

Index

- A_n -singularity, 411
 - and double points, 411
 - blow-up of a curve at a singularity, 423–424
 - Abel's theorem, 572–573
 - Abel–Jacobi maps, 572
 - and projective bundles, 573–574, 587–589
 - birationality onto image, 573
 - class of image, 585
 - fibers, 587
 - used to describe cohomology of the Jacobian, 583
 - abstract secant variety, 371
 - of a rational normal curve, 374
 - adjunction formula, 40–41
 - gives genus formula, 69
 - statement*, 41
 - used to calculate canonical classes of hypersurfaces, 41
 - adjunction map, 526–527
 - affine space, 24
 - affine stratification, 26–28
 - definition*, 27
 - gives basis for Chow group, 28
 - of $\mathbb{G}(1, 3)$ via Schubert cells, 102, 126
 - of $G(k, n)$, 135–137, 161
 - of a blow-up of \mathbb{P}^n , 58, 80
 - of flag manifolds, 160
 - of projective space, 44
 - of the product of projective spaces, 51
 - affine tangent space, 10
 - algebraic cycles modulo algebraic equivalence, 551
 - algebraic cycles modulo homological equivalence, 380, 552
 - algebraic stacks, 502
 - alternative cycle theories, 550–552
 - a comparison, 553
 - advantages over Chow ring, 564–565
 - analytic topology, 344
 - apparent nodes, 114
 - asterisk, 65
 - Atiyah class, 251
 - Azumaya algebras, 346
 - Bézout's theorem, 14–15, 17, 46–48
 - for dimensionally transverse intersections, 14
 - general statement*, 46
 - base change, 524
 - associated natural map, 524–525
 - commuting with direct image, 525–526
 - for finite morphisms, 525–526
 - for flat base change, 525
 - of an affine morphism, 525
 - commuting with higher direct image, 531–533
 - Bertini's theorem, 11, 230
 - application to lines on a cubic, 221
 - extension, 237–238
 - strong form, 169–170
 - Betti numbers
 - odd Betti numbers of a smooth projective variety are even, 549
 - of a hypersurface, 182
 - of a K3 surface, 192
 - of the quadric line complex, 192
 - binodal curves
 - in a net, 425
 - birational equivalence
 - between the Hilbert scheme and Kontsevich space, 315
 - blow-up, 36
 - applied to the five conic problem, 291–292
 - as projective bundle, 337–338
 - Chow ring, 471–473
 - generators, 473
 - relations, 476–477
 - of \mathbb{P}^2 at a point, 339
 - of \mathbb{P}^3 along a line, 81
 - class of proper transform of a smooth surface, 357
 - of \mathbb{P}^3 along a smooth curve, 473–475, 478–479
 - of \mathbb{P}^4 along a line, 357
 - of \mathbb{P}^5 , 301–302
 - of \mathbb{P}^n along a linear space, 337–339, 357, 358
 - Chow ring, 479
 - various classes, 357
 - of \mathbb{P}^n at a point, 56–60, 72, 80
 - of a curve at an A_n -singularity, 423–424
 - of a singular curve, 74
 - of a surface at a point, 72–74
 - of the Veronese surface in \mathbb{P}^5 , 480
 - applied to the five conic problem, 480
 - resolving indeterminacy of a rational map, 282
 - utility in calculating intersection multiplicities, 61–62
- boundary, 289
- branch divisor, 278
- Brauer group, 346
- Brauer–Severi variety, 344–346
- Brill–Noether locus, 573

- as degeneracy locus, 580–583, 591–592
- class, 577, 590–592
- description, 578–579
- dimension, 576, 579, 590
 - bound for a general curve, 579–580
- nonemptiness, 583, 590
- scheme structure, 583
- Brill–Noether theorem, 564
 - existence part, 578–579
 - used to count linear series, 577
 - version 1, 566
 - version 2, 576
 - comparison with Castelnuovo’s bound, 576
 - version 3, 590–592
 - via curves on a K3 surface, 580
 - via degeneration to singular curves, 579
- bundle of k -frames, 561
- bundle of principal parts, 244, 247–248
 - advantage, 254
 - Chern class, 254
 - via Grothendieck Riemann–Roch, 508
 - definition* and properties, 248–250
 - does not behave well in families, 272
 - of a line bundle on \mathbb{P}^n , 251
- bundle of relative principal parts, 391–392
 - Chern class, 508
 - properties, 392
- canonical bundle, 39
- canonical class, 14, 39–40
 - of a blow-up, 73–74
 - of a subvariety, 40
 - of hypersurfaces and complete intersections, 41–42
 - of projective space, 39–40
- Cartesian flex, 271
- Cartier divisor, *see* divisor, Cartier
- Castelnuovo bound, 386
- Castelnuovo’s theorem, 570–571
 - open problem, 570
- Castelnuovo–Mumford regularity, 261
- Catalan number, 149, 577
- Čech complex, 528–529
- characteristic classes, 559
- Chern character, 177, 484–486
 - computes the Chern class of a tensor product, 486
 - definition*, 485–486
 - is a ring homomorphism, 485–486
 - is an isomorphism up to torsion, 486–487
 - of the tangent bundle of $\mathbb{G}(1, 3)$, 507
- Chern class, 14, 134, *see also* topological Chern class
 - alternative *definition* via Grassmannian, 170
 - and the Chow ring of a projective bundle, 331
 - as degeneracy locus, 167–168, 177, 426
 - characterization, 167–169
 - coincides with topological Chern class for an algebraic vector bundle, 561
 - construction, 170–172
 - definition* for smooth quasi-projective varieties, 169
 - first Chern class, 37, 167
 - is a homomorphism, 37
 - of a line bundle, 37–42
 - on a singular variety, 38
 - general definition*, 171
 - Grothendieck’s *definition* via projective bundles, 332
 - in connection with lines on a cubic, 199–200
 - in connection with the degree of a discriminant hypersurface, 253
 - information about the Grothendieck ring, 486–487
 - introductory example, 166
 - is the reciprocal of the Segre class, 364
 - of $\mathcal{E} = \text{Sym}^3 \mathcal{S}^*$, 200
 - via Grothendieck Riemann–Roch, 490–493
 - of $\mathcal{O}_{\mathbb{P}^r}(1)$, 177
 - of $\Phi(3, 3, 1)$, 232
 - of $\text{Sym}^d \mathcal{S}^*$, 227
 - of a bundle of principal parts, 252–255
 - on \mathbb{P}^n , 254
 - of a coherent sheaf is well-defined, 507
 - of a determinant, 173–174
 - of a direct sum, 169, 174
 - of a Fano scheme, 198
 - of a smooth curve, 179
 - of a symmetric power tensored with a line bundle, 255
 - of a symmetric square, 174
 - of a tensor product with a line bundle, 174–176
 - of tautological quotient bundle, 347
 - of the dual bundle, 173
 - of the relative tangent bundle of a projective bundle, 394
 - of the structure sheaf of a point in \mathbb{P}^n , 507
 - of the structure sheaf of a smooth curve in \mathbb{P}^3 , 507
 - of the tangent bundle, 179–183
 - of $\mathbb{G}(1, 3)$, 507
 - of a Grassmannian, 183
 - of a hypersurface, 179–180
 - of a product of projective spaces, 192
 - of a quadric in \mathbb{P}^5 , 192
 - of products of projective spaces, 508
 - of projective space, 179
 - of the Grassmannian of $G(2, 4)$, 191
 - of the tensor product of a vector bundle and a dual bundle, 428
 - of the tensor product of bundles, 176, 191
 - of the universal bundles on the Grassmannian, 178–179
 - of the universal quotient bundle on projective space, 177–178
 - parallel with Segre class, 364
 - top Chern class of a tensor product via the resultant, 428
 - topological, *see* topological Chern class
 - vanishes above the rank, 173
 - via Grothendieck Riemann–Roch, 490
- Chern polynomial, 427
- Chinese remainder theorem, 11
- chords, 113
 - of two twisted cubics, 85, 115, 129
 - via specialization, 122, 130
 - to a curve, 113–115
 - class, 163
 - computing the class via specialization, 121–122, 130

