

FILTERING COMPLEX TURBULENT SYSTEMS

Many natural phenomena ranging from climate through to biology are described by complex dynamical systems. Getting information about these phenomena involves filtering noisy data and making predictions based on incomplete information, and often we need to do this in real time, e.g. for weather forecasting or pollution control. All this is further complicated by the sheer number of parameters involved, leading to further problems associated with the “curse of dimensionality” and the “curse of small ensemble size”.

The authors develop, for the first time in book form, a systematic perspective on all these issues from the standpoint of applied mathematics. Their approach follows several strands:

- blending classical stability analysis of partial differential equations and their finite difference approximations;
- extending classical Kalman filters and applying them to stochastic models of turbulence to deal with large model errors;
- developing test suites of statistically exactly solvable models and new SPEKF algorithms for filtering slow–fast systems, moist convection, turbulent tracers, and geophysical turbulent systems.

The book contains enough background material from filtering, turbulence theory, and numerical analysis to make the presentation self-contained, and is suitable for graduate courses as well as for researchers in a range of disciplines across science and engineering where applied mathematics is required to enlighten observations and models.

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Preface

This book is an outgrowth of lectures by both authors in the graduate course of the first author at the Courant Institute during spring 2008 and 2010 on the topic of filtering turbulent dynamical systems as well as lectures by the second author at the North Carolina State University in a graduate course in fall 2009. The material is based on the authors' joint research as well as collaborations with Marcus Grote and Boris Gershgorin; the authors thank these colleagues for their explicit and implicit contributions to this material. Chapter 1 presents a detailed overview and summary of the viewpoint and material in the book. This book is designed for applied mathematicians, scientists and engineers, ranging from first- and second-year graduate students to senior researchers interested in filtering large-dimensional complex nonlinear systems.

The first author acknowledges the generous support of DARPA through Ben Mann and ONR through Reza Malek-Madani which funded the research on these topics and helped make this book a reality.