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PART I

INTRODUCTION AND OVERVIEW

1 Qualitative description of single and binary star evolution

Although locally conspicuous, stars are but one of many forms in which matter and energy in the Universe can manifest themselves; and there is a continuous interaction between these other forms and the stars. Stars are made out of diffuse interstellar matter that gathers itself into the condensations seen as giant molecular clouds; the stars lose mass as they evolve, returning to the interstellar medium material which they have enriched in heavy elements, thus causing a gradual change in the composition and cooling characteristics of the interstellar medium. Radiation from the stars interacts with interstellar matter in the immediate environment, changing the thermodynamic characteristics of this matter. Ejection at high velocity of stellar envelope material, such as occurs in supernova explosions, makes another important contribution to the energetics of the interstellar medium. The change in the composition, thermodynamic, and dynamical characteristics of the interstellar medium then alters the character and dynamics of the star formation process. Thus, although one may concentrate on the evolutionary behavior of stars as if they were isolated entities, stars are actually in a state of interaction with their environment, both feeding and feeding upon the matter in this environment. There is, however, a degree of asymmetry in the interaction. Although one cannot understand the evolution of the interstellar medium without taking into account the influence of stars, one can understand the evolution of a star without worrying about how it was made. Thus, in this book, stars are viewed as more or less isolated entities, their existence being accepted as a given and the circumstances that lead to their formation being examined only to the extent that they offer insight as to an appropriate initial configuration with which to begin evolutionary model calculations.

The growth in our understanding of stellar evolution has been a joint venture between observational astronomers, who have been aided immensely over the years by the development of more powerful telescopes and detectors at all wavelengths, and computational astronomers, who have been aided by the development of ever more powerful digital computers. It is safe to say that each new advance in detector or in computer technology and in marketing this technology has led to an understanding of at least one facet of stellar evolution which eluded understanding prior to this new advance. In this sense, the science of stellar evolution owes much of its success to engineers in both the academic and industrial sectors, and to the business community.

A vast amount and variety of observational and theoretical information goes into providing the foundation on which the edifice of stellar evolution is built. From the Sun and from stars near enough to permit a determination of distance by trigonometry, it has been established that the bulk of the stars lie within a well-defined band or main sequence of low dispersion in a Hertzsprung–Russell (HR) diagram. The HR diagram can take many forms. From an observer's perspective, color and visual magnitude can be a convenient coordinate pair; from a theoretician's perspective, bolometric luminosity and surface temperature are the logical coordinate pair. The orbital characteristics of wide non-interacting binary systems in which at least one of the components is a main sequence star establish a relatively well-defined correlation between the luminosity and mass for a star in the main sequence band. Analysis of the strengths of lines of different atomic and molecular species in stellar spectra demonstrates that hydrogen and helium are by far the dominant constituents of matter at the surfaces of main sequence stars, and permits quantitative estimates of the relative abundances of the heavier elements. Finally, solutions of the equations of stellar evolution for stellar models which are initially of homogeneous composition (with the distribution of elements like that seen at the surfaces of main sequence stars) and which are burning hydrogen in their cores mimic in fair quantitative detail the observed properties of real main sequence stars, thereby solidifying the identification of the main sequence as the residence of core hydrogen-burning stars.

Among the visually brightest stars in the solar vicinity are many which are intrinsically much brighter and considerably cooler at their surfaces than main sequence stars of the same mass. These include stars known as red giants and supergiants, blue supergiants, and RR Lyrae and Cepheid variables. There are others which are intrinsically fainter, but have higher surface temperatures than main sequence stars of the same mass. These include white dwarfs and some subdwarf O and B (sdO and sdB) stars. Still others exhibit at their surfaces abundance distributions which are significantly different from those in main sequence stars. Examples are: red (super) giant carbon stars which show spectral lines of molecules containing the element technicium (all isotopes of which are radioactive with half lives much shorter than the ages of the stars); Wolf–Rayet stars which appear to lie near the main sequence in the luminosity–surface temperature plane but whose spectra show unusual abundances of helium, carbon, and nitrogen; FG Sge stars which have been observed to alter their surface abundances in real time; and hydrogen-deficient supergiant R CrB stars and helium-rich PG 1159 white dwarfs. Still others are both brighter and hotter at their surfaces than are main sequence stars of the same mass. Examples are the central stars of planetary nebulae and some sdO and sdB stars.