- Chow cohomology, 77
- Chow group, 16–17
 analogy with (co)homology, 15, 28
definition, 16
 grading, 17
 of a Grassmannian via Young diagrams, 153
 of a projective bundle, 331
 of a subset of affine space, 26
 of an affine space, 24
 theory carries over to cycles modulo algebraic equivalence, 551
 via affine stratification, 28
- Chow ring, 14
 computation, 22–36
definition, 19
 examples, 44–62
 existence, 19
 introduction, 15–19
 of a 0-dimensional scheme, 22
 of a blow-up, 471–473
 generators, 473
 of \mathbb{P}^3 along a smooth curve, 474–475
 of \mathbb{P}^n along a linear space, 338–339
 of \mathbb{P}^n at a point, 56–61
 of a surface, 72
 of the Veronese surface in \mathbb{P}^5 , 480
 relations, 476–477
 of a flag variety, 356, 359
 of a Grassmannian bundle, 346–347
 of a parameter space, 62, 86
 of conics in \mathbb{P}^3 , 349
 of a product of projective spaces, 51–52, 79
 of a projective bundle, 331–335
 of projective space, 44
 of the Grassmannian, 137, 183–187
 of lines in \mathbb{P}^3 , 105–109
 of the space of complete conics, 306–309
 of the universal hyperplane, 336
 of the universal line, 337, 394
 of the universal plane, 335–336
 relation to transversality, 17–19
- circle
 circular points, 66
 in complex projective space, 66–67
 tangent to a given circle, 67–68
 tangent to three general circles, *see* circles of Apollonius
- circles of Apollonius, 66–68
- classical topology
 and algebraic geometry, 544
 is finer than the Zariski topology, 543
- Clemens conjecture, 239
- Clifford's theorem, 569–570
- codimension
definition, 10
 expected, *see* expected codimension
 of a Schubert cycle, 133
 of a subvariety, 17
- cofactor map, 297
- Cohen–Macaulay variety
 and Bézout's theorem, 46–47
 and intersection multiplicity, 32
 and the theory of liaison, 71
 pullback, 30
- coherent sheaf
 criterion to be a vector bundle, 536
 has a locally free resolution, 486
- cohomology
 group vs. ring, 192
 of \mathbb{P}^3 , 192
 of a smooth quadric threefold, 192
- collinearity, 80
- compactification, 290, 292
 of the total space of a vector bundle, 343
- complete conic, *see* space of complete conics
- complete flag, 102, 132
- complete intersection, 109
 and the excess intersection formula, 455
 Chern class, 180
 counting lines, 240
 homology and cohomology, 555
 hyperplane section with triple points, 286, 421
 normal bundle, 212
 singular curves on a complete intersection, 286
 subvariety contained in no smooth hypersurface, 555
 subvariety of codimension-1 is intersection with hypersurface, 556–557
- complete linear series
 allowed by Castelnuovo, 576
 corresponding to a line bundle on a curve, 567
- completeness of the adjoint series, 386
- complex of flat modules
 approximation by finitely generated free modules, 537
 for quasi-projective schemes, 541
- complex projective variety
 as holomorphic subvariety/submanifold, 543
- complexification
 and Hodge theory, 546
 of the cotangent bundle of a complex manifold, 546
- composition series, 11
- cone, 83, 257, 453
- cone construction, 512
- conic curve
 in \mathbb{P}^4 , 361
- conormal variety, 380
- contact problem, 389
- cotangent bundle, 179
- cubic surface
 can have at most four isolated singular points, 236, 241
 cannot have three collinear isolated singular points, 241
 must contain lines, 201
- curvature form, 562
- curvature matrix, 562
- curve of genus 4
 expressed as a 3-sheeted cover of \mathbb{P}^1 , 564, 577–578
- curve of genus 6
 expressed as a degree-6 curve in \mathbb{P}^2 , 564
- curve of genus 8

- embedded as a degree-9 curve in \mathbb{P}^3 , 564
- image curve of embedding does not lie on a cubic, 593
- curves expressed as covers of \mathbb{P}^1 , 566
- cuspidal, 34, 410
 - in a net of plane curves, 413–419
 - ordinary, *see* ordinary cuspidal
- cycle, 15
 - definition*, 15
 - of plane conics in \mathbb{P}^3 meeting a given line, 348
 - class, 349, 359
 - tangent spaces, 352–353
- Debarre–de Jong conjecture, 227, 236–239, 243
- defective variety, 371
 - curves are not defective, 372
 - defective Veronese varieties, 373, 385–386
 - equivalent characterization via tangent spaces, 372
- deformations, *see* first-order deformations
- degeneracy locus, 167, 168, 426–427
 - class, 429, *see also* Porteous’ formula
 - is independent of sections chosen, 187
 - expected codimension, 427
 - from pencils, 251–252
 - geometry, 168
 - reducedness, 252
- degeneration to singular curves
 - in connection with Brill–Noether, 579
- degree map, 29
- degree of a covering, 28
- del Pezzo surface, 288
- derivations
 - identification with Zariski tangent space, 100
- derived category, 534
 - is formal, 535, 540–541
- determinant of a bundle, 173
- determinantal variety, 430
 - degree, 433, 442
 - scheme structure on the Brill–Noether locus, 583
- diagonal of $\mathbb{P}^r \times \mathbb{P}^r$
 - class, 53–54
 - generalization, 189
 - via specialization, 79
- dimensional transversality, 31–33, 46
 - definition*
 - for cycles, 32
 - for subschemes, 32
 - fails for cycles in a proper subvariety, 465
 - to a morphism, 518
 - sufficient condition, 518
 - weaker than generic transversality, 33
- dimensionally proper intersection, 14
- direct image, 520–521, 523–526
 - conditions to be a vector bundle, 525–526
 - definition*, 523
 - for a projective morphism, 523
 - for an affine morphism, 523
 - for finite morphisms, 525
 - higher, *see* higher direct image
 - of a line bundle, 541
 - direct image complex, 533–534
 - explicit computation, 534, 541–542
 - with terms given by sums of line bundles, 541
 - directrix, 357
 - discriminant, 244, 245, 418
 - definition*, 258
 - degree in the space of forms of degree d , 253
 - is an irreducible hypersurface, 245–246
 - linearizing the description, 247
 - multiplicity, 280–281
 - at a double conic, 288
 - at a double line, 288
 - of a net
 - of plane curves, 274–276
 - of a quartic polynomial, 246, 247
 - of a very ample linear series, 258
 - smooth locus, 284
 - tangent cone, 284
 - tangent space, 282–284
 - of discriminant of degree- d polynomials, 285, 304
 - divisor, 15
 - associated to a rational function, 23
 - Cartier, 37, 41, 165
 - cohomology of an ample divisor, 553
 - of a function, 23
 - of a nonaffine variety, 23
 - divisor class group, 22, 306
 - divisor classes in the space of complete conics, 307–309
 - Dolbeault complex, 548
 - double conic, 478
 - double point, 34, 411
 - is an A_1 -singularity, 411
 - of a curve, 411–412
 - dual conic
 - degeneration, 293–295
 - dual variety, 259, 297, 380
 - of a hypersurface, 49–50, 78
 - degree, 50, 382
 - of a quadric, 296–299
 - of a smooth complete intersection, 388
 - of a smooth conic, 293
 - divisor, 298
 - of a smooth hypersurface, 381
 - in char p , 387
 - of a smooth variety failing to be a hypersurface, 382, 388
 - of a smooth variety tends to be singular, 382, 388
 - of the Veronese embedding, 247
 - reflexivity of projective varieties, 294, 381
 - dualizing sheaf, 503
 - dynamic projection
 - in a family of projective spaces, 333–334
 - introduction, 117–119
 - dynamic specialization, 150–152, 162–163
 - Eckhart point, 420
 - elementary symmetric functions, 169, 175, 485
 - elliptic curve
 - elliptic quintic lies on no planes or quadrics, 452

