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Conventionalism and the linguistic doctrine of logical truth

1.1 Introduction

Quine's work on analyticity, translation, and reference has sweeping philosophical implications. In his first important philosophical publication on these topics, however, Quine focused on a particular problem, the problem of the basis of logical and mathematical truth, and on a particular putative solution to that problem, the solution which says that such truths are grounded in linguistic conventions.[1] In this chapter we shall consider Quine's criticisms of this solution, as well as his criticisms of the more general view of which this conventionalistic doctrine is a special case, the linguistic doctrine of logical and mathematical truth. Before examining Quine's writings in detail, however, we should

[1] The term 'logic' has been used in the philosophical literature in different senses. Some writers, for example, treat set theory as a part of "logic," whereas others do not. We shall introduce these distinctions into the text as the need arises.

It is also ambiguous to speak of logic or mathematics as being "grounded" or "based" on conventions. Is the grounding or basing to be taken as epistemological or metaphysical? Is the conventionalist claiming that our knowledge of logic and mathematics is justified on the basis of our knowledge of linguistic conventions? Or is he claiming that logical and mathematical truths are, in some sense, truths about linguistic conventions? In answering these questions it is important to realize that the existence of the ambiguity does not imply that the conventionalist must hold no more than one of the doctrines in question. Not only are the doctrines not mutually exclusive, they are mutually complementary. The epistemic connection between conventions and our knowledge of logic and mathematics can be explained on the basis of the metaphysical connection; conversely, the existence of the metaphysical connection would suggest the existence of the epistemic connection. Thus the conventionalist's best answer to the foregoing questions would appear to be "Both." In fact, it is one of the attractions of conventionalism that it offers answers to both the metaphysical and the epistemological questions about mathematics and logic. In what follows, therefore, I treat the conventionalist as holding both the metaphysical and the epistemological doctrines alluded to above, and I mean the expressions 'based on' and 'grounded on' to have both their metaphysical and epistemological senses.

consider how they are related to earlier philosophical discussions of the basis of logical and mathematical truth.[2]

On the face of it there seem to be some fundamental differences between, on the one hand, statements of logic and mathematics, such as 'All bachelors are bachelors' or '5 + 7 = 12', and, on the other hand, many statements not belonging to either of these disciplines, such as 'Pierre is the capital of South Dakota' or 'All US presidents elected before 1900 were male'.[3] For one thing, there seems to be a kind of necessity, or inevitability, in truths of the first kind, which is lacking in truths of the second kind. That all bachelors are bachelors, that 5 + 7 = 12: These are statements which, it seems, would hold under any conceivable circumstances, or in any possible world; not so for our statements about geography and history. Statements of the first kind also seem to differ from statements of the second kind in that we can know them to be true without checking them against observation. To know that all bachelors are bachelors, it is not necessary to check all, or even a representative sample of, bachelors, and find that they are all bachelors. Perhaps one *could* come to know the truth of this statement by observing bachelors, but the point is that one *need not* do so. There seems to be, in these cases, an alternate route to knowledge. In taking this alternate route, we have recourse to experience only in learning what the words in our sentences mean, not in learning that the sentences are true. The alternate route is not available, however, in the case of sentences of the second kind. To learn these truths, we need experiences beyond those involved in learning their component expressions. In this sense our knowledge of statements of the second kind may be said to be *grounded in experience* in a way in which our knowledge of statements of the first kind is not grounded in experience.

But what is involved in this alternate path to knowledge? If our knowledge of statements of logic and mathematics is not grounded in

[2] There is a good discussion of pre-Quinean accounts of a priori knowledge in Orenstein, *W. V. Quine*, pp. 75–79. The account in the text was written before I had seen Orenstein's discussion.