White dwarfs and red giants define distinct, well populated sequences in the HR diagram. A typical white dwarf in the observed sample is roughly the size of the Earth and one hundred times less luminous than the Sun. A typical red giant in the nearby sample is one hundred times larger and one hundred times brighter than the Sun. White dwarfs can be easily detected from the Earth only within 100 parsecs of the Sun (a parsec, or pc for short, is approximately 3.26 light years), but bright giants can be observed throughout the Galaxy. Altogether in the entire Galaxy, red giants are roughly one hundred times less numerous than white dwarfs, and white dwarfs are roughly ten times less numerous than main sequence stars more massive than 0.1 $M_\odot$ (one tenth of a solar mass). Hence, a random sampling of stars chosen from within a sphere of radius one hundred pc about the Sun will turn up a few red giants and a modest number of white dwarfs relative to the number of main sequence stars, and a random sampling of visually bright stars will turn up many giants and intrinsically bright main sequence stars, but very few white dwarfs, if any.

In the next two sections, properties of theoretical models of evolving stars are used to interpret the observed characteristics of real single (Section 1.1) and binary (Section 1.2) stars. Although making use of theoretical results before describing these results might seem to be putting the cart before the horse, the objective has been to emphasize that a full appreciation of the luminous objects in the sky that we gaze at in awe and wonder requires the knowledge gained by staring into the bowels of a computer, as the observational astronomer Olin Eggen once characterized my approach to astronomy. It should be noted that, until he died, Olin and I remained good friends and collaborators. In a more serious vein, the following two sections are recommended reading for those who have had an elementary course in astronomy. Those who have not had such a course might find it more profitable to skip immediately to Chapter 2 and then return to Sections 1.1 and 1.2 in this chapter.

1.1 On the evolutionary status of real single stars

The relative characteristics of stars in stellar aggregates both in our Galaxy and in several nearby galaxies, especially in the nearby Magellanic Clouds, have played crucial roles in helping us to determine the internal evolutionary state of many of the stellar types which are not main sequence stars. This determination has been accomplished by comparing the observed properties of stars in these aggregates with theoretical stellar models which have been constructed by taking into account changes in composition wrought by nuclear transformations in the interior. Basic to the exercise is the fact that, if the aggregate is far enough away from us, the distance between stars in the aggregate is sufficiently small compared with the distance from us to the center of the aggregate that the apparent luminosity of one star relative to the apparent luminosity of another is essentially identical with the ratio of the intrinsic (absolute) luminosities of the two stars. The second basic element in the exercise is the assumption that the main sequence defined by stars in the cluster is roughly the same as the main sequence defined by stars in our vicinity for which trigonometric parallaxes can be obtained; this permits an estimate of the distance to the cluster and therefore an estimate of the absolute brightness of each star. The third basic element is the assumption that all of the stars in an aggregate are of essentially the same age and initial composition; if the period of time over which the stars in the aggregate have been formed is small compared with the time which has elapsed since star formation in the aggregate ceased, then the only attribute which differentiates one star from another at birth is stellar mass. Since initially more massive stars evolve more rapidly than less massive ones, the most massive cluster stars which are still burning nuclear fuel will be in the most advanced evolutionary state prior to becoming white dwarfs, neutron stars, or black holes; since the lifetime in phases more advanced than the main sequence phase is short compared with the lifetime of the main sequence phase, the distribution in the color–magnitude plane of the stars in the aggregate which are not on the main sequence effectively defines the evolutionary path of an individual star of an initial mass which slightly exceeds the initial mass of the most luminous stars still on the main sequence. This path may be compared with the path obtained by solving the equations of stellar evolution.