- secant varieties, 388
- symmetric power, 388
- elliptic normal curve, 287
 - characterized by independence of divisors, 386
- embedded component, 9
- embedded point, 121, 129
- enumerative formula, 85, 87–88
 - for singular elements of linear series, 258
- enumerative geometry, 62
 - 19th century achievements, 2
 - applications of intersection theory, 289
 - aspects of problems, 88
 - used to analyze geometric questions, 564
- enumerative problem, 86
 - generality, 88
 - geometry of set of solutions, 87
 - negative expected dimension analog, 397
 - steps to solving, 86–88
- étale equivalence relations, 575
- étale topology, 344
- Euler characteristic, *see also* topological Euler characteristic
 - constancy for a sheaf on a family, 534–535
 - of a coherent sheaf, 482
 - of the structure sheaf of a smooth projective threefold, 508
 - via the Todd class, 487
- Euler sequence, 97–99, 179
- evaluation map, 582–583
- exceptional divisor, 36, 57, 60, 61, 72
 - normal bundle, 472
 - of a blow-up of \mathbb{P}^n along a linear space, 338, 357
- excess intersection
 - elementary examples, 447–452
 - in a subvariety, 465–466
 - of a pullback to a subvariety, 469–470
 - of hypersurfaces, 464–465, 477, 478
 - via blowing up, 479
 - of three surfaces intersecting in a curve and a 0-dimensional scheme, 445, 449–451
 - via blowing up, 475–476
- excess intersection formula, 5, 292, 446, 454–456
 - applied to the five conic problem, 462–464
 - does not extend to irreducible components, 458
 - for a pullback via an inclusion, 470
 - for cycles on a subvariety, 465
 - via specialization to the normal curve, 468–469
 - for hypersurfaces, 465
 - for three surfaces meeting in a curve and a 0-dimensional scheme, 451
 - for Veronese surface in \mathbb{P}^5 , 478
 - heuristic explanation, 456–458
 - statement*, 454–455
 - three-surface case restated in general form, 452–453
 - utility, 455–456
- excision, 25
- expected codimension, 19, 187
 - of a degeneracy locus, 427
- expected dimension, 446
 - negative, 396
 - of the secant variety, 371
- family of bundles, 489
 - on \mathbb{P}^1 , 494–497
- family of divisors, 571–576
- family of lines, 230–233
- Fano scheme, 193, 194
 - as special case of Hilbert schemes, 201
 - bounds on k, n necessary to obtain expected dimension, 237
 - Chern class, 198
 - definition*, 196–197
 - dimension, 194
 - bound via the normal bundle, 209
 - expected dimension, 195
 - expression via the Grassmannian, 198–199
 - has expected dimension when $d \ll n$, 237
 - of 2-planes on a quadric, 240
 - of a cubic surface
 - with one ordinary double point, 234–236
 - with two ordinary double points, 241
 - must have ≤ 4 isolated singular points, 241
 - of a smooth cubic surface, 226
 - of a smooth degree-4 hypersurface, 238
 - of a smooth quintic threefold, 228
 - of a smooth surface, 212
 - of lines on a quadric surface, 197
 - of lines on a smooth hypersurface, 226, 243
 - of planes of maximal dimension, 222
 - of the Fermat quartic in \mathbb{P}^4 , 226
 - potential generalization, 239
 - reducedness, 197, 208, 228, 238
 - scheme structure, 196–197
 - smoothness and the normal bundle, 208–212
 - the nonsmooth case, 234–236, 241, 242
 - universal property, 203
 - upper bound on dimension, 197
 - Zariski tangent space, 208
- Fermat surface
 - Fermat quartic, 238, 243
 - hyperflexes, 422
 - lines contained in, 421
- fiber of a vector bundle, 10
- fine moduli space, 575
 - of degree- d line bundles, 573, 575–576
- first-order deformations, 212–219
 - geometric view of lines on a cubic, 218
 - identification with global sections of the normal sheaf, 214–215
 - identification with morphisms from a double point, 213–214
 - utility in identifying tangent spaces, 213
 - vector space structure, 215–216
- five conic problem, 3, 289
 - answer, 308
 - generalization, 321
 - naive approach, 290–291

- transversality in the variety of complete conics, 303–306
- via blowing up the excess locus, 291, 480
- via the excess intersection formula, 292, 462–464
- via the space of complete conics, 302–308
- flag bundle, 347
- flag variety, 125, 159–160
 - Chow ring, 356, 359
 - class, 356
- flecnodal locus, 398–401
 - degree, 399
 - general point of a smooth surface of degree ≥ 3 is not flecnodal, 399
 - geometry, 421
- flecnode, 398
- flex, 266
 - flex tangent, 266
 - flexes approaching a cusp, 423
 - hyperflex, 244, 266, 271–272
 - in a general pencil of curves, 405
 - in a pencil of plane curves, 287
 - in families, 401
 - of a family of curves, 405–408
 - of a general curve, 286
 - of a plane curve, 270–271
 - alternate notion, 271
 - via defining equations, 401–409
- flex line, 401–402
 - geometry of the curve of flex lines, 408–409
 - hyperflex lines, 403–405
 - on a curve defined by a homogeneous form, 402–403
- flip, 315
- footprint of a subvariety, 333
- form of type (p, q) , 547
- frame manifolds, 561
- fundamental class, 544
 - of a codimension- k subvariety, 549
 - of a scheme, 22
- fundamental class map, 545
 - applied to the Chern class, 559
 - image, 545, 552
 - is a ring homomorphism, 545
- fundamental cycle, 544
- GAGA theorems, 543–544
- Gauss map, 49–50, 218, 219, 263, 439
 - definition, 263–264
 - from a surface to its dual, 260
 - of a hypersurface is either birational or has positive-dimensional fibers, 381
 - of a smooth hypersurface is finite and birational, 381
- general polynomial of degree $2m - 1$
 - unique expression as a sum of $m d$ -th powers, 362, 376–377
- general position lemma, 370
- general quadratic polynomial has no rational solution, 346, 356
- generality, 9, 88
 - of a curve, 565–566
 - of a rational curve, 441
 - very general, 9
- generalized principal ideal theorem, 11, 363
- generic finiteness, 28
- generic transversality, 46
 - definition
 - for a subvariety and a function, 30
 - for subvarieties, 18
 - is stronger than dimensional transversality, 33
 - of Schubert cycles, 139
 - reasons for nontransversality, 516–517
 - to a cycle, 512–517
 - to a morphism, 518–519
 - necessity of char 0, 518
 - sufficient condition, 518
- generically finite morphism
 - degree, 470–471
- genus formula, 69–70
 - applications, 70–71
 - for singular curves, 74
- geometric genus, 74
- Giambelli's formula, 157–159
 - in conjunction with Pieri's formula, 158–159
 - inductively via Pieri's formula, 164
 - statement, 158
- graph of a map, 54–55
- Grassmannian
 - as Hilbert scheme, 96, 201
 - Chow ring, 137
 - generators and relations via Chern classes, 183–187
 - is generated by special Schubert classes, 158
 - class of the pullback to the product of Grassmannians, 189
 - covering by affine spaces, 92–94
 - cut out by quadrics, 91, 94, 125
 - definition, 89
 - degree, 150, 164
 - differential of a morphism into the Grassmannian, 99–100
 - generalizations, 159–160
 - Lagrangian Grassmannian, 160
 - natural identification with Grassmannian of dual space, 89, 134
 - notation, 89
 - of lines in \mathbb{P}^3
 - Chow ring, 105–109
 - of lines in \mathbb{P}^n , 126, 147–150
 - degree, 131, 149–150
 - of 2-planes, 91–92
 - orthogonal Grassmannian, 160
 - smoothness, 91
 - tangent bundle, 96–99
 - expression via the universal bundles, 96
 - tangent space, 129
 - via the universal property, 100–102
 - universal bundles, 95–96
 - universal property, 201–203
 - universal quotient bundle, 95
 - Chern class, 178

