

## Stellar Evolution Physics

### Volume 1: Physical Processes in Stellar Interiors

This volume describes the microscopic physics operating in stars and demonstrates how stars respond from formation, through hydrogen-burning phases, up to the onset of helium burning. Intended for beginning graduate students and senior undergraduates with a solid background in physics, it illustrates the intricate interplay between the microscopic physical processes and the stars' macroscopic responses. The volume examines the gravitationally contracting phase which carries the star from formation to the core hydrogen-burning main sequence, through the main sequence phase, through shell hydrogen-burning phases as a red giant, up to the onset of core helium burning. Particular emphasis is placed on describing the gravothermal responses of stars to nuclear transformations in the interior and energy loss from the surface, responses which express the very essence of stellar evolution. The volume is replete with many illustrations and detailed numerical solutions to prepare the reader to program and calculate evolutionary models.

The processes in this volume are added to in Volume 2 of *Stellar Evolution Physics: Advanced Evolution of Single Stars* (ISBN 978-1-107-01657-6), which explains the microscopic physics operating in stars in advanced stages of their evolution and describes how they respond to this microphysics. *Stellar Evolution Physics* is also available as a 2-volume set (ISBN 978-1-107-60253-3). Taken together, the two volumes will prepare a graduate student for professional-level research in this key area of astrophysics.

**Icko Iben, Jr.** is Emeritus Distinguished Professor of Astronomy and Physics at the University of Illinois at Urbana-Champaign, where he also gained his MS and PhD degrees in Physics and where a Distinguished Lectureship in his name was established in 1998. He initiated his teaching career at Williams College (1958–61), engaged in astrophysics research as a Senior Research Fellow at Cal Tech (1961–4), and continued his teaching career at MIT (1964–72) and Illinois (1972–99). He has held visiting professorships at over a dozen institutions, including Harvard University, the University of California at Santa Cruz, the University of Bologna, Italy, and Niigata University, Japan. He was elected to the US National Academy of Sciences in 1985, and his awards include the Russell Lectureship of the American Astronomical Society (1989), the George Darwin Lectureship (1984) and the Eddington Medal (1990) of the Royal Astronomical Society, and the Eminent Scientist Award of the Japan Society for the Promotion of Science (2003–4).

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## Volume 1: Physical Processes in Stellar Interiors



ICKO IBEN, JR.  
University of Illinois at Urbana-Champaign



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Shaftesbury Road, Cambridge CB2 8EA, United Kingdom  
One Liberty Plaza, 20th Floor, New York, NY 10006, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India  
103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment,  
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[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9781107016569](http://www.cambridge.org/9781107016569)

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First published 2013

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloging-in-Publication data*

Iben, Icko, 1931–

Stellar evolution physics / Icko Iben, Jr.

p. cm.

Includes bibliographical references and index.

ISBN 978-1-107-01656-9 (Hardback)

1. Stars–Evolution. 2. Stellar dynamics. I. Title.

QB806.I24 2012

523.8’8–dc23

2012019504

ISBN 978-1-107-01656-9 Hardback

Also available as part of a two-volume set, *Stellar Evolution Physics* ISBN 978-1-107-60253-3

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## Preface

The bright stars in the familiar constellations of the Milky Way have intrigued mankind for millennia. Over the past several centuries we have obtained by observations a quantitative understanding of the intrinsic global and surface characteristics of these stars, and over the past century we have learned something about their internal structure and the manner in which they change with time. An awareness that one kind of star can transform into another kind of star and an appreciation of how this transformation is achieved have been accomplishments of the last half of the twentieth century. One of the objectives of this monograph is to describe some of the transformations and to understand how they come about.