In this way, it is learned from the globular clusters in our Galaxy that, after spending approximately 10^{10} yr converting hydrogen into helium on the main sequence, stars of near solar mass exhaust hydrogen at the center and evolve into red giants which are approximately 2000 times brighter than the Sun before igniting helium in a hot white-dwarf-like core composed of nearly pure helium. The helium nuclei have average energies of $\frac{3}{2}\,kT$, where k is Boltzmann's constant and T is the temperature, but the electrons are forced by the Pauli exclusion principle into higher energy states of average energy much larger than kT, so the electrons, which are said to be degenerate, provide most of the pressure support for the core. The ignition of helium is initially explosive, with the explosion continuing until the kinetic energy imparted to the degenerate electrons by the nuclear energy released as helium is fused into carbon exceeds the degeneracy energy of the electrons, forcing the hydrogen-exhausted core to expand. In response, the envelope of the star contracts. The process continues until the model star has reached a new equilibrium configuration in which helium burns quiescently in a core in which electrons are not degenerate and hydrogen burns in a shell. The new equilibrium configuration lasts for approximately 10^8 yr, or approximately as long as it takes a precursor to ascend the red giant branch.

A quiescent helium-burning model evolves within a region in the HR diagram defined by horizontal branch (HB) stars in globular clusters. Stars on this branch are approximately twenty-five times more luminous than the most luminous main sequence stars in the clusters and the spread in surface temperatures along observed HBs, when compared with model evolutionary tracks, demonstrates that approximately $0.2 \pm 0.1\,M_\odot$ is lost by precursor red giants in a stochastic wind, the physics of which is not fully understand. The identification of HB stars as core helium and shell hydrogen burning stars is strengthened by the fact that the number of HB stars and the number of ascending red giant stars are comparable, in agreement with the theoretical result that the lifetimes of the two phases are comparable. The near constancy of the luminosity of HB models is a consequence of the fact that the mass of the hot white dwarf core of the model which ignites helium explosively is essentially independent of the initial mass of a main sequence precursor.

If their evolution carries them into a pulsational instability strip, the location of which can be estimated from theory and determined from observation, HB model stars pulsate acoustically as RR Lyrae stars. Such stars are typically of about the same surface temperature as the Sun. For stars pulsating in the fundamental radial mode, typical pulsation periods are in the range 6–9 hours and, for stars pulsating in the first overtone, periods are typically in the range 10–17 hours. The eponymous variable star RR Lyra is a first overtone pulsator with a period of 13.4 hours. By comparing observed properties with the results of pulsation calculations, one can corroborate that the masses of HB stars in the instability strip are less massive than their main sequence precursors by an amount which varies between about $0.1\,M_\odot$ and $0.2\,M_\odot$.

After hydrogen is exhausted at the centers of model stars initially more massive than about $2\,M_\odot$, the hydrogen-exhausted core contracts and heats rapidly enough that helium is ignited at the center before electrons have become degenerate. The transition to a new equilibrium configuration proceeds in a way which is qualitatively similar to the way in which low-mass models reach the HB branch except that the location in the HR diagram of the quiescent core helium burning and shell hydrogen burning model depends strongly on the mass of the main sequence precursor. In fact, the models define a core helium-burning band which is roughly parallel to the hydrogen-burning main sequence band and can be identified with a similar band defined by nearby intrinsically bright stars such as Rigel, Canopus, and Capella. Just as along the hydrogen-burning main sequence band, the luminosity and surface temperature in the core helium-burning band increase with increasing model mass.

