

Stellar Evolution Physics

Volume 2: Advanced Evolution of Single Stars

This volume explains the microscopic physics operating in stars in advanced stages of their evolution and describes with many numerical examples and illustrations how they respond to this microphysics. Models of low and intermediate mass are evolved through the core helium-burning phase, the asymptotic giant branch phase (alternating shell hydrogen and helium burning), and through the final cooling white dwarf phase. A massive model is carried from the core helium-burning phase through core and shell carbon-burning phases. Gravothermal responses to nuclear reaction-induced transformations and energy loss from the surface are described in detail. Written for senior graduate students and researchers who have mastered the principles of stellar evolution, as developed in the first volume of *Stellar Evolution Physics*, sufficient attention is paid to how numerical solutions are obtained to enable the reader to engage in model construction on a professional level.

The processes in this volume build upon those in Volume 1 of *Stellar Evolution Physics: Physical Processes in Stellar Interiors* (ISBN 978-1-107-01656-9), which describes the microscopic physics operating in stars and demonstrates how stars respond from formation, through hydrogen-burning phases, up to the onset of helium burning. *Stellar Evolution Physics* is also available as a 2-volume set (ISBN 978-1-107-60253-3). Taken together, the two volumes will prepare a graduate student for professional-level research in this key area of astrophysics.

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Preface

One might think that the most appropriate division of topics in a two volume book on stellar evolution physics would be the placement of all chapters describing the input physics required for the construction of stellar models in the first volume and the placement of all chapters describing stellar evolutionary models in the second volume. However, such a division disguises the fact that it is the operation of the input physics in stars that explains why they shine and evolve and that, therefore, both the input physics and the response of stars to the operation of this physics comprise the science of stellar evolution physics.

In preparing this book, after describing much of the input physics required for the construction of stellar models during early evolutionary stages, I constructed models in these early stages of evolution. Then, after describing some of the more complicated physical processes that play important roles during more advanced stages of evolution, I constructed models in these more advanced stages. The ordering of topics in the two volumes of this book reflects this chronological development.

After providing a general introduction to the observed properties of real stars and to the results of stellar evolution calculations, the first volume focusses on equations of state, energy generation by hydrogen-burning reactions, energy transport by radiation and convection, and on the elementary equations of stellar evolution and methods of solution. This is followed by a description of stellar models evolving during gravitationally contracting phases onto the main sequence, during the main sequence phase when core hydrogen burning is the primary source of surface luminosity, and during shell hydrogen-burning phases up to the onset of helium burning as the primary factor in controlling the evolutionary time scale and as a source of surface luminosity second only to hydrogen burning.

In the first part of this second volume, which is divided into three parts, input physics of a somewhat more subtle nature than addressed in the first volume is presented – diffusion, heat conduction by electrons, beta decay and electron capture at high densities, and weak interaction processes that are responsible for the production of neutrino–antineutrino pairs. The first part ends with a discussion of helium-burning nuclear reactions, which control the evolutionary time scale during the quiescent core helium-burning phase and play starring roles in the intricate dance between hydrogen burning and helium burning during the thermally pulsing asymptotic giant branch phase, leading to the formation of carbon and neutron-rich s-process elements that make their way from interior regions of production to the surface and thence into the interstellar medium.

All of the input physics developed in the first volume plays a role in the evolutionary models described in both volumes. Some of the input physics developed in this volume also plays a role in several evolutionary models presented in the first volume. For example, electron conduction and energy loss by neutrinos and antineutrinos play decisive roles in

establishing the thermal structure of the hydrogen-exhausted, electron-degenerate cores of low mass post-main sequence stars as they ascend the red giant branch for the first time, with the consequence that helium is ignited off center and proceeds in a series of flashes until a quiescent core helium-burning phase is reached. Evolution to the off center ignition of helium is described in Chapter 11 at the end of the first volume, whereas the helium flashing phase and the subsequent quiescent core helium-burning phase are described in Chapter 17 in this volume.

Another example of the relevance of the input physics developed in this volume to models described in the first volume is the discussion in Chapter 15 of the interaction Hamiltonian for weak interactions. The interaction between electron neutrinos and electrons predicted by this Hamiltonian has been used in Chapter 10 in Volume 1 to demonstrate that some of the electron neutrinos generated in the Sun are converted into muon neutrinos on their passage outward through the Sun, a demonstration which contributed significantly to the resolution of the solar neutrino problem which plagued stellar astrophysics for many years.

Results of stellar model calculations during helium-burning phases for models of mass $1 M_{\odot}$, $5 M_{\odot}$, and $25 M_{\odot}$ are described in the middle part of this second volume in Chapters 17, 18, and 20, respectively. In the case of the two lower mass models, special attention is given to the thermally pulsing AGB (TPAGB) phase when the main nuclear burning source of energy alternates between hydrogen burning and helium burning. In the case of the $5 M_{\odot}$ model, special attention is given in Chapter 19 to the activation of s-process nucleosynthesis and to the dredge-up of freshly made carbon and s-process elements during the TPAGB phase. In the case of the $25 M_{\odot}$ model, evolution is carried into the core and shell carbon-burning stages.

In the third and last part of this volume, consisting of Chapter 21, attention is focussed on the final stages of evolution of low and intermediate mass stars which become white dwarfs, with the evolution of a model of initial mass $1 M_{\odot}$ being highlighted. Described in the first three sections of Chapter 21 are (1) the wind mass loss which a TPAGB star experiences in consequence of radiation pressure on grains in a shock-inflated atmosphere, the shocks being due to Mira-like acoustical pulsations, and (2) the resulting planetary nebula stage, when the contracting remnant emits radiation of sufficient energy to cause the ejected material to fluoresce. After a discussion in the next two sections of the equation of state and the specific heat of stellar matter in the solid and liquid states, the final evolution of the remnant as it cools as a white dwarf is described in the sixth section. In concluding sections, the formation of monoelemental surface layers due to diffusion, the dependence of final surface abundances on evolutionary history, and quantitative estimates of the birthrate of low mass stars in the Galaxy and of the age of the Galactic disk are presented.