The microscopic and macroscopic physics that enters into the construction of the equations of stellar structure and evolution is described in many other monographs and texts. For highly personal reasons, this physics is nonetheless developed here in some detail. My undergraduate and graduate training was in physics, but I did not fully appreciate the beauty of physics until, just prior to my second year of college teaching, during an enforced sedentary period occasioned by a collision between myself on a bicycle and an automobile, I discovered the book *Frontiers of Astronomy* by Fred Hoyle and became entranced with the idea that the evolution of stars could be understood by applying the principles of physics. During my next two years of teaching, I embarked on a self study course heavily influenced by the vivid description of physical processes in stars by Arthur S. Eddington in his book *The Internal Constitution of the Stars* and by the straightforward description of how to construct solutions to the equations of stellar structure by Martin Schwarzschild in his book *The Structure and Evolution of the Stars*. These books taught me that stars provide a context for understanding physics on many different levels.

Stars are fascinating objects in their own right and the marvelous transformations which they experience as they evolve provide a captivating story worth telling over and over again. In this book, the distinction between the microscopic physics that occurs in stars and the macroscopic response of the stars to the operation of this physics is highly blurred. In short, physics is physics and every effort has been made to understand the relationship between the microscopic and macroscopic aspects of this physics. As Eddington put it, “It would be hard to say whether the star or the electron is the hero of our epic.”

In the world of single stars and of stars in wide binaries, focus has been placed on how a star of low to intermediate mass reaches the main sequence, then evolves successively through the main sequence phase, into a red giant, through the core helium-burning phase, into and through the shell helium- and hydrogen-burning asymptotic giant branch phase, and finally into a white dwarf. Attention has also been given to the evolution of stars massive enough to experience central and shell carbon burning, experience a type II supernova explosion, and then evolve into a neutron star or black hole.

A basic objective has been to provide some feeling for what stars really are – not just how big, how bright, and how old they are, but what is going on inside them. How hot are they inside, how dense are they, what sources of pressure do they rely on to balance gravity, what are their energy sources, what are the dominant modes of energy transport, what has been the extent of nuclear transformations in their interiors, and so forth, are questions that have been deemed of central importance. Although one needs to understand physics deeply in order to address these questions properly, and although one can solve the equations based on this physics, many aspects of the solution must be described in a very qualitative way. In an effort to offset the impression of remoteness between the input physics and the set of numbers which constitutes a model star, some partial solutions have been constructed which attempt to tie together in a simple and straightforward way the macroscopic characteristics of stars with the microscopic physics occurring in their interiors.

Just as it has benefited from other disciplines, the science of stellar evolution has contributed to other disciplines. For example, much of the light coming from distant galaxies is the sum of the contributions of individual stars which are themselves unresolvable, but knowledge of whose characteristics is essential for understanding the structure and evolution of these galaxies. By the same token, understanding cosmology requires understanding the characteristics of galaxy evolution. Thus, to fully understand both galaxy evolution and cosmology, one needs also to understand stellar evolution, and so the colors (surface temperatures), luminosities, and evolutionary time scales which characterize different evolutionary stages receive careful attention.

The heavy elements which are essential for the formation of terrestrial planets and for the existence of life on such planets have been constructed in stars and ejected by them into the interstellar medium out of which successive generations of stars are born. Hence, to truly understand the origins of the Solar System and of life, one needs to understand the manner in which stars contribute to the build-up of the heavy elements in the Galaxy. A description of element building in stars is therefore a central theme.

The logic of the presentation, including the grouping of the observational evidence, has evolved through almost five decades of emersion in theoretical model building and comparison with the observations. The observations have, in some instances, eliminated possible theoretical paths, made multiple by uncertainties in and incompleteness of the input physics, and, in others, it has opened up new possibilities for theoretical exploration not previously envisioned.

One of the greatest pleasures in writing this book has been to experience moments of revelation when insight comes for the first time. At such moments, I recall the words of T. S. Eliot in *Four Quartets* to the effect that it is in the nature of mankind to explore, and that the end result of all our exploring is to return to the place from which we started and to understand it for the first time. The exact quote, near the end of the fourth quartet (*Little Gidding*), is:

“We shall not cease from exploration  
And the end of all our exploring  
Will be to arrive where we started  
And know the place for the first time.”

