission of Chapman and Hall, Ltd., London. Figs 8.6*a* and *b*. Also, with kind permission from Springer Science and Business Media.

Chicago University Press. Reproduced by kind permission of Chicago University Press. Figs 1.5*a* and *b*.

Creative Commons. Reproduced by kind permission of Creative Commons. Figs 7.6*b* (Diagram drawn for wikipedia by Theresa Knott) and 18.3*b* (<http://labspace.open.ac.uk/mod/resource/view.php?id=431626>).

Dover publications. Reproduced by kind permission of Dover publications. Fig. 16.1.

Edinburgh Philosophical Journal. Reproduced by kind permission of Edinburgh Philosophical Journal. Fig. 1.6.

Gyldendal: London and Copenhagen. Reproduced by kind permission of Gyldendal: London and Copenhagen. Figs 8.2, 8.4 and 8.5.

Institut International de Physique Solvay. Reproduced courtesy of the Institut International de Physique Solvay: Photograph by Benjamin Couprie. Fig. 3.3.

Journal of the Franklin Institute. Reproduced by kind permission of Journal of the Franklin Institute. Fig. 7.3*b*. Also with kind permission from Elsevier.

MacMillan and Company. Reproduced by kind permission of MacMillan and Company. Figs 1.4 and 2.4*a* and *b*.

Methuen: London. Reproduced by kind permission of Methuen: London. Figs 4.4, 5.3, 5.4 and 7.4*a* and *b*.

Museum Boerhaave, Leiden. Reproduced by kind permission of the Museum Boerhaave, Leiden. Fig. 4.1.

Naturwissenschaften, Die. Reproduced by kind permission of Die Naturwissenschaften. Fig. 14.2*a* and *b*. Also, with kind permission from Springer Science and Business Media.

Niels Bohr Archive. Reproduced by kind permission of the Niels Bohr Archive of the Niels Bohr Institute. Fig. 7.7.

Nuovo Cimento, II. Reproduced with kind permission of the Società Italiana di Fisica. Fig. 15.1.

Oversigt over det Kgl. Danske Videnskabernes. Reproduced by kind permission of the Oversigt over det Kgl. Danske Videnskabernes. Fig. 8.1.

Pergamon Press. Reproduced by kind permission of the Pergamon Press. Figs 6.1*a* and *b* and 6.2.

Philosophical Magazine. Reproduced by kind permission of the Philosophical Magazine. Figs 4.2*a*, 4.3, 4.5 and 8.7.

Philosophical Transactions of the Royal Society of London. Reproduced by kind permission of the Royal Society of London. Fig. 1.3*b*.

Pic Du Midi Observatory. Reproduced by kind permission of Drs Rozelot, Desnoux and Buil of the Pic Du Midi Observatory. Figs 7.1*a*, *b* and *c*.

Physical Review. Reproduced by kind permission of the American Physical Society. Figs 9.1, 9.2 and 6.2. Copyright 1923, 1927 and 1933 by the American Physical Society.

Proceedings of the Royal Society of London. Reproduced by kind permission of the Royal Society of London. Figs 9.3 and 18.7.

Verhandlungen der Deutschen Physikalischen Gesellschaft. Reproduced by kind permission of the Deutschen Physikalischen Gesellschaft. Fig. 4.6.

Wolski, Andrzej. Reproduced by kind permission of Andrzej Wolski from his lecture notes. Fig. 1 of Notes (page 396).

Zhurnal Russkoe Fiziko-Khimicheskoe Obshchestvo. Reproduced by kind permission of the Zhurnal Russkoe Fiziko-Khimicheskoe Obshchestvo. Fig. 1.2.

Zeitschrift für Physik. Reproduced by kind permission of the Zeitschrift für Physik. Figs 7.6*a*, 8.3, 18.1*a*, 18.1*b*, 18.2 and 18.3*a*. Also, with kind permission from Springer Science and Business Media.