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978-1-107-01490-9 - Seismic Imaging and Inversion: Application of Linear Inverse Theory

Robert H. Stolt and Arthur B. Weglein

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SEISMIC IMAGING AND INVERSION

Application of Linear Inverse Theory

Extracting information from seismic data requires knowledge of seismic wave propagation and reflection. The commonly used method involves solving linearly for a reflectivity at every point within the Earth. The resulting reflectivity, however, is not an intrinsic Earth property, and cannot easily be extended to nonlinear processes which might provide a deeper understanding and a more accurate image of the subsurface.

In this book, the authors follow an alternative approach which invokes inverse scattering