

Index

- abelian Hilbert ternary algebra, 244
- abelian Jordan triple system, 21
- abelian Lie algebra, 40
- abelian tripotent, 211
- absolute zero divisor, 236
- act transitively, 87
- Ad , 55, 78
- ad , 41, 55, 79
- adjoint representation of Lie algebra, 41, 79
- adjoint representation of Lie group, 55, 78
- admissible neighbourhood, 90
- Albert type Jordan algebra, 13
- almost complex structure, 144
- alternative algebra, 5
- analytic (Banach) manifold, 68
- analytic function, 64
- analytic Killing field, 93
- analytic Riemannian manifold, 84
- analytic structure, 67
- analytic vector field, 73
- anisotropic, 26
- antiautomorphism, 1
- associative H^* -triple system, 250
- associative inner product, 104, 143, 228
- associative part of JH-triple, 239
- associator, 5
- atlas, 67
- atomic part, 215
- aut M , 76
- Aut M , 69
- automorphism group of Lie algebra, 40
- automorphism group of manifold, 69
- automorphism of manifold, 69
- Banach Lie algebra, 79
- Banach Lie group, 76
- Banach Lie subgroup, 82
- Banach manifold, 66
- Bergmann metric, 188
- Bergmann operator, 22, 188, 191
- bianalytic function, 64
- biholomorphic function, 65
- biholomorphic isometry, 89
- box operator, 2, 21
- brackets of vector fields, 74
- C^* -algebra, 103
- Campbell–Baker–Hausdorff series, 99
- canonical Hermitian triple product, 14
- canonical Jordan triple product, 14
- canonical part of TKK Lie algebra, 43
- canonical TKK Lie algebra, 43
- canonical trace, 114
- Carathéodory metric, 150, 152
- Carathéodory norm, 153
- Cartan domains, 151
- Cartan factor, 168
- Cartan involution, 43
- Cartan’s uniqueness theorem, 88
- Casimir polynomial, 41
- Cauchy–Schwarz inequality, 218
- Cayley algebra, 4
- centre, 103
- centre of a Jordan algebra, 4
- centre of a Lie algebra, 41
- Clifford type Jordan algebra, 13
- commutative Jordan triple system, 21
- commutator of vector fields, 74
- compact holomorphic map, 197
- compact Jordan triple, 142
- complemented subspace, 70, 216
- complete dual symmetric part, 124

complete symmetric part, 124
 complete tripotent, 33, 185
 complete vector field, 75
 completely additive, 209
 completely positive definite, 83
 complex extreme point, 184
 complex manifold, 67
 complex structure, 144
 complex triple spin factor, 151, 231
 complexification, 17, 65
 conjugation, 176
 connection, 85
 continuous inner derivation, 122
 continuous JH-triple, 136
 continuous von Neumann algebra,
 216
 continuously differentiable, 62
 contractive projection, 202
 coset space, 82
 covariant tensor, 82

 $D(p, r)$, 90
 Denjoy–Wolff theorem, 200
 derivation, 40
 derivative, 62
 derived Hilbert ternary algebra, 246
 diffeomorphism, 69
 differentiable function, 62
 differentiable structure, 68
 differential, 70
 dimension of manifold, 67
 dual involution, 44
 dual symmetric part, 44

 ellipsoid, 199
 Euclidean Jordan algebra, 104
 exceptional Cartan factor, 168
 exceptional domains, 151
 exceptional Jordan algebra, 2
 exp, 76
 exponential map, 56, 78
 ext \overline{D} , 184
 extreme point, 184

 factor, 211
 factor representation, 215
 finite rank projection, 112
 flat Jordan triple system, 21
 formally real Jordan algebra, 11

 $G(M)$, isometry group of M , 87
 $G_a(M)$, 89

$\mathfrak{g}(V)$, 121
 $\mathfrak{g}^*(V)$, 121
 geodesic, 86
 G -invariant Riemannian metric, 129
 graded Lie algebra, 42