An even greater pleasure would be realized if perusal of this book were to inspire some in future generations to contribute to the description of the physics that constitutes stellar evolution in simple and transparent ways, free of jargon and complex mathematics. I would encourage those so motivated to actually construct a stellar evolution code to explore a phase of evolution not yet properly addressed. One of the most exciting moments in my life occurred when in 1963, after almost a year of effort as a senior research fellow at Cal Tech, an evolution program I was constructing finally worked. The music and words of Franz Joseph Haydn's *The Creation* exploded in my brain: "The heavens are telling the glory of God, the wonder of His work displays the firmament."

Due to its length, the book is broken into two volumes and the text is divided into six parts, three parts in the first volume and three in the second volume. In Part I, a qualitative description of single and binary star evolution is followed by a quantitative description of the global characteristics (luminosity, radius, and mass) of stars provided by the observations. In Part II, the basic physical processes occurring in main sequence stars are outlined and the stage is set for constructing realistic mathematical models. Equations of state are constructed by exploring the consequences of momentum and energy balance under conditions of thermodynamic equilibrium and simple stellar models are constructed by demanding only a balance between gravity forces inward and pressure gradient forces outward and a one-parameter relationship between pressure and density. Estimates of the rates of hydrogen-burning reactions and of cross sections for photon-matter interactions are obtained. Part II ends with a description of the equations necessary for constructing theoretical models and of methods for solving the equations. In Part III, solutions of the equations as applied to the evolution of  $1 M_{\odot}$ ,  $5 M_{\odot}$ , and  $25 M_{\odot}$  models approaching the main sequence are presented. Solar models are constructed and a discussion is given of how the initial discrepancy between a predicted and detected solar neutrino flux forced scientists to rethink neutrino physics and resolve the discrepancy, thus vindicating the basic validity of the input physics on which astrophysicists had relied for decades in constructing stellar models. Part III ends with the solutions of the equations of stellar structure as applied to the evolution of  $1 M_{\odot}$ ,  $5 M_{\odot}$ , and  $25 M_{\odot}$  models through all phases of hydrogen burning until the onset of helium burning in the hydrogen-exhausted core.

The second volume begins with a description, in Part IV, of physical processes that are particularly important during advanced phases of evolution. These processes are diffusion, heat conduction by electrons, neutrino losses by beta-decay reactions of the sort occurring in terrestrial laboratories and of a sort uniquely occurring in stars, and helium-burning reactions. In Part V, models of  $1 M_{\odot}$  and  $5 M_{\odot}$  are evolved through the core helium-burning phase and into the thermally pulsing AGB (TPAGB) phase, with a special chapter devoted to the production of s-process elements. The  $25 M_{\odot}$  model is evolved through the core helium-burning phase and into the shell carbon-burning phase. In the sixth and final part, the  $1 M_{\odot}$  TPAGB model is subjected to wind mass loss, evolved through the planetary nebula phase, and, after an exploration of the physics of solids and liquids, is evolved into the white dwarf stage through phases of liquefaction and solidification.

The presentation in the book assumes a solid grounding in basic physics at the advanced undergraduate and early graduate levels and a firm grasp of calculus. The reader would benefit from having had an elementary course in astronomy, but this is not an essential requirement. The content and level of presentation of Volume 1 are suitable for a one-semester beginning graduate-level course which is also within the capabilities of well-prepared senior undergraduate physics students. Volume 2 discusses somewhat more sophisticated physical processes, and carries the discussion of stellar evolution to more advanced stages than described in the first volume. It could form the basis of a second one-semester graduate level course in which students might be encouraged to write and/or make use of a stellar evolution code to construct their own evolutionary models.

My original intent was to provide a definitive account of the evolution from start to finish of all stars whose evolution is essentially quasistatic. To my surprise, I have found that there remain as many unanswered questions regarding this evolution as there have been answers, a discovery that makes me quite sanguine about the future of the field. The story which I tell is far from complete and there are mountains of rewarding work remaining for future generations to tackle.

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