- is globally generated, 178
 - universal subbundle, 95
 - Chern class, 178–179
 - existence, 95
 - lacks nonzero global section, 178
 - used to define Fano schemes, 198–199
- Grassmannian bundle, 346
 - Chow ring, 346–347
- Grothendieck ring of vector bundles, 484
- group of cycles, 15
- “hairy coconut” theorem, 179
- hard Lefschetz theorem, 558
- Hessian, 271
- higher direct image, 489, 498, 528–532
 - computation via Serre’s coherence theorem, 531
 - definition*, 528–529
 - is coherent for a projective morphism and coherent sheaf, 530
 - natural map to Čech cohomology group, 529
 - properties, 529–530
 - relation to direct image, 528
- higher direct image functors, 528
- Hilbert polynomial, 35, 45, 204–207
 - of a hypersurface, 206
 - subschemes with polynomial $2m + 1$, 350–352
- Hilbert scheme, 3, 83, 201, 203–207, 310–312
 - advantages, 311–312
 - as better compactification than symmetric power, 370
 - construction, 205–207
 - definition* and uniqueness, 204–205
 - disadvantages, 312
 - extraneous components, 312
 - mysterious closure of locus of smooth curves, 312
 - of a hypersurface, 206
 - of conics and cubics in \mathbb{P}^2 , 311
 - of divisors on a smooth scheme identified with symmetric power, 575–576
 - of hypersurfaces in \mathbb{P}^n , 207
 - of plane conics in \mathbb{P}^3 , 311
 - of subschemes of \mathbb{P}^3 with Hilbert polynomial $2m + 1$, 350–352
 - as projective bundle, 350
 - of twisted cubics, 311
 - singularities, 312
- Hilbert series
 - of a graded complete intersection, 185
- Hodge bundle, 503
 - of a pencil of quartics, 505, 510
- Hodge conjecture, 545, 549–550
 - integral codimension-1 case, 550
- Hodge decomposition, 546–548
 - algebraic computation of $H^{p,q}(X)$, 547–548
- Hodge diamond, 548–549
 - of a smooth quartic surface, 548
 - symmetries, 549
- Hodge number, 549
 - as invariant, 549
- Hodge structure, 548
 - polarization, 548
- Hodge–Riemann bilinear relations, 558–559
- holomorphic
 - map is algebraic, 544
 - subvariety is algebraic, 543
- homological equivalence, 545
- hook formula, 164
- Hopf index theorem, 506
- hyperelliptic curves, 570
- hyperplane
 - contact with a curve, 244, 265
- hypersurface
 - criterion to contain a line, 240
 - in \mathbb{P}^4 containing a complete intersection, 192
 - lines having point of contact of order 7, 420
 - of sufficiently low degree contains lines, 226
- ideal sheaf
 - of three points in \mathbb{P}^2 , 521–522
 - direct image, 526
 - direct image complex, 534–535
 - higher direct images, 532–533
 - of two points in \mathbb{P}^2 , 521
- inflection point, 265–273
 - weight on a general curve, 268
- integrals of algebraic functions, 571–572
- interpolation problem, 373
- intersection multiplicity, 14, 31–33
 - coinciding with the order of contact, 389
 - connection with multiplicity of a scheme at a point, 36
 - definition* and existence, 32
 - in the Cohen–Macaulay case, 32
 - necessity, 19
 - of a curve and a Cartier divisor, 265
 - of a divisor and a subvariety, 447–448
 - Serre’s formula, 48
 - via blow-ups, 61–62
- intersection number, 69
- intersection product, 15, 20, 41, 76
 - correction terms, 47–48
 - existence, 7, 19
 - for curves on surfaces, 68–74
 - in Chow ring corresponds to cup product in cohomology, 545
 - necessity of smoothness, 20
 - of a Cartier divisor and a subvariety, 447
 - geometric view, 447–448
 - of cycles on a proper subvariety, 465–466
 - on singular varieties, 38, 75–77, 455
 - semi-refined version via the strong moving lemma, 511
 - via the basic moving lemma, 511
- intersection theory
 - applications to enumerative geometry, 289
 - dependence on the moving lemma, 511, 512
 - goals, 14–15
 - influence on algebraic geometry, 1–2, 5
 - motivation, 1–2
 - on algebraic stacks, 502
- invariants of families of curves, 502–504