 \mathbb{H} , 5
 $H_3(\mathbb{O})$, 6
 $H_3(\mathcal{O})$, 19
 H^* -triple system, 248
 hermitian element in Banach algebra, 156
 Hermitian Jordan Banach triple, 172
 Hermitian Jordan triple, 13
 Hermitian metric, 145
 hermitian operator, 156
 Hermitian structure, 144
 Hermitian symmetric space, 145
 Hermitian type Cartan factor, 168
 Hermitian type Jordan algebra, 12
 hermitification, 17
 Hilbert manifold, 67
 Hilbert space over \mathbb{H} , 51
 Hilbert ternary algebra, 244
 Hilbert–Schmidt class, 229
 holomorphic extreme point, 184
 holomorphic function, 63
 holomorphic invariant domain, 198
 homogeneous domain, 153
 homogeneous polynomial, 63
 homogeneous space, 82
 homomorphism of Lie algebra, 40
 homotope of Jordan triple system, 20
 hyperbolic metric, 149

 ideal of Jordan algebra, 12
 ideal of Lie algebra, 40
 idempotent, 7
 identity component, 127
 immersion, 70
 infinite rank projection, 112
 inner derivation, 41, 135
 inner derivation pair, 49
 inner ideal, 39, 216
 integral curve, 75
 inverse function theorem, 66
 invertible element, 107
 involution of Jordan pair, 17
 involution of Lie algebra, 42
 involutive isometry, 101
 involutive Lie algebra, 42
 isolated fixed point, 101
 isometry group of manifold, 87

- isometry of manifold, 87
- isometry of normed vector space, 87
- isotropic, 246
- isotropy representation, 48, 125
- isotropy subgroup, 125
- J^* -algebra, 169
- J^* -triple, 172
- Jacobi identity, 39, 74
- JB*-algebra, 173
- JB*-triple, 164
- JB-algebra, 102
- JB-triple, 176
- JBW*-algebra, 173
- JBW*-triple, 167
- JBW-algebra, 103
- JBW-factor, 103
- JBW-triple, 218
- JC*-triple, 169
- JH*-triple, 143
- JH-algebra, 104
- JH-triple, 136
- Jordan algebra, 1
- Jordan C*-algebra, 173
- Jordan equivalent projections, 112
- Jordan Hilbert triple, 135
- Jordan homomorphism, 12
- Jordan isomorphism, 12
- Jordan pair, 17
- Jordan symmetric space, 140
- Jordan triple, 13
- Jordan triple system, 13
- JW*-triple, 169
- \mathfrak{k} , 1-eigenspace, 42
- Killing field, 89
- Killing form, 41
- $L(H)$, 81
- $L(V)$, 51
- $L(H, K)$, 18
- $L_2(V)$, 229
- $L_2(V, W)$, 229
- $\mathfrak{L}(V)$, 44
- left invariant vector field, 77
- left multiplication, 2
- Levi–Civita connection, 86
- Lie algebra, 39
- Lie algebra of Lie group, 55, 77
- Lie brackets, 39
- linear isometry, 87
- local chart, 66
- local coordinates, 67
- local flow, 75
- locally uniform action, 93
- Lorentz cone, 105
- $M_m(\mathbb{C})$, 18
- $M_{mn}(\mathbb{C})$, 18
- $M_{12}(\mathcal{O})$, 18
- main involution, 46
- main triple identity, 14
- manifold modelled on Banach space, 67
- maximal idempotent, 10
- maximal tripotent, 33
- mean value theorem, 63
- minimal projection, 112
- minimal tripotent, 211
- Möbius transformation, 148, 165, 188, 192
- model space for manifold, 67
- negative tripotent, 240
- nilpotent, 7
- non-degenerate Jordan triple system, 25
- non-degenerate TKK Lie algebra, 46
- non-degenerate trace form, 25
- normal functional, 211
- normal neighbourhood, 87
- normed Jordan triple system, 121
- normed Lie algebra, 121
- numerical range, 156
- \mathcal{O} , 4
- \mathbb{O} , 5
- octonions, 4
- one-parameter group, 76
- one-parameter subgroup, 78
- open mapping theorem, 83
- operator commute, 4
- orthogonal idempotents, 7
- orthogonal Lie algebra, 125
- orthogonal tripotents, 28
- \mathfrak{p} , (-1) -eigenspace, 42
- paracompact, 84
- Peirce k -projection, 32
- Peirce k -space, 9, 32
- Peirce decomposition, 9, 32, 38
- Peirce multiplication rules, 36
- Peirce symmetry, 185
- Poincaré distance, 148
- Poincaré metric, 148
- polarisation identity, 16
- positive definite, 25