The instability strip defined by pulsation theory passes through the theoretical core helium-burning band defined by evolutionary calculations. Observational counterparts are the Cepheids which, in our Galaxy, have radial pulsation periods between about two and thirty days, peaking in frequency at about six days. The locations of both the band and the strip are functions of the composition, and theory predicts that the functional dependences are such that the intersection between the band and the strip occurs at lower luminosities and lower periods as the metallicity is decreased. Since lifetime increases with decreasing luminosity, one infers that the relative number of stars in an aggregate which are Cepheids should increase with decreasing metallicity. The mean metallicity of stars decreases in going from our Galaxy to the Large Magellanic Cloud (LMC) and then to the Small Magellanic Cloud (SMC). The decrease in the average pulsation period and the increase in the relative number of Cepheids in passing from our Galaxy to the Large and Small Magellanic Clouds is consistent with these predictions.

After exhausting helium at the center, stars of near solar mass evolve again into a red giant configuration, alternately burning hydrogen and helium in separate shells above a hot white-dwarf-like core in which the electrons are degenerate. The bare nuclei in the core are either a mixture of carbon and oxygen (stars less massive than $\sim 8\,M_\odot$) or oxygen and neon (stars of mass in the range 8–10 $M_\odot$). The asymptotic giant branch (AGB) phase, as this phase is called, lasts for a few times 10^5 years (for stars initially of intermediate mass, 2–10 $M_\odot$) to a few times 10^6 years (for stars initially of low mass $\lessapprox 2\,M_\odot$).

AGB stars experience thermonuclear flashes which are also called thermal pulses. Most of the time a model burns hydrogen quiescently in a shell, depositing the resultant hot helium ashes onto a layer of increasing mass above the electron-degenerate carbon–oxygen (CO) core. The helium layer becomes dense and heats as it is compressed, and, once it becomes hot enough, helium is ignited and helium burning begins to produce energy more rapidly than this energy can be carried away by photon diffusion. A thermonuclear runaway (helium shell flash) ensues and continues until the extra pressure built up by heat deposition in the burning region has forced a significant expansion of matter in the helium layer and in overlying layers. With expansion comes cooling; temperatures at the base of the hydrogen-containing portion of the model drop to such low values that hydrogen ceases to burn and the rate at which helium burning produces energy comes into equilibrium with the rate at which photon diffusion can carry energy outward. The model embarks on a phase of quiescent helium burning which lasts about one-tenth as long as the preceeding hydrogen-burning phase.

Eventually, the temperatures in the transition layer between hydrogen-rich matter and helium- and carbon-rich matter decrease to such an extent that helium burning ceases. The decrease in the outward flux of helium-burning energy is compensated for by the reinstitution of contraction and release of gravothermal energy in immediately overlying hydrogen-rich layers, and the concomitant heating in these layers means that hydrogen burning recommences. The duration of the thermonuclear runaway which punctuates each cycle is only a few dozen years and the time between thermal pulses for a typical AGB star of low mass is of the order of 100 000 yr, so the probability of detecting a real star in the process of undergoing the first stages of a thermal pulse is not very large. However, there are several stars which may be identified as being in the process of relaxing after the thermonuclear runaway has taken place. Stars undergoing these alternating hydrogen- and helium-burning episodes are called TPAGB (thermally pulsing AGB) stars.

The lifetimes of real stars in the AGB phase can be estimated by comparing the distribution in luminosity achieved by real AGB stars with that predicted by model AGB stars. If no mass loss were to occur to terminate the ascent of the AGB branch, a model star of near solar mass would achieve a luminosity which is $\sim 3 \times 10^4$ times the Sun's luminosity as its carbon–oxygen core grows to contain most of the mass of the model. Since the maximum luminosity of a globular cluster AGB star is only a few thousand times the Sun's luminosity, one infers that mass loss must abstract from real AGB stars most of their remaining hydrogen-rich envelope before the carbon–oxygen core exceeds about 0.5–0.6 $M_\odot$, or after only about 10^6 years of evolution along the AGB. The short time scale for the mass-loss phase and the large amount of mass loss have prompted the name superwind. The atoms in the shell of matter that is lost by the AGB star are eventually excited into fluorescence by photons from the still bright stellar remnant which contracts to high surface temperature before cooling into a white dwarf. The fluorescing shell is known as a planetary nebula.