- degree of the Hodge bundle, 503
 for a pencil of plane quartics, 504–505
 inequalities, 504, 510
 number of nodes, 503
 of a pencil of curves of bidegree (a, b) , 510
 of a pencil of plane curves, 510
 of a pencil of plane sections of a smooth surface, 510
 relations, 504
 self-intersection of the relative canonical divisor, 504
- irreducible component, 9
- isotropic, 155
- Jacobi inversion theorem, 573
 classical form, 592
- Jacobian, 571–572
 Chow vs. cohomology rings, 583
 cohomology ring, 583–584
 classes of interest, 584
 cotangent space, 574
definition, 572
 identified with linear equivalence classes of effective divisors, 573
 isomorphism with $\text{Pic}^d(C)$, 575
- join, 9
- Jordan–Hölder theorem, 11
- jumping lines, 225, 493–494
 examples, 501–502
 on a bundle of rank 2 on \mathbb{P}^2 , 497–502
 on a bundle of rank 2 on \mathbb{P}^3 , 508–509
- K3 surface, 192
 curves on a very general K3 surface, 580
- Kleiman’s transversality theorem, 20–21
- Kontsevich space, 292
 and tangency conditions, 317
 application, 322
 birationality to the Hilbert scheme, 313
 cross-ratio, 318
 disadvantages, 316–317
 extraneous components, 316
 introduction, 312–313
 is proper, 313
 mysterious closure of locus of smooth curves, 317
 number of PGL_4 -orbits, 322
 of plane conics, 314
 in \mathbb{P}^3 , 315
 of plane cubics, 315–316
 is not irreducible, 315
 of rational plane curves, 317–321
 of twisted cubics, 316
 singularities, 317
- Krull’s principal ideal theorem, 16, 23
statement, 11
- Künneth formula, 51
- Lefschetz $(1, 1)$ -theorem, 550
- Lefschetz decomposition, 558
- Lefschetz hyperplane theorem, 182, 222, 228, 553–554
 applied to complete intersections, 554–557
 extensions, 557
statement, 553
- Lefschetz principle, 245, 277
- lexicographic ordering, 207
- liaison, 71
- line bundle
 generated by global sections, 362–363
 on the projectivization of a bundle, 327–328
 products of line bundles, 191
 restriction to pullback, 527–528
 tautological line bundle of a projective bundle, 171
 twisting a vector bundle, 335, 355, 356
- linear series, 10, 566
 complete, *see* complete linear series
 discriminant of a very ample linear series, 258
 maps to \mathbb{P}^n given by a linear series, 568–570
 by a general series, 567
 embeddings, 570–571
 existence, 576
 number, 577, 590
 present on a general curve, 570–571
 singular elements, 258–265
 characterization of tangency of hyperplanes, 265
 via the topological Hurwitz formula, 277
 singular elements of a pencil, 259–260
- linear subspaces
 characterized by Fano scheme, 197
 characterized by Hilbert polynomial, 204
- linearization, 5, 166
- lines
 and curves in \mathbb{P}^3 , 110–115
 and surfaces in \mathbb{P}^3 , 122–125
 have no sixth order contact with a general surface, 420
 meeting a curve, 111–112
 via specialization, 120–121, 128
 meeting a curve in \mathbb{P}^4 , 161
 meeting a line on a quadric, 286
 meeting a smooth rational curve four times, 426
 meeting a surface in \mathbb{P}^4 , 161
 meeting a surface to high order, 390–391, 420
 meeting four curves, 85, 112
 transverse intersection of cycles, 127–128
 meeting four lines, 85, 110–111, 127
 meeting four planes, 131, 150, 162
 on a complete intersection, 240
 on a cubic surface, 199–201, 212
 geometric viewpoint via first-order deformations, 218
 the smooth case, 212
 via the map α and Bertini’s theorem, 221
 on a cubic with a double point, 234–236
 on a hypersurface, 240
 on a pencil of quartic surfaces, 231–233
 alternative approach, 233–234
 on a quadric, 122–123
 on a quintic threefold, 193, 227–229
 in algebraic geometry and string theory, 228
 on a smooth cubic, 165, 166, 493
 on a smooth hypersurface
 odd behaviors, 243
 on a smooth surface in \mathbb{P}^3 , 396–399
 bounds, 399, 421

- number, 422
- on the intersection of two quadrics, 131, 157
- tangent to a surface, 161
- tangent to four quadrics, 85, 125
- tangent to four spheres, 480
- linked curves in \mathbb{P}^3 , 70–71
- Littlewood–Richardson coefficients, 143
 - appearing with multiplicity >1 , 164
- locally closed subscheme, 9
- loci in space of plane cubics, 81–83
- locus of d -fold lines, 479
- locus of m -tuples of points of a curve lying on a plane, 377
- locus of asterisks, 65–66, 82
- locus of bundles of splitting type a , 495
 - as pullback of strata in a family of vector bundles, 496
- locus of Castelnuovo curves, 571
- locus of chords tangent to a curve, 113
 - alternative characterization, 129
 - class, 114
- locus of cones, 83, 257
- locus of conics containing a point in \mathbb{P}^3 , 360
- locus of cubics of the form $2L + M$, 82
- locus of curves tangent to a smooth curve degree, 479
 - multiplicity along locus of d -fold lines, 479
- locus of curves with a triple point, 244, 257
- locus of degeneracy, *see* degeneracy locus
- locus of double lines, 81
 - class, 360
- locus of flecnodal curves
 - dimension and irreducibility, 422
- locus of hyperflex lines, 403
- locus of jumping lines, 494
 - as a nonsingular cubic curve, 509
 - degree in the even first Chern class case, 499
 - degree in the odd first Chern class case, 501
 - for a bundle of rank 2 on \mathbb{P}^3 , 508–509
 - of a bundle defined by a bilinear form, 509
 - scheme structure, 499, 501
- locus of matrices of rank $\leq k$
 - degree, 426, 433–436
- locus of planes on a quadric, 155–157
 - class, 157
- locus of reducible cubics, 64–65, 81
- locus of reducible cubics composed of a smooth conic with a tangent line, 83
- locus of secant planes, 369
 - class, 378–380
 - improper secants, 385
- locus of singular conics, 81, 360
- locus of singular plane cubics, 62–66, 409
- locus of singular plane curves, 412, 423–424
- locus of smooth conics with a tangent line, 83
- locus of smooth curves
 - closure in the Hilbert scheme, 312
- locus of smooth curves with a hyperflex, 287
- locus of special lines, 444
- locus of triangles, 65, 82
- locus of triple lines, 82
- locus of trisecant lines, 443
- locus where global sections do not generate a bundle, 366
 - codimension and class, 363
- Macaulay2*, 2
 - applied to direct image complex, 534, 541–542
 - calculation of $c_1(\mathcal{E})$, 233
 - calculation of $c_4(\Phi(3, 3, 1))$, 232
 - calculation of $c_4(\text{Sym}^3 \mathcal{S}^*)$, 200
 - calculation of $c_{d+1}(\text{Sym}^d \mathcal{S}^*)$, 227
- maps from a curve to \mathbb{P}^n , 565–566
 - all curves vs. general curves, 565
 - birationally very ample maps, 569–571
 - correspond to pairs (\mathcal{L}, V) , 566
 - ease of finding high-degree maps and embeddings, 566
 - embedding in \mathbb{P}^3 , 564, 567
 - existence condition, 566, 576
 - to \mathbb{P}^1 , 564
 - to \mathbb{P}^2 , 564, 567
- Mayer–Vietoris, 25
- method of undetermined coefficients, 53–55, 79, 80, 107, 156
 - applied to the cycle of plane conics in \mathbb{P}^3 meeting a line, 349, 359
 - applied to the Grassmannian, 143–144
 - applied to the product of Grassmannians, 189
- minimal model program, 552
- minor, 90
- mirror symmetry, 228
- moduli space, *see also* fine moduli space
 - parametrizing smooth projective genus- g curves, 566
- moduli stack of stable curves, 503
- moving lemma, 7, 19–21
 - basic version, 511
 - via the cone construction, 512–517
 - bypassed via the Fulton–MacPherson approach, 512
 - direct proofs of the second part, 512
 - failure for singular varieties, 75
 - necessity of smoothness, 20
 - proof of basic version, 511–512
 - statement*, 19
 - strong version, 511
 - when one cycle is a first Chern class, 38
- multiplicity
 - of a hypersurface in smooth variety, 34
 - of a scheme, 15
 - of a scheme at a point, 33–36
 - connection with intersection multiplicity, 36
 - of a variety at a point, 411
 - of an intersection, 14
- Mumford relation, 504
 - in the case of a pencil of plane quartics, 505
 - proof via Grothendieck Riemann–Roch, 506–507
- Nakayama’s lemma, 101
- nested pairs of divisors, 55–56, 339–340
 - as projective bundle, 340
- net of cubic surfaces, 240
- net of curves