[3] Note that this allows for the possibility of there being statements outside of logic and mathematics that are necessarily true or knowable a priori. A number of philosophers in recent years have argued that the class of statements that are knowable a priori is not coextensive with the class of statements that are necessarily true. This issue is not discussed here, because the exposition is intended to evoke the philosophical climate of the 1930s, when "Truth by Convention" was written. At that time, it was generally assumed that the a priori coincides with the necessary.

experience, what *is* it grounded in? A number of different answers to these questions have been suggested. Kant proposed his famous theory of the synthetic a priori, according to which some of our knowledge is grounded in certain characteristics of the human mind. Various objections have been raised against this theory, of which it will suffice to mention two. One objection is that the theory cannot account for the necessity of logical and mathematical statements. The mental characteristics to which it appeals would seem to reflect merely contingent facts about the structure of our minds: it seems quite possible that our minds might have had a character different from the one they actually have. Consequently, if the truths of logic and mathematics hold because our minds have certain characteristics, as the theory maintains, it would seem that the truths of logic and mathematics would not hold necessarily, because they might fail to hold if our minds were different. If we say that $5 + 7$ equals 12 because of the way in which our minds operate, this seems to raise the possibility that, if our minds had operated differently, $5 + 7$ might have equaled 13.[4] A second objection to Kant's theory is that it applies only to some of the statements with which we are concerned. While the truths of arithmetic and geometry are, for Kant, synthetic a priori, the truths of logic are not. Thus, even if we were to grant that Kant's theory explains our knowledge of mathematics, we would still be without an explanation of our knowledge of logic.

A different account was offered by J. S. Mill, who held that we learn the truths of logic and mathematics by generalizing from experience. Put five praying mantises together with seven praying mantises and you have (for a while at least) twelve praying mantises. It is by generalizing from such facts as these that we discover that $5 + 7 = 12$. Some critics have felt that Mill's account fails to do justice to the necessity of mathematical statements. An empirical generalization, however strong might be the evidence in its favor, could, conceivably, be false. Not so the statement that $5 + 7 = 12$. If some of our praying mantises were to eat some of the other praying mantises (as praying mantises are wont to do), we would not regard this event as refuting the claim that $5 + 7 = 12$.[5] A further problem with Mill's view is that it does not seem to be able to account for the truth of statements involving infinities. There are infinitely many natural numbers and infinitely many real numbers. It thus seems hopeless

4 Cf. Russell, *The Problems of Philosophy*, p. 87.
5 For information about the habits of preying mantises I am indebted to Bill Willis.

to try to establish, by means of some empirical process such as counting, the mathematical truth that the reals outnumber the naturals.

For philosophers who are of an empiricist turn of mind, the rejection of Mill's view leads to an embarrassing question: If all knowledge is grounded in observation, and yet our knowledge of logical and mathematical truth is not based upon generalization from experience, then what *is* the foundation for our knowledge of those truths? The situation becomes even more critical if one's empiricism includes the positivistic doctrine that statements which cannot conceivably be refuted by experience are without meaning, for by this standard of significance logic and mathematics would seem to make no more sense than the ruminations of the most benighted metaphysician.

In an attempt to escape these difficulties many modern empiricists have embraced some version of the linguistic theory of logical and mathematical truth, according to which the statements of logic and mathematics are rendered true by the very language in which they are couched. The truth of 'All bachelors are bachelors' is guaranteed by the meanings, or uses, of its component expressions, and similarly for the other truths of logic and mathematics. If the linguistic theory is correct, a logical or mathematical statement could not be made false except by a change in the meanings of its component expressions. This explains why these statements are true under any conceivable circumstances (except, of course, circumstances in which their words have different meanings). It also follows from the linguistic theory that knowledge of the meanings of the expressions which make up these statements, in combination with knowledge of how the meanings of compound expressions depend upon the meanings of their parts, would be a sufficient basis for coming to know that the statements are true. This explains the possibility of our knowing these truths without checking them against experience.

Another advantage of the linguistic theory is that it partakes of the spirit of empiricism. Knowledge of the meanings of words, however much it may presuppose in the way of innate mechanisms, is empirical knowledge: we acquire it through experience. In explaining our knowledge of logic and mathematics by means of the linguistic theory, therefore, we represent that knowledge as being founded on empirical knowledge. This is not to say that we represent that knowledge as being itself empirical, in the sense of being based on some sort of observational check made subsequent to the learning of its words; but it *is* to say that we attempt to explain it as arising from ordinary empirical processes, rather than from some mysterious source such as intellectual intuition.