The Magellanic Clouds contain aggregates also called globular clusters, but these aggregates typically have only a few times 10^4 stars, over ten times fewer stars than are found in most galactic globular clusters, and their ages span a much larger range. Thus, there are clusters containing AGB stars whose progenitors are of near solar mass, as in the galactic globular clusters, but there are also clusters containing AGB stars whose progenitors have masses near 5 $M_\odot$. One may therefore learn how AGB evolution depends on progenitor mass. The most important lesson is that real AGB stars deriving from intermediate-mass stars expel their hydrogen-rich envelopes within only a few times 10^5 yr after arriving on the AGB; therefore, the mass of the white dwarf remnant of such stars is only $\sim$0.05–0.1 $M_\odot$ larger than the mass of the CO or ONe core at the beginning of the thermally-pulsing AGB phase. Since, as is learned from model studies, this largest initial CO core mass is about $\sim 1.1 M_\odot$, and since an electron-degenerate configuration can burn carbon explosively only if its mass exceeds the Chandrasekhar mass ($\sim$1.4 $M_\odot$), it has been learned that all single stars initially less massive than about 8 $M_\odot$ become CO white dwarfs; they do not explode as supernovae. Single stars of initial mass in the range $\sim$8–10 $M_\odot$ burn carbon non-explosively during the AGB phase and evolve into white dwarfs with substantial ONe cores topped by CO shells. More massive single stars ultimately develop cores composed of iron peak elements which collapse into neutron stars or black holes; the envelopes of such stars are ejected in type II supernova explosions.

As AGB stars evolve to larger luminosities, their envelopes begin to pulsate acoustically, with pulsation periods which can be typically between several hundreds of days and about two thousand days and with amplitudes up to several magnitudes. It is suspected that shock waves formed during pulsations inflate the atmosphere into an extended envelope in which grains can form, and that radiation pressure on the grains converts the envelope into an escaping superwind. It is found empirically that, for periods larger than about 400 days, mass loss from the surface occurs typically at a rate of $10^{-4}\ M_\odot\ \mathrm{yr}^{-1}$. The huge, rapidly expanding OH/IR sources, which emit maser light of 18 cm wavelength from the OH molecule and are strong sources of infrared emission, contain hidden in their interiors an object which produces energy at a rate typical of an AGB star and are themselves composed of matter expelled by this interior object. It is evident that the central object was once an AGB star or is still an AGB star in the process of losing mass and that OH/IR sources are destined to become planetary nebulae, forced into fluorescence by radiation from the central object which has maintained a high luminosity but has contracted into a compact object of high surface temperature.

The evolution of the stellar remnant can be understood with the help of stellar models and the properties of large samples of planetary nebulae and white dwarfs. After the ejection of most of its hydrogen-rich envelope is completed, the remnant star, with a fuel-exhausted core of mass between about $0.6\ M_\odot$ and something short of $1.4\ M_\odot$, rapidly evolves into a compact object of size comparable to that of the Earth. For approximately 10 yr ($M_{\rm core} \sim 1.1\ M_\odot$) to 10^4 yr ($M_{\rm core} \sim 0.6\ M_\odot$), it burns hydrogen or helium, or both, in a thin layer near its surface, remaining bright enough and hot enough (surface temperature higher than 30 000 K) to cause the fluorescence of ejected material, which is seen as a planetary nebula 3000–30 000 times more luminous than the Sun. Following the extinction of all nuclear fuels, the remnant cools along the white-dwarf sequence in the HR diagram at nearly constant radius, but at steadily decreasing luminosity and surface temperature. It requires approximately 10^8 yr for the luminosity to drop to about $10^{-2}\ L_\odot$ (one one-hundredth of the Sun's luminosity), and another 10^{10} years to drop two more decades in luminosity to $10^{-4}\ L_\odot$.