- binodal elements, 425
- geometric view, 417–419
- net of plane curves, 273–276
 - cusps, 413–419
 - discriminant, 274–276
 - flecnodes, 422
 - hyperflexes of quartics, 405
 - of cubics, 389, 409
 - singular points, 244, 275
- node, 411
- Noether's formula, 483
- nondegenerate map, 565
 - degree, 565
- nondegenerate quadratic form, 34
- nontransversality, 47–48
- normal bundle, 180, 208
 - computation, 209
 - definition*, 40
 - for arbitrary schemes, 209
 - lines on a hypersurface with prescribed normal bundle, 225
 - of k -planes on hypersurfaces, 219–220
 - of complete intersections, 212
 - of the diagonal, 249
 - of the exceptional divisor, 472
 - used for excess intersections, 447, 451
- numerical equivalence, 552, 565
 - and the Hodge conjecture, 552
 - yields ring structure, 552
- one-parameter family, 10
- order of contact, 265, 389
 - of a plane curve and a hyperflex, 422
- order of vanishing, 23
- ordinary m -fold point, 72
- ordinary cusp, 409–411
- ordinary double point, 34
- ordinary tacnode, 411
- ordinary triple point, 256
- oscnode, 414
- osculating plane, 267
 - rotation, 381, 388
- parallel transport, 561
- parameter space, 1, 62
 - alternative choices, 292–293
 - as projective bundle, 348–349
 - Chow ring of a parameter spaces, 62
 - desired attributes, 86
 - necessity of compactification, 289–290
 - of conics in \mathbb{P}^3 , 348–349
 - of curves, 310–317
 - utility in enumerative problems, 289
- pencil of cubic surfaces, 193
- pencil of curves
 - on a quadric surface, 285, 288, 510
 - on a surface, 260–262, 279–280
 - singular at a point, 286, 288
- pencil of plane curves, 422–423
 - curve traced out by flexes, 389, 405–409, 422–423
 - flex lines through a point, 422
 - hyperflexes, 389, 422
 - invariants, 504–505, 510
- pencil of plane sections of a smooth surface, 510
- pencil of quartic surfaces containing a line, 193, 231–233
- Pfaffians, 92
- Picard group, 37
 - of degree- d line bundles, 567
 - class of subvarieties, 585
 - is a fine moduli space, 573, 575
 - is an algebraic variety, 575
 - of the space of complete conics, 306
 - tangent space can be identified with cohomology, 544
- Pieri's formula, 145–147
 - for other special Schubert classes, 154–155
 - interpreted via Young diagrams, 154
 - statement*, 145
- pinch point
 - of a projection of the Veronese surface, 439
 - of a smooth surface in \mathbb{P}^3 , 442
 - of the projection of a cubic scroll, 442
 - of the projection of a rational normal surface scroll, 442
 - of the projection of a smooth surface, 436–440
- plane conics in \mathbb{P}^3
 - form a projective bundle, 347
 - locus of conics meeting a given line, 348
 - meeting eight general lines, 323
 - finite expected answer, 347
 - must be smooth, 352
 - outline of proof, 347–348
 - solution, 354
- plane curve
 - cusps, 389, 409
 - singularities, 410–412
 - triple points, 256–257
- plane sextic with four nodes, 592–593
- planes on the intersection of two quadrics, 157
- Plücker coordinates, 90
 - ratios as determinants of submatrices, 94
- Plücker embedding, 89–92, 131, 229
 - image, 90–91
 - Schubert cycles as intersections, 138
- Plücker formula, 268–270
 - analog in higher dimension, 272–273
- Plücker formula for plane curves, 418
- Plücker relations, 91
 - for the Grassmannian of 2-planes, 92
 - general case, 94
- Poincaré bundle, 575, 580–582
 - as direct image, 581
 - pushforward, 587–589
 - Chern class, 589–590
- Poincaré duality, 15
 - for the Chow ring of projective space, 45
- Poincaré's formula, 565, 585–586
- Poincaré–Hopf theorem, 181
- Porteous' formula, 427–429
 - applied to m -secant lines to curves, 444
 - applied to pinch points of a projection, 438

- applied to quadrisecant lines, 440
- as general case of Theorem 10.2, 364
- for $\mathcal{M}_0(\varphi)$, 429–430
- geometric applications, 433–442
- linearizing the problem, 431–432
- reduction to a generic case, 430–431
- statement*, 429
- used to calculate class of the Brill–Noether locus, 591
- primary decomposition, 9
- primitive cohomology groups, 558
- principle of specialization, 462
- projection of a smooth curve
 - singularities, 128
- projection of a smooth surface, 436
 - pinch points, 426, 438–440
 - singularities, 436–437
- projection of the Veronese surface, 439
- projective bundle, *see also* vector bundle, projectivization
 - can be written as projectivization, 324
 - characterization, 329
 - Chow ring, 331–335
 - definition*, 323
 - is a Brauer–Severi variety, 344
 - is the projectivization of a bundle, 327–330
 - over \mathbb{P}^1 , 324–327
 - over an arbitrary scheme, 331
 - pushforward, 332
 - utility, 363
 - weakening of definition, 344
- projective tangent space, 10
- projectivization
 - of a bundle, 96, 324
 - degree, 335
 - identifying isomorphic projectivization, 330
 - recovering original data, 327–328
 - of a subbundle, 340–341
 - class, 341
 - line subbundle, 341
 - normal bundle, 341
 - of a vector space, 9
- projectivized tangent cone, *see also* tangent cone, 61
 - characterization via blow-ups, 36
 - extension to arbitrary schemes, 35
 - of a hypersurface, 34
- proper transform, 57, 72
 - class, 61
- Prym map, 471
- pullback, 29–31
 - existence and uniqueness, 30
 - flat, 31
 - flat pullback, 25
 - general definition via the excess intersection formula, 456
 - is not defined on a singular variety, 77
 - to a subvariety, 469–471
- push-pull formula, 30
- pushforward
 - for a projective bundle, 332
 - from the Grassmannian bundle, 432–433
 - of a cycle, 28
 - proper pushforward, 28–29
- quadric cone, 479
- quadric line complex, 192
- quadric surface
 - curves lying on a smooth quadric, 69–70
 - defined by a symmetric map $V \rightarrow V^*$, 297–298
 - intersection of quadrics containing a linear space, 445, 460–462
 - linear subspaces, 155–157
 - ruling, 53, 69
 - tangency of two smooth quadrics, 298
 - two rulings, 123
- quadrisecant
 - condition to be simple, 444
 - to a curve in \mathbb{P}^3 , 377
 - to a curve of higher genus, 441–442
 - to a general rational curve is simple, 444
 - to a rational curve, 387
 - to a rational space curve, 440–441, 444
- quartic curve
 - double at five specified points, 385
 - hyperflexes of a plane quartic, 389, 405
 - reducible quartics in \mathbb{P}^2 , 83
- quartic surface
 - containing a conic, 361
 - containing a line, 165
 - double at nine specified points, 386
 - in \mathbb{P}^3 , 242
- quasi-affine stratification, 27, *see also* affine stratification
- quasi-isomorphism, 534
- quintic surface
 - homological equivalence of curves on a smooth threefold, 551
 - lines on a quintic threefold, 3, 193, 227–229
 - lines with high-order contact, 389–391, 394–396
- ramification
 - divisor, 278
 - index, 278
 - points on \mathbb{P}^1 , 287
 - sequence, 266, 268
- rational curve
 - in projective space via the Kontsevich space, 317–321
 - is the projection of a rational normal curve, 373
 - on a hypersurface, 239
 - quadrisecant lines via Porteous’ formula, 440–441, 444
- rational equivalence, 16
 - definition*, 16
 - failure to preserve genera, 45
 - generation by divisors of rational functions, 23
 - of two 0-cycles on a curve, 24
 - preservation by pushforward, 29
 - via divisors, 22–24
- rational normal curve, 324, 355
 - abstract secant variety, 374
 - characterized by independence of divisors, 386
 - curve of pure d -th powers, 376
 - independence lemma, 373

