Contents

Pre	eface	pag	e xiii			
	Biblio	graphy and references	xvi			
		Part I Introduction and overview	1			
1	Qualita	tive description of single and binary star evolution	3			
	1.1	On the evolutionary status of real single stars	5			
	1.2	Close binary stars and evolutionary scenarios	14			
	Biblio	graphy and references	28			
2	Quantit	Quantitative foundations of stellar evolution theory				
	2.1	Observed properties of the Sun	30			
	2.2	Nearby stars in the Hertzsprung-Russell diagram	38			
	2.3	Mass-luminosity relationships	41			
	2.4	Evolutionary paths of theoretical models in the HR diagram	45			
	2.5	The evolutionary status of familiar stars	51			
	Biblio	graphy and references	54			
		Part II Basic physical processes in stellar interiors	55			
3	Propert	ties of and physical processes in main sequence stars – order of magnitude estimates	57			
	3.1	Particle numbers and separations, pressures and temperatures	58			
	3.2	Departures from a classical perfect gas: electrostatic interactions,				
		electron degeneracy, and radiation pressure	61			
		3.2.1 Electrostatic forces	61			
		3.2.2 The exclusion principle and electron degeneracy	62			
		3.2.3 Radiation pressure	64			
	3.3 Virial theorems relating kinetic, gravitational binding, and net bindir					
	energies					
	3.4	Energy transport: radiation, convection, and conduction	68			
		3.4.1 Radiative flow and mass-luminosity relationships	69			
		3.4.2 Opacity sources	71			
		3.4.3 Convection	73			

۷

vi	Contents	
	3.4.4 Heat conduction by electrons	78
	3.4.4 Heat conduction by electrons3.4.5 Summary	80
	3.5 Nuclear energy-generation rates and evolutionary time scales	80
	3.6 The static electrical field	86
	Bibliography and references	87
	4 Statistical mechanics, thermodynamics, and equations of state	88
	4.1 Quantum-mechanical wave functions and the unit cell in phase space	89
	4.2 The connection between thermodynamics and Fermi–Dirac statistics	
	for particles which obey the Pauli exclusion principle	91
	4.3 Calculation of pressure and energy density	97
	4.4 Equation of state for non-degenerate, non-relativistic ions	99
	4.5 Equation of state for weakly degenerate, non-relativistic electrons	101
	4.6 Equation of state for strongly degenerate, non-relativistic electrons	105
	4.7 Equation of state for non-relativistic electrons of intermediate	
	degeneracy	110
	4.8 Equation of state for relativistically degenerate electrons at zero	
	temperature	116
	4.9 Equation of state for relativistically degenerate electrons at finite	
	temperatures	121
	4.10 High temperatures and electron–positron pairs	128
	4.11 Indistinguishable particles, Bose–Einstein statistics, and the	1.10
	electromagnetic radiation field	149
	4.12 Maxwell–Boltzmann statistics and entropy	154
	4.13 Ionization equilibrium and the Saha equation for pure hydrogen	158
	4.14 Thermodynamic properties of partially ionized hydrogen	165
	4.15 The generalized Saha equations, with application to pure helium	170
	4.16 Thermodynamic properties of hydrogen- and helium-rich matter	174
	in stellar envelopes	174
	4.17 The effect of Coulomb interactions on the equation of state for a gas	179
	4.17.1 Modifications when electrons are modestly degenerate	184
	4.17.2 When electrons are significantly degenerate	187
	Bibliography and references	189
	5 Polytropes and single zone models	191
	5.1 The basic structure equation when pressure is proportional to a fixed $(1 + 1/1)$.	102
	power $(1+1/N)$ of the density	193
	5.2 Several properties of solutions as functions of N	197
	5.3 Additional properties of solutions when an equation of state and a law	010
	of nuclear energy generation are assumed	210
	5.4 Luminosity as a function of position in core nuclear burning models	215
	5.5 Polytropic characteristics of zero age main sequence models	222
	5.6 Models in adiabatic equilibrium: existence of a maximum temperature	224
	and decrease of entropy with time	224