Another lesson learned from Magellanic Cloud AGB stars, in conjunction with model stars, is that the ^{12}C made in the interior during a thermal pulse is dredged to the surface following the pulse. Also made in the interior during a pulse and brought to the surface after a pulse are about 200 varieties of neutron-rich isotopes called s-process isotopes. The neutrons necessary for neutron capture on the seed nucleus ^{56}Fe and on its progeny are a consequence of reactions between ^{4}He and ^{13}C (in AGB stars of small core mass) or of reactions between ^{4}He and ^{22}Ne (in AGB stars of both large and small core mass). The most striking demonstration of carbon production and dredge-up is the fact that, in Magellanic Cloud clusters of intermediate age (3×10^8–3×10^9 yr), there is a fairly sharp demarcation in luminosity between AGB stars which are also carbon stars and those which are not. The carbon stars are brighter and, at their surfaces, the abundance of carbon exceeds the abundance of oxygen, in contrast with the normal situation during less advanced evolutionary stages when oxygen is more abundant than carbon. This agrees qualitatively with the facts that AGB model stars: brighten as their CO core grows during the quiescent hydrogen-burning phase; do not dredge up freshly synthesized material until the mass of the CO core exceeds a critical value; and, depending on initial composition and total mass, may not develop a surface ratio of carbon to oxygen greater than one until several dozens of carbon dredge-up episodes have occurred. Theory and observation combine to demonstrate that AGB stars are the source of most of the ^{12}C and s-process elements in the Universe.

Dramatic demonstrations of the occurrence of neutron-capture nucleosynthesis and dredge-up are the presence of unstable technetium at the surfaces of several galactic AGB stars and the presence of unusually strong lines of ZrO in the spectra of long period variables in the Magellanic Clouds which are also AGB stars. The long period variable AGB stars have large core masses and their paucity, relative to their precursors which have passed through the Cepheid instability strip, reinforces the inference from the Magellanic Cloud globular clusters that AGB stars deriving from intermediate-mass main sequence stars eject a nebular shell within roughly 10^5 yr after reaching the thermally-pulsing phase. Their short lifetimes as AGB stars is further strengthened by the fact that these stars are not carbon stars, even though the presence of strong ZrO lines indicates that dredge-up is occurring; models suggest that typically 10^6 yr of dredge-up is necessary to develop the carbon-star syndrome.

Since stars take time to form, the assumption that all stars in an aggregate are of the same age is obviously inappropriate for a sufficiently young aggregate. This lesson is most clearly demonstrated by the properties of stars in young open clusters and in superclusters and groups in the disk of our Galaxy. Instead of finding just one cluster locus of low dispersion, one sometimes finds evidence for two or more bursts of star formation occurring at distinctly different times. In the very youngest of open clusters it is clear that the lowest mass stars in clusters have not yet reached the main sequence, but are still gravitationally contracting toward the main sequence; the dispersion about the mean cluster locus suggests, on comparison with models of contracting stars, that a burst of star formation lasts for of the order of 10^7 years.

Galactic globular clusters (halo clusters) and the less tightly bound open clusters (disk clusters) occupy distinctly different locations in our Galaxy and the stars in the two cluster types have quite different metallicities (roughly solar for the disk clusters and 10–100 times less than solar for the halo clusters). The striking dichotomy in abundances has led to the concept of two distinct populations having quite different histories. The conditions at the time of the formation of the metal-poor (population II) globular clusters must have been quite different from the conditions at the time of the formation of the metal-rich (population I) open clusters. Comparison with theoretical stellar models shows that the disk clusters, which orbit the galactic center in nearly circular paths, range in age from 10^7 yr to 10^{10} yr. The age of the oldest white dwarfs in our vicinity, as estimated by comparing with theoretical white dwarf cooling models, is about 9×10^9 yr, and this is close to the age of the oldest known disk cluster, NGC 188. The globular clusters, which traverse highly elliptical orbits within a huge halo of diameter about 30 kpc, have a mean age near 14×10^9 yr. These differences in ages, based on comparisons between theoretical stellar models and real stars, offer intriguing clues as to how galaxies evolve.