- passing through seven general points, 386
 - secant variety
 - degree, 375–376
- rational normal scroll, 53, 324–327, 355
 - degree, 335
 - pinch points, 442
- rational quartic
 - containing 11 points, 289, 321
- rational quartic containing 11 points, 318
- reducible cubic, 64
- Rees algebra, 471
- reflexivity, 380–382
- regular 1-forms, 481
- relative duality, 328
- relative dualizing sheaf, 503
- relative Euler sequence, 394
- relative tangent bundle, 393
 - of a projective bundle, 393–394
 - Chern classes, 394
- representable functors, 205
- resultant, 427
 - formula via Δ_f^e , 428–430
- Riemann–Hurwitz formula, 68, 278
- Riemann–Roch theorem
 - applied to linear series on curves, 567–568
 - for smooth curves, 481–482
 - for smooth projective surfaces, 483
 - Grothendieck vs. Hirzebruch, 489
 - Grothendieck’s version, 489–490
 - applied to $\mathcal{E} = \text{Sym}^3 \mathcal{S}^*$, 490–493
 - applied to bundle of principal parts, 508
 - for a submersion, 490
 - Hirzebruch’s version, 488–489
 - reduces to classical versions, 508
 - motivation, 481
 - original formulation, 481–482
 - produces high-degree maps and embeddings, 566
- ruled surface, 341–343
 - containing curve of negative self-intersection, 323, 341–342
 - sections, 359
- Sard’s theorem
 - algebraic version, 519
- scheme, 9
- Schubert calculus, 2, 131
 - by static specialization, 115–117
- Schubert cell, 135
 - in $\mathbb{G}(1, 3)$, 102, 104
 - tangent space, 135
- Schubert class, 160
 - and the Chern class, 134
 - as fundamental invariant, 134
 - basis for Chow ring, 142–143
 - closed-form expression for multiplication, 148
 - combinatorial formula for multiplication, 149
 - counting via Young diagram, 153
 - definition, 132
 - notation, 132
- product formula, 143
- relation among special classes, 147
- representation via Young diagram, 152
- special classes, 145, 155
 - generate Chow ring of Grassmannian, 158, 183
- Schubert cycle, 4, 132
 - common cases, 132–133
 - definition, 132
 - equations, 138–139
 - generic transversality, 139
 - in $\mathbb{G}(1, 3)$, 102–103
 - intersection in complementary dimension, 141–142
 - notation, 103
 - benefits, 133–134
 - partial ordering, 133
 - relative to transverse flags, 140
 - special cycles, 133
 - tangent space, 108–109, 126
- Schubert index, 135
 - dual index, 142
- Schubert variety, *see* Schubert cycle
- scroll, 388
- secant line, 113, *see also* secant variety
 - stationary, 128
- secant plane, *see* secant variety
- secant plane map, 369
 - birationality, 370, 385
 - composed with the Plücker embedding, 378, 386
 - extends to an embedding, 374
 - improper secants, 385
 - is never regular for $n, m > 1$, 370
- secant variety, 367–371
 - abstract, *see* abstract secant variety
 - definition, 370
 - dimension, 370
 - expected dimension, 371
 - general point lies on unique secant plane to a curve, 386–387
 - of a curve of positive genus, 380
 - of a curve that is not a rational normal curve, 377–380
 - of a rational normal curve, 373–377
 - degree, 375–376
 - of an elliptic curve, 388
 - of the Veronese surface, 439
 - proper, 386
 - secant plane, 367
 - used to study pure d -th powers, 376–377
- second fundamental form, 243, 261–265
 - of a smooth hypersurface, 264
- section, 58
- Segre class, 350
 - as locus where global sections do not generate a bundle, 362, 363
 - definition, 363
 - generalized definition, 453–454
 - gives degree of the variety swept out by a linear space, 367, 385
 - is the reciprocal of the Chern class, 364
 - of the dual of a bundle, 364

- parallel with Chern class, 364
- used to obtain degrees of secant varieties, 375
- Segre map, 78
- Segre variety
 - as determinantal variety, 434
 - definition*, 52
 - degree, 52–53
 - Segre threefold, 286, 326
- self-intersection, 83
 - of a 2-plane on a fourfold, 478
 - of a curve on a surface, 70
 - of the zero section, 343–344
 - question of boundedness below, 359
- Serre duality, 328, 482
- Serre's coherence theorem, 530
- Serre's formula, 48
- sheaf, 9
- sheaf with specified fiber, 520
 - for 3-point ideal sheaf example, 522
- singular curve
 - conic meeting seven general lines in \mathbb{P}^3 , 360
 - cubic, 62
 - in a general pencil, 165, 244, 253, 279–280
 - of conics, 253
 - of higher degree, 64
 - on a quadric, 285
- singular elements
 - of linear series, *see* linear series, singular elements
- singular hypersurface, 34, 245–247
 - in a general pencil of hypersurfaces, 253
- singularity
 - of plane curves, 410–412
 - of plane sections of a general surface, 420
 - of the curve traced out by flexes of a pencil, 423
- smooth curve
 - conic tangent to a singular curve, 321
 - conic tangent to five conics, *see* five conic problem
 - conic tangent to five general curves, 321
 - conics degenerating to a double line, 294–295
 - conics degenerating to a rank-2 conic, 294
 - in \mathbb{P}^3 as intersection of three surfaces, 445, 452
 - quintic of genus 2 is the intersection of three surfaces, 477
 - quintics lying on surfaces, 83
 - with no inflection points is the rational normal curve, 287
- smooth hypersurface
 - cannot contain a plane of more than half its dimension, 222
 - containing a 2-parameter family of lines, 193, 238
 - quintic containing a 2-plane, 243
- smooth locus of discriminant, 284
- smooth plane curve
 - divisor of a conic, 298
 - genus, 69
 - smooth cubics, 62
- smooth rational curve
 - meeting lines four times, 426
 - quintic as intersection of three surfaces, 477
- smooth surface
 - class of a curve squared, 559
 - containing a curve
 - points of tangency, 445, 476
 - containing infinitely many irreducible curves of negative self-intersection, 343, 358
 - finitely many hyperplane sections with triple points, 420–421
 - finitely many lines on, 70, 83
 - of degree 3 in \mathbb{P}^4
 - lies on no smooth hypersurface, 556
- snake lemma, 187
- socle, 184
- space of complete conics, 290, 292
 - as compactification of smooth conics, 293
 - Chow ring, 306–309
 - classification of smooth conics, 296, 301
 - complete conic tangent to five general conics is smooth, 302–303
 - divisor class of complete conics tangent to a conic, 307–308
 - equations, 299–301
 - informal introduction, 293–296
 - is smooth and irreducible, 299
 - other divisor classes, 308–309
 - relation to blow-up, 301–302
 - rigorous description, 296–301
 - smooth complete conics, 296
 - used in solution of five conic problem, 302–308
- space of complete quadrics, 309–310
 - stratification, 310
- special divisors, 569
- special secant plane, 377
 - examples, 377–378
 - expected dimension, 387
- specialization, 62, 115–122
 - appearance of multiplicities, 121–122, 129
 - dynamic, *see* dynamic specialization
 - introductory example, 115
 - relations between singular plane cubics, 63
 - static vs. dynamic, 116–117
 - utility in projective space, 117
- specialization to the normal cone, 5
- specialization to the normal curve, 466–468
 - applied to excess intersection of cycles on a subvariety, 468–469
- sphere in complex projective space, 480
- splitting principle, 172–173
 - splitting construction, 172
 - statement*, 172
 - with Whitney's formula, 173–174
- splitting type, 494
- stability of fibers, 502, 503
- stable map, 313
- Steiner construction, 460
- stratification, 27
 - of \mathbb{P}^9 , 62–64
 - of the space of complete quadrics, 310
- strict transform, *see* proper transform