vii	Contents				
	5.7	White dwarf properties revealed by polytropes: radius-mass relation-			
		ships and the maximum mass	231		
		5.7.1 Consequences of the $N = 3/2$ polytropic approximation	232		
		5.7.2 The Chandrasekhar mass limit from the $N = 3$ polytrope	235		
	5.8	Insights from one zone models: white dwarf radius versus mass,			
		heating and cooling in a low mass star as functions of radius, and			
		compression and ion cooling as luminosity sources in white dwarfs	237		
		5.8.1 The temperature maximum and the nature of energy sources			
		in cooling white dwarfs and completely convective low mass			
		stars	242		
	5.9	One zone models of neutron stars, neutron star composition,			
	~	and neutron star masses	248		
	Biblic	ography and references	256		
	6 Hydrod	gen-burning reactions and energy-generation rates	258		
	6.1	The nature and energetics of the pp reaction	260		
	6.2	Ingredients of the pp-reaction probability	260 261		
	6.3	An estimate of the weak interaction coupling constant	262		
	6.4	The nuclear matrix element	264		
	6.5	A numerical estimate of the cross section	268		
	6.6	The pp-reaction rate and proton lifetime	269		
	6.7	Other hydrogen-burning reactions – laboratory cross sections			
		and extrapolation to stellar conditions	271		
	6.8	The pp-chain reactions	275		
	6.9	Equilibrium abundances and energy-generation rates for pp-chain			
		reactions	280		
	6.10	The CN-cycle reactions	284		
	6.11	The effect of electrostatic screening on nuclear reaction rates	287		
	6.12	Polytropic models for zero age main sequence stars	291		
	Biblic	ography and references	297		
	7 Photor	n–matter interaction probabilities, absorption cross sections, and opacity	298		
	7.1	Photons and the electron-photon interaction Hamiltonian	300		
	7.2	First order perturbation theory and the golden rule for a radiative			
		transition between two matter eigenstates	303		
	7.3	The relationship between emission and absorption probabilities	307		
	7.4	The cross section for bound-free (photoelectric) absorption			
		from the K shell	312		
		7.4.1 On the determination of the associated opacity coefficient	319		
	7.5	The matrix element for free-free (inverse bremsstrahlung) absorption	319		
	7.6	The cross section for free-free absorption	325		
	7.7	The Kramers semiclassical approximation and Gaunt factors			
		for free-free absorption	331		
	7.8	Spontaneous emission between bound atomic states	337		

viii		Contents	
	7.9	Detailed balance, stimulated emission, bound-bound cross sections,	
	1.)	and line broadening	341
	7.10	The Rosseland mean opacity	347
	7.11	Sample calculations of the Rosseland mean opacity	350
	/.11	7.11.1 Hydrogen and helium completely ionized, oxygen with zero	550
		to two bound electrons	351
		7.11.2 Anatomy of an opacity when density = 1 g cm ⁻³ and	
		temperature varies from 10^7 to 10^6 K	354
		7.11.3 Arbitrary states of ionization for hydrogen, helium, and oxyge	en 361
		7.11.4 Number abundances and opacities as functions of temperature	
		when density = 0.01 g cm^{-3}	363
		7.11.5 Number abundances and opacities as functions of temperature	
		when density = 10^{-4} g cm ⁻³	371
		7.11.6 Number abundances and opacities as functions of temperature	
		when density $= 10^{-6} \text{ g cm}^{-3}$	376
		7.11.7 The effect on the Rosseland mean opacity of using Coulomb-	
		distorted plane waves for electrons to obtain the free-free	
		absorption coefficient	377
		7.11.8 Concluding comments	379
	7.12	Analytical approximations to results of opacity calculations	
		for intermediate to high temperatures	380
		7.12.1 Keller–Meyerott opacities	381
		7.12.2 Metal-free opacities at high temperatures	382
		7.12.3 Cox–Stewart opacities at intermediate to high temperatures	
		for mixtures of hydrogen and helium when $Z \le 0.02$	384
		7.12.4 Cox–Stewart opacities at high temperatures for mixtures	
		of helium, carbon, and oxygen	385
	7.13	Convective cores in stars burning nuclear fuel at the center	388
		7.13.1 The criterion for convection at the center and evidence	
		for the composite nature of models relying on CN-cycle	200
		energy generation	389
		7.13.2 Estimates of the size of a convection core in CN-cycle-burning	
		main sequence stars	392
		7.13.3 Convective regions in realistic models of zero age main sequence models	395
	7.14	Algorithms for interpolation in opacity tables	399
	/.14	7.14.1 Linear interpolation	399
		7.14.2 Quadratic interpolation	401
		7.14.3 Cubic spline interpolation	401
		7.14.3 Cubic spline interpolation 7.14.4 Bicubic spline interpolation	404 410
	7.15	Interpolation in opacity tables: a concrete example	410
	7.15	Absorption by the negative hydrogen ion	413
		ography and references	419
	DIDIO	graphy and tototototos	433

