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To my family

Contents

Preface

Despite the rapid advances in Jordan theory and its diverse applications in the last two decades, there are few convenient references in book form for beginners and researchers in the field. This book is a modest attempt to fill part of this gap.

The aim of the book is to introduce to a wide readership, including research students, the close connections between Jordan algebras, geometry, and analysis. In particular, we give a self-contained and systematic exposition of a Jordan algebraic approach to symmetric manifolds which may be infinite-dimensional, and some fundamental results of Jordan theory in complex and functional analysis. In short, this book is about *Jordan geometric analysis*.

Although the concept of a Jordan algebra was introduced originally for quantum formalism, by P. Jordan, J. von Neumann and E. Wigner [64], unexpected and fruitful connections with Lie algebras, geometry and analysis were soon discovered. In the last three decades, many more applications of Jordan algebraic structures have been found. We expose some of these applications in this book. Needless to say, the choice of topics is influenced by the author's predilections, and regrettable omissions are inevitable if the length of the book is to be kept manageable. Nevertheless, an effort has been made to cover sufficient basic results and Jordan techniques to provide a handy reference.

We begin by discussing the basic structures of Jordan algebras and Jordan triple systems in Chapter 1, and the connections of these Jordan structures to Lie theory. An important link is the Tits–Kantor–Koecher construction, which establishes the correspondence between Jordan triple systems and a class of graded Lie algebras. We discuss some details of classical matrix Lie groups and their Lie algebras and use them as examples to illustrate these connections, as well as preparation for the introduction of Banach Lie groups in the following chapter.

Since É. Cartan's seminal work, Lie theory has been an important tool in the study of Riemannian symmetric spaces and their classification. It was found relatively recently that Jordan algebras and Jordan triple systems can be used to give an algebraic description of a large class of symmetric spaces which is also accessible in infinite dimension. This is the subject of Chapter 2. We give a concise introduction to Banach manifolds and Banach Lie groups. We show the connections between Jordan algebras and symmetric cones, and the correspondence of Jordan triple systems and Riemannian symmetric spaces in the infinite-dimensional setting. We complete the discussion by showing that the bounded symmetric domains in complex Banach spaces are exactly the open unit balls of JB*-triples which are complex Banach spaces equipped with a Jordan triple structure.

A large part of Chapter 3 is devoted to the study of JB*-triples. They play an important role in geometry and analysis, as informed by the previous result. The open unit balls of JB*-triples can be regarded as an infinite-dimensional generalisation of the open unit disc in the complex plane and provide a natural setting for complex function theory. As examples, we discuss distortion theorems and iterations of holomorphic maps on these open balls, where Jordan techniques come into play. In a functional-analytic vista, JB*-triples form an important class of Banach spaces, which includes C*-algebras, spaces of operators between Hilbert spaces and some exceptional Jordan algebras. We present a sufficient number of basic properties of JB*-triples as research tools, but a complete treatment would lengthen the book to excess. From the viewpoint of JB*-triples, many results in C*-algebras, for example, those on contractive projections and isometries, can be explained simply in a geometric perspective. Finally, we discuss Jordan structures in Hilbert spaces, which are important in the geometry of infinite-dimensional Riemannian symmetric spaces; for instance, the curvature tensor is related to the Jordan triple product. The last chapter contains some new results.

It is a great pleasure to thank many colleagues and friends for valuable conversations concerning the subject matter of this book. I have benefited especially from inspiring discussions with Wilhelm Kaup and the late Issac Kantor on many occasions. I thank Pauline Mellon for reading part of the manuscript and for her useful comments. I much appreciate the sabbatical leave from Queen Mary College in 2010, which enabled me to complete the manuscript. I would also like to thank my wife, Yen, and my daughter, Clio, for their constant support and encouragement.