In sum, the linguistic theory has a number of advantages. It avoids treating logical and mathematical statements as meaningless or without cognitive content. It explains the two most prominent features of the statements with which it is concerned, their necessity and their a priori knowability. And it does all this in a manner congenial to the spirit of empiricism.

Enter Quine. Despite the apparent advantages of the linguistic theory, Quine rejects it.[6] If the theory is to serve as a principle of empiricist philosophy, then, he maintains, it should itself meet the empiricist standard of significance, i.e., there should be some way of testing it against experience.[7] He then proceeds to argue that the theory has never been given a formulation under which it is both testable and true. Moreover, Quine holds that the distinction which the theory invokes, the distinction between "analytic" truths, true purely because of language, and "synthetic" truths, true because of how the world is, also falls short of empiricistic standards. According to Quine, the distinction has never been drawn in such a way that (a) the things people have wanted to say about analytic and synthetic statements turn out to be true, and (b) it is possible to determine empirically whether a given truth is analytic or synthetic.[8]

Having rejected the linguistic doctrine, Quine offers an alternative account of our knowledge of logic and mathematics. His view resembles Mill's to the extent that it treats the statements of these disciplines as differing only in degree, not in kind, from other statements; but unlike Mill Quine does not regard the truths of logic and mathematics as empirical generalizations, and he regards them as testable only insofar as they are included in testable theories. For Quine, the statements of logic and mathematics are similar, in point of cognitive status, to the statements of theoretical physics.[9]

All of these matters will be discussed in this chapter. We shall begin with the first publication in which Quine addressed the issues that are

[6] See, for example, "Truth by Convention" and "Carnap and Logical Truth."

[7] See "Carnap and Logical Truth," section III. We shall discuss this passage in more detail later, but it is clear even from a superficial reading that Quine is here concerned with the empirical content of the linguistic doctrine and that he is skeptical of the prospects of finding an interpretation of the doctrine under which it has empirical meaning. The essay as a whole develops Quine's arguments for the thesis stated in the next sentence of the text.

[8] See "Carnap and Logical Truth," section IX, and "Two Dogmas of Empiricism."

[9] For Quine's positive account of logical and mathematical truth, see *The Ways of Paradox*, pp. 120–22, and *Philosophy of Logic*, second edition, pp. 97–102.

the subject of this book, his "Truth by Convention," originally published in 1936.[10] This essay is one of Quine's most penetrating, and also one of his most difficult, works. We shall examine it in detail.

1.2 Conventionalism in "Truth by Convention"

The content of conventionalism

The linguistic doctrine of logical and mathematical truth, as we have so far formulated it, is vague. It says merely that the statements of logic and mathematics are true because of the way in which people use language, without saying how linguistic usage produces truth or which aspects of such usage are responsible for its production. Conventionalism is a more precise and specific version of the linguistic doctrine: It says that logical and mathematical statements owe their truth to the adoption of certain linguistic *conventions*. Quine's penetrating critique of conventionalism will be the focus of our discussion in this section. Before considering his treatment of this topic, it will be useful to try to arrive at a better understanding of what conventionalism says and of why many philosophers have found it to be a plausible doctrine.

The conventionalist doctrine can be understood in such a way that the conventions which give rise to truth can be adopted by a speaker without any conscious decision on his part and, indeed, even without his having formulated them. Just as it might be said, of speakers untutored in the rules of grammar, that they nevertheless *obey* those rules when they talk, so it might be said that speakers can *follow* conventions without realizing that they are doing so. Tacit conventions will not, however, be considered in this section, for when Quine discusses conventionalism, he generally

[10] Although Quine's views on the analytic/synthetic distinction were less radical at the time he wrote "Truth by Convention" than they were fifteen years later, when he wrote "Two Dogmas," Quine himself has traced his doubts about the analytic/synthetic distinction back to "Truth by Convention." He wrote: "My misgivings over the notion [of a sweeping epistemological dichotomy between analytic truths as by-products of language and synthetic truths as reports on the world] came out in a limited way in 'Truth by Convention' (1936) and figured increasingly in my lectures at Harvard" (*Word and Object*, p. 67, footnote 7). This comment, together with the fact that "Truth by Convention" is such a brilliant but difficult work, justifies our beginning our study of Quine's views with this essay. This is not to say, however, that Quine's comment justifies our interpreting "Truth by Convention" as an attack on Carnap's views about analyticity and a priori knowledge. As Richard Creath has argued, the break with Carnap over these issues did not come until later. (For more on this point see Creath's Introduction to *Dear Carnap–Dear Van*, esp. pp. 28–31.)