ix		Contents	
	• • • •		105
	•	ons of stellar evolution and methods of solution	435
	8.1	Consequences of the conservation of mass, momentum, and energy	437
	8.2	Examples of the creation–destruction potential for ions and electrons	446
	8.3	The quasistatic equations of stellar structure in spherical symmetry	453
	8.4	The photospheric boundary condition	457
	8.5	The classical fitting technique for model construction	459
		8.5.1 Development close to the model center and near the surface	459
	2 (8.5.2 Matching results of inward and outward integrations	460
	8.6	On the construction of integration algorithms	462
	- -	8.6.1 Application to stellar structure	476
	8.7	The relaxation technique for model construction	477
	8.8	Composition changes in radiative regions due to nuclear transformations	493
	8.9	Solution of linear equations by Gaussian elimination and LU	
		decomposition	500
	8.10	Composition changes in convective regions	506
		8.10.1 Time scales of relevance	507
		8.10.2 Convective diffusion in the mixing length approximation	508
		8.10.3 Solution of the convective diffusion equation: conversion	
		to a difference equation and construction of recurrence	511
		relationships	511
		8.10.4 The outer boundary condition and the quantities a_K and b_K	513
		8.10.5 The inner boundary condition and determination of new	C1C
		composition variables	515
	0.11	8.10.6 Composition in the static envelope	516
	8.11	Remarks on mixing in radiative zones due to particle diffusion:	
		gravitational settling, abundance-gradient induced diffusion and	510
	0.10	rotation-induced diffusion	516
	8.12	Zoning considerations and choice of time step	519
	8.13	On the evolution of the computing environment	521
	Biblio	ography and references	525
	Pa	art III Pre-main sequence, main sequence, and shell hydrogen-burning	
		evolution of single stars	527
	9 Star fo	rmation, pre-main sequence evolution, and the zero age main sequence	529
	9.1	Some concepts relevant to star formation	531
		9.1.1 The Jeans criterion	531
		9.1.2 The roles of magnetic fields and cosmic rays	534
		9.1.3 Other studies of collapse and formation of a quasistatic core	535

- 9.1.3 Other studies of collapse and formation of a quasistatic core9.1.4 Further description of the accretion phase
- 9.1.4Further description of the accretion phase5389.2Pre-main sequence quasistatic evolution of a solar mass population I
model with deuterium burning539

x		Contents	
		9.2.1 Input physics and initial abundances for evolutionary calculations	540
		9.2.2 Structure and gravothermal characteristics of a Hayashi-band	
		model	543
		9.2.3 The deuterium-burning phase	548
		9.2.4 Evolution in the HR diagram and characteristics of models along the approximately vertical portion of the track	556
		9.2.5 Development of a radiative core and the transition from vertically downward to upward and leftward in the HR diagram	563
		9.2.6 Characteristics of a model in transition from the convective phase to the predominantly radiative phase	566
	9.3	Approach of a solar mass model to the main sequence: the onset	
		and ascendancy of hydrogen burning by pp-chain reactions	
		and properties of a zero-age main sequence model	573
		9.3.1 Hydrogen burning begins	573
		9.3.2 A model in which nuclear burning and gravitational work contribute comparably to the surface luminosity	579
		9.3.3 A zero age main sequence model	587
	9.4	Evolution of a 5 M_{\odot} population I model to the main sequence: gravitational contraction, C \rightarrow N burning, CN-cycle burning, and properties	
		of a zero age main sequence model	597
	9.5	Evolution of a 25 M_{\odot} gravitationally contracting population I model through deuterium burning and two C \rightarrow N burning phases,	
		and properties of a CN-cycle burning zero age main sequence model	622
	Biblio	graphy and references	635
10	Solar st	ructure and neutrino physics	637
	10.1	Construction and properties of a $Z = 0.01$ solar-like model	
		with the Sun's luminosity, radius, and estimated age	639
	10.2	Construction and properties of a $Z = 0.02$ solar-like model	
		with the Sun's luminosity, radius, and estimated age, and comparisons	CRO
	10.2	with the $Z = 0.01$ solar-like model	658
	10.3	Contributions to photon and neutrino luminosities and to neutrino fluxes at the Earth	668
	10.4	The solar neutrino problem	673
	10.5	Neutrino oscillations in vacuum	682
	10.6	The MSW effect	689
	10.7	Numerical solutions for neutrinos produced at the center of a simple	
	10.0	solar model	699
	10.8	Solutions for neutrinos produced in realistic solar models	703
	10.9	Summary and conclusions	710
	10.10	Postscript	711
	Biblio	graphy and references	712

xi	Contents				
	11 Evoluti 11.1	on through hydrogen-burning phases of models of mass 1, 5, and 25 M_{\odot} Evolution of a 1 M_{\odot} model during core and shell hydrogen burning on the main sequence, formation of an electron-degenerate core,	714		
		and core growth during shell hydrogen burning on the red giant branch	716		
	11.2	Evolution of a 5 M_{\odot} model during core hydrogen burning, development of a thick hydrogen-burning shell, and shell hydrogen-burning			
		evolution up to the onset of core helium burning as a red giant	768		
	11.3	Evolution of a 25 M_{\odot} model without mass loss during core hydrogen burning on the main sequence and during shell hydrogen burning near			
		the main sequence up to the onset of core helium burning as a blue giant	820		
	11.4	Global properties of main sequence models as functions of model mass and estimates of surface mass loss during pure hydrogen-burning			
		phases	836		
		11.4.1 Global main sequence properties	836		
		11.4.2 Mass loss during hydrogen-burning phases	840		
	Biblio	graphy and references	845		
	Index		846		