In disk clusters, the brightness of the brightest red giants relative to the brightness of the brightest main sequence stars undergoes a dramatic change with increasing cluster age. In the youngest clusters, such as the Pleiades and Alpha Persei, the two types of star are of comparable luminosity. In the oldest clusters, such as NGC 188 and M67, the brightest red giants are much brighter than the brightest main sequence stars, as is also the case in the halo globular clusters. Comparison with models shows that this change in morphology has to do with whether or not, after a star leaves the main sequence, electrons in its hydrogen-exhausted core become degenerate before helium is ignited in the core. If they do not, then ascent along the giant branch is aborted at a luminosity not much above that which the star had on leaving the main sequence. If they do, then the giant branch extends to high luminosity as the electron-degenerate core grows in mass to about 0.45 $M_\odot$, when conditions become ripe for helium to ignite explosively in the core. Models of a solar-like initial abundance show that the transition between the two types of behavior occurs at a critical stellar mass which is somewhere between 2 $M_\odot$ and 2.5 $M_\odot$, the imprecision being a consequence of the uncertainty in the extent to which matter is mixed outward beyond the boundary of the formal convective core found in these models while on the main sequence.

Since the luminosity of a main sequence star is not overly sensitive to this uncertainty in the degree of convective overshoot, there is the possibility of ascertaining, by comparison with the observations, the value of the critical mass, thereby learning something about the physics of convective overshoot. The mass of the most luminous main sequence star in the Hyades cluster is about 2.3 $M_\odot$ and the brightest red stars are the population I analogues of HB stars in galactic globular clusters, namely core helium-burning, shell hydrogen-burning stars. They are called clump stars because, instead of being distributed in the HR diagram over a wide range in surface temperature, they form a nearly constant luminosity peak in the distribution along the first red giant branch. The larger opacity in the metal-rich population I disk stars relative to the metal-poor population II globular cluster stars is responsible for the different HR-diagram morphology of the clump and HB stars. The Hyades cluster does not exhibit an extended giant branch and one may infer only that the critical mass for metal-rich stars is less than about 2.5 $M_\odot$, the mass of the progenitors of the clump stars, and that, therefore, the degree of convective overshoot may be quite modest.

Within the Galaxy there are of the order of 10^4 Cepheids, luminous stars (typically 10^3–10^4 $L_\odot$) of low surface temperature (typically 5000–6000 K) which pulsate at periods of a few days to several tens of days. They are rare stars, but are conspicuous because of their large-amplitude variability. Only about a dozen disk clusters contain a Cepheid. The distances of the parent clusters can be estimated by the main sequence fitting technique and so both the luminosity and the radius of each Cepheid can be estimated. On comparing with evolutionary models of the appropriate mass and composition which pass near the location of the Cepheids in the temperature–luminosity plane, it is evident that Cepheids are in the core helium-burning stage of evolution. A quantitative fit between the estimated luminosity of each Cepheid and the luminosity–mass relationship given by evolutionary calculations allows one to determine an evolutionary mass for each Cepheid. A pulsation mass can be assigned as well by making use of a relationship between mass, radius, and period obtained by theoretical pulsation calculations (which do not require a specification of the evolutionary state of the interior) and fitting with the luminosity and radius estimated from the observations. On first inspection, the evolutionary masses do not agree in detail with the pulsation masses. That is, the mass–luminosity relationship obtained by using the theory of evolution differs from the mass–luminosity relationship obtained by combining pulsation theory with the luminosities estimated by the main sequence fitting technique. Similarly, the period–luminosity relationship obtained by combining the results of evolution and pulsation theory calculations differs from that obtained by adopting the luminosities estimated by main sequence fitting. A resolution of these discrepancies can be achieved by adjusting the luminosities of the relevant clusters by approximately 20% (accomplished by increasing the estimated distances of the clusters by 10%) or by assuming that the radiative opacities in the envelopes of the theoretical models have been