assumes that the conventions in question are explicitly formulated and deliberately adopted. We shall, therefore, deal only with conventions having these features. (Quine does discuss the sort of position that arises from taking conventions as tacit, but usually not under the heading, "conventionalism." His views on this matter will be taken up below and in Chapter 2.)

While conventionalism attributes the truth of logical and mathematical statements to conventions, it does not deny that conventions can play a role in determining the truth of other statements. The conventionalist doctrine says that the distinguishing mark of logical and mathematical statements is that they are true not just *partly* because of conventions but *purely* because of conventions.

The conventions that are said to produce truth are of two kinds: definitions and postulates. An example of truth produced by definition is the definition of the tangent, which verifies '$\tan \pi = \frac{\sin \pi}{\cos \pi}$'. An example of truth by postulation is afforded by the geometer who chooses to regard as true the statement that through a point not on a given line there is only one line parallel to the given line.

To summarize: Conventionalism, as Quine generally understands it, and as we shall understand it in this chapter, is the doctrine that the truths of logic and mathematics, in contrast to those of other disciplines, are true *purely* in virtue of explicitly formulated, deliberately adopted linguistic conventions. The conventionalist does not maintain that the truths of other disciplines owe nothing to conventions, but only that they are determined, in part, by non-conventional factors. The truths of logic and mathematics, on the other hand, are determined entirely by one or both of two kinds of conventions, definitions and postulates.

The plausibility of conventionalism

The thesis that the truths of mathematics, or at least of number theory, are definitional received powerful support from the reduction of arithmetic to set theory. Kant had argued that '$5 + 7 = 12$' is not analytic, since the concept $5 + 7$ does not contain the concept 12. But the work of Gottlob Frege, Bertrand Russell, and Alfred North Whitehead showed that numbers can be defined as sets, and operations on numbers as operations on sets, in such a way that statements like '$5 + 7 = 12$' are reduced to truths of set theory. If this reduction is taken to show that arithmetical truths *are* set theoretical truths, then, given the additional premises that set theory is part of logic and that logical truths are analytic, it follows that

Kant was wrong: The truths of arithmetic are analytic. In any event, whether or not we take the reduction of arithmetic to set theory to have refuted Kant, it is natural in the light of the reduction to say that arithmetical truths are true by definition. Moreover, since all of number theory proves to be, in turn, definitionally reducible to arithmetic, we are led to conclude that the truths of number theory are also true by definition, and therefore, given that definitions are conventions, by convention.

Further support for conventionalism was found in the development of non-Euclidean geometries. Euclidean geometry had been a paradigm of a priori knowledge since its development in ancient Greece, but the nineteenth century saw the development of various alternative geometrical systems, all of them based on some postulate setting the number of parallels that can be drawn through a point not on a given line as equal to some number other than one. When to the surprise of many it was established that if Euclidean geometry is internally consistent, then all of these alternative systems are also internally consistent, the claim that the Euclidean system embodies a priori knowledge no longer seemed defensible. For if the non-Euclidean geometries were internally consistent, then, it would seem, one could accept one of them, and reject Euclidean geometry, without sinning against reason. And if it is not *unreasonable* to reject Euclidean geometry, how could the theorems of such geometry be known a priori by reason's light?

Given that geometrical knowledge is not a priori, the obvious alternative is to regard it as empirical. Many philosophers, however, found the latter option to be no more inviting than the former. Poincaré, for example, argued that, unlike empirical statements, geometrical statements cannot be refuted by observation. If we were to attempt to test the Euclidean theorem that the sum of the angles of a triangle equals 180 degrees by setting up light sources on three adjacent mountain tops and measuring the angles formed by the light beams, and if we found that the sum of the measures of the angles did not equal 180, we could save the Euclidean principle by attributing the result to the bending of the light beams by unknown physical forces; and in general, any observations which might seem to refute Euclid could be accommodated by changes in our physics.[11]

[11] See Salmon, *Space, Time and Motion*, p. 16.

If geometrical truths are known neither by reason nor by experience, what basis do we have for accepting them? The answer proposed by Poincaré and others was that our acceptance of geometrical principles is based on nothing more than our having adopted *conventions* according to which such principles are true, the decision to adopt such conventions being based ultimately on simplicity, convenience, and other such pragmatic considerations. Geometrical statements do not describe the characteristics of any existing entities, either in the physical world or in some Platonic heaven, but they do serve as a powerful instrument in organizing experience. Whether we follow Poincaré in preferring the Euclidean postulates, or follow Einstein in opting for one of the non-Euclidean systems, our choice will be based entirely on a judgment as to which system will best facilitate our attempts to understand the world.

Combining the results of our discussion of geometry with the results of our discussion of number theory, we get the conclusion that the truths of the former are true by postulation and those of the latter are true by definition. Thus if we are prepared to describe both postulates and definitions as "conventions," number theory and geometry would both be true by convention. It does not follow, of course, that *all* of logic and mathematics are true by convention. Nevertheless, our discussion shows that conventionalism can claim, with some plausibility, to derive support from important developments in the history of mathematics.

Definitional conventionalism

As we have seen, the conventionalist may adhere to either or both of two theses: *definitional conventionalism*, which says that logical and mathematical truths are true by definition; and *postulational conventionalism*, which says that such truths are true by postulation. In considering Quine's views on conventionalism, it will be convenient to give these two conventionalist claims separate treatment. We shall first consider definitional conventionalism, which, in its application to the truths of mathematics, is discussed by Quine in the first part of his classic, but now somewhat neglected, article, "Truth by Convention." In this penetrating but difficult section, Quine presents an analysis of definitional conventionalism that both clarifies what this thesis says, and shows why, even if it is true, it cannot, by itself, vindicate the conventionalist claim that all of logic and mathematics is true by convention. We shall consider this material in detail.

Part I of the essay begins with a discussion of the nature of definitions.[12] "A definition, strictly," says Quine, "is a convention of notational abbreviation" (p. 78).[13] Such a convention, he tells us, may simply abbreviate one expression by another, as when we define 'kilometer' as 'a thousand meters'; or it may be a *contextual definition*, such as '$\tan \pi = \frac{\sin \pi}{\cos \pi}$', in which each of the indefinitely many expressions having a certain form (in this case 'tan π') is equated with an expression having another form. Anything which is to qualify as a definition must, according to Quine, satisfy a certain requirement which is *formal* in the sense that it refers only to the forms or shapes of expressions, and not to their meanings. This is the requirement of eliminability: Given any context in which the expression being defined, the definiendum, occurs, the definition must allow us to eliminate the expression from that context in favor of the expression that it abbreviates, the definiens; thus the definition of 'kilometer' must allow us to eliminate that expression from any context in the language in favor of 'a thousand meters'. As long as the requirement of eliminability is satisfied, any expression may be introduced as definiendum for a given definiens. Hence, "From a formal standpoint the signs thus introduced are wholly arbitrary." (p. 78).

Against this account of definition the reader may be inclined to object that some definitions are neither conventional nor abbreviatory. When a dictionary defines 'difficult' as 'hard', it is not offering the former as an abbreviation of the latter. Moreover, it seems incorrect to describe this and other dictionary definitions as conventions, since their purpose is not to *stipulate* a meaning for the defined term but simply to record one of the meanings that it already has.

[12] The introductory paragraph that precedes section I explains the contrast claimed by the conventionalist between the purely conventional truths of logic and mathematics and the partly non-conventional truths of the other sciences and concludes with the remark that

> It is less the purpose of the present inquiry to question the validity of this contrast than to question its sense.
>
> (*The Ways of Paradox*, p. 77)

This remark raises the question: In what sense of 'sense' is Quine questioning the sense of the contrast? This difficult and important question will have to be faced eventually, but for the time being I shall set it aside.

[13] Page references in the text in this section and the following section are to *The Ways of Paradox*.