- positive J*-triple, 172
- positive Jordan triple system, 28
- positive quaternion, 6
- positive tripotent, 240
- power series, 64
- primitive idempotent, 10
- primitive signed tripotent, 243
- principal inner ideal, 39
- principle of analytic continuation, 66
- projection, 107
- quadratic algebra, 6
- quadratic operator, 2, 22
- quasi-normed TKK Lie algebra, 121
- quaternion algebra, 5
- quaternion Hilbert space, 51
- quaternion unitary, 52
- range projection, 116
- rank of projection, 112
- rank-one operator, 230
- real C*-algebra, 103
- real form, 176
- real JB*-triple, 176
- real JBW*-triple, 218
- real manifold, 67
- real restriction of Hilbert space, 67
- real ternary algebra, 249
- real triple spin factor, 231
- rectangular type Cartan factor, 168
- reduced Lie algebra, 48
- Riemann mapping theorem, 151
- Riemannian connection, 86
- Riemannian distance, 85
- Riemannian manifold, 84
- Riemannian metric, 83
- Riemannian symmetric pair, 129
- Riemannian symmetric space, 101
- right invariant vector field, 77
- s-identity, 7
- Schwarz–Pick lemma, 148, 195
- self-adjoint part, 174
- semidirect product, 88
- semisimple Jordan triple system, 26
- semisimple Lie algebra, 41
- signed tripotent, 240
- simple Hilbert ternary algebra, 250
- simple Jordan algebra, 12
- smooth (Banach) manifold, 68
- smooth curve, 69
- smooth function, 62
- solvable Lie algebra, 41
- special Jordan algebra, 2
- spectral decomposition, 31
- spin factor, 13, 106
- standard Hermitian form, 51
- structural projection, 216
- submanifold, 72
- submersion, 70
- subtriple, 13
- support tripotent of a functional, 214
- support tripotent of an element, 208
- symmetric cone, 105
- symmetric domain, 150
- symmetric part of Banach space, 171
- symmetric part of TKK Lie algebra, 44
- symmetric R-space, 142
- symmetric space, 101
- symmetrised ternary product, 245
- symmetrised triple spectrum, 180
- symmetry s_p at p , 101
- symmetry in \mathcal{A} , 112
- symmetry s_a , 151
- symplectic group, 54
- symplectic inner product, 51
- symplectic type Cartan factor, 168
- tangent bundle, 73
- tangent map, 73
- tangent space, 69
- tangent vector, 69
- ternary annihilator, 247
- ternary ideal, 244
- ternary product, 244
- ternary structure, 227
- ternary subalgebra, 244
- Tits–Kantor–Koecher construction, 39
- Tits–Kantor–Koecher Lie algebra, 43
- TKK Lie algebra, 43
- topological boundary ∂A , 90
- torsion-free, 86
- trace form, 25
- triple annihilator, 235
- triple homomorphism, 13
- triple ideal, 38
- triple identity, 14
- triple isomorphism, 13, 175
- triple monomorphism, 13, 217
- triple orthogonal, 234
- triple product, 13
- triple spectrum, 31, 180

Index

261

- triple spin factor, 168, 230
tripotent, 28
type 1 Cartan factor, 168
type 2 Cartan factor, 168
type 3 Cartan factor, 168
type 4 Cartan factor, 168
type 5 Cartan factor, 168
type 6 Cartan factor, 168
type I JBW*-triple, 211
type I JBW-factor, 113
 $U(H)$, unitary group, 81
 $\mathfrak{u}(H)$, Lie algebra of $U(H)$, 81
ultraproduct of Banach spaces, 205
unit extension of an algebra, 1
unital algebra, 1
unitary tripotent, 33
 V' , dual of a real Banach space V , 176
 V_{00} , 49
vector field, 73
vector field along a curve, 85
von Neumann algebra, 106, 113
w*-topology, 206
weak* topology, 210
weakly semisimple, 26
weakly sequentially complete, 211
 x^3 , the triple product $\{x, x, x\}$, 16