- swallowtail singularity, 247
- sweeping out
- by a subscheme, 217
 - bound on dimension of tangent space, 217
 - by linear spaces, 144–145, 161, 366–367
 - degree via the Segre class, 367
 - by lines on a pencil of hyperplane sections of a cubic, 240
 - by lines on a quartic threefold, 240
 - by lines with specified order of contact, 421
 - by the 2-planes of an irreducible surface, 443
 - by trisecants, 362, 379–380
 - by twisted cubic, 131, 144–145, 162
- symmetric power, 367–369
- affineness and projectivity, 368
 - as a projective bundle, 587–589
 - is a fine moduli space, 575–576
 - maps to Jacobian, 572
 - of \mathbb{A}^1 , 368
 - of \mathbb{P}^1 , 368
 - smoothness, 368–369
- tacnode, *see also* ordinary tacnode, 424–425
- tangent bundle, 179
- of projective space, 179
 - of projective spaces
 - is not the sum of line bundles, 192
 - to hypersurface, 179–180
 - to the Grassmannian, 96–99
- tangent cone
- extension to arbitrary schemes, 35
 - of a hypersurface, 34
 - to the discriminant, 284
- tangent hyperplane
- to a quadric, 297
 - to two smooth quadrics, 298
- tangent lines to a surface, 123–125
- tangent space
- of a cycle of tangent conics, 304
 - of the Fano scheme, 208
 - to a Schubert cycle, 108–109
 - to cycle of plane conics in \mathbb{P}^3 meeting a given line, 353
 - to the discriminant, 282–284
 - of degree- d polynomials, 285, 304
 - to the Fano and Hilbert schemes, 208–227
 - to the Grassmannian, 129
 - to the Picard group can be identified with cohomology, 544
- tangent vector
- rank, 97, 126
- tangential variety of a surface, 161, 286, 387, 439, 443
- is 4-dimensional, 440, 443
- tautological bundle, 177–179
- on the universal k -plane, 336
- tautological class, 336
- tautological family of plane conics, 348, 350
- tautological quotient bundle
- of a Grassmannian bundle, 347
- tautological subbundle, 324
- of a Grassmannian bundle, 346
 - Chern class, 347
- Terracini's lemma, 372–373
- theorem on cohomology and base change, 520
- necessity of flatness in version 3, 535
 - proof via approximation of a complex and the vector bundle criterion, 535–540
 - version 1, 526
 - version 2, 531–532
 - version 3, 533
 - alternative formulation, 541
- theta divisor, 573, 585
- Todd class, 487–488
- as polynomial in the Chern classes, 508
 - multiplicativity, 488
- topological Chern class
- algebraic Chern class results carry over, 563
 - and curvature, 561–563
 - as obstruction to a nowhere-zero section, 560–561
 - coincides with Chern class for an algebraic vector bundle, 561
 - definition*, 560–561
- topological Euler characteristic, 39, 179–182, 280, 482
- additivity, 277
 - and multiplicity of the discriminant, 281
 - determines the middle Betti number, 182
 - of a blow-up of a surface, 181
 - of a hypersurface, 181–182
 - of a smooth hypersurface of bidegree (a, b) , 192
 - of projective space, 181
 - via top Chern class, 181
- topological genus, 482
- topological Hurwitz formula, 277–285
- applied to pencils of curves on a surface, 279, 280
 - statement*, 278
- total inflection, 268
- transversality
- generic, 18
 - of eight cycles corresponding to general lines, 354
 - of five cycles in the variety of complete conics, 303–306
- transverse flags, 139
- transverse intersection
- definition*, 18
 - of Schubert cycles, 141
- triangle, 65
- triple point, 34
- of plane curves, 256–257
- triples of collinear points, 79
- class via Porteous' formula, 442
- trisecant
- surface in \mathbb{P}^3 swept out by, 362, 379–380
 - to a curve in \mathbb{P}^3 , 377
 - to a curve in \mathbb{P}^4 , 378
 - to a rational curve in \mathbb{P}^4 , 362, 378–379
 - to a space curve, 443
- Tschirnhausen transformation, 412
- tubular neighborhood theorem, 466
- fails to generalize, 466, 478
- twisted cubic

- common chords of two twisted cubics, 122
 - positive-dimensional component, 480
- is the unique curve whose secants sweep out \mathbb{P}^3 once, 374, 386
- tangent to 12 quadrics, 2
- two surfaces intersecting in a curve and a 0-dimensional scheme, 445, 459–460
 - dependence on geometry of the surfaces, 460
 - examples, 478
- 2-planes meeting three quadrics, 164
- universal divisor, 576, 581
- universal family of conics in the plane, 345
- universal family of subschemes, 204
- universal Fano scheme, *see also* Fano scheme, 230
 - and families of lines, 229–234
 - classes of universal Fano schemes of lines on surfaces in \mathbb{P}^3 , 231
 - defining equations, 197
 - definition*, 194
 - dimension, 194
 - global view as the zero locus of a section of a vector bundle, 229–231
 - of lines on cubic surfaces, 230
 - reducedness, 230
- universal flex, 423
- universal hyperplane, 382
 - Chow ring, 336
- universal hypersurface, 359
- universal line
 - Chow ring, 337, 394
 - in \mathbb{P}^n , 356
- universal line bundle, 575, 580–582
- universal plane, 96, 125, 144, 159
 - as projectivization of the universal subbundle, 336
 - Chow ring, 335–336
 - class via Porteous' formula, 442
- universal property of Proj, 327
- universal singular point, 245, 254, 260
 - class, 273–274
 - is a complete intersection, 245
- universality of a map, 484
- vanishing sequence, 266
 - geometric meaning, 267
- variety, 9
- vector bundle
 - complete classification over \mathbb{P}^1 , 223–224
 - generated by global sections, 362
 - mysteries on higher-dimensional projective space, 224
 - of rank 2 on \mathbb{P}^2
 - with even first Chern class, 497–499
 - with odd first Chern class, 499–501
 - on \mathbb{P}^1 splits, 223
 - on projective space
 - behavior in families, 494–497
 - projectivization, 171, 172
 - twisting by a line bundle, 335, 355, 356
- vector field, 99
- Veronese map, 48, 78
 - relation to discriminant, 246–247
 - used to prove the moving lemma, 514
- Veronese surface, 82
 - as variety of minimal degree, 335
 - hyperplane section with triple points, 421
 - in \mathbb{P}^5 as defective variety, 371–372
 - in \mathbb{P}^5 is the intersection of \mathbb{P}^5 with a Segre variety, 478
 - projection from a general line, 439
- Veronese variety, 48, 78
 - degree, 48–49
 - tangent planes, 385
 - which are m -defective, 373, 385–386
- very ample line bundle, 424
- web of quadrics in \mathbb{P}^3 , 444
- Weierstrass point, 287
- weight of a point with respect to a linear system, 268
- Weil divisor, *see* divisor
- Whitney's formula, 169
 - for globally generated bundles, 187–190
 - with the splitting principle, 173–174
- Young diagram, 132, 152–154
 - transposition and Grassmannian duality, 153–154, 163
- Zariski tangent space, 34, 208
 - algebraic descriptions, 100–102
 - identified with first-order deformations, 213
 - of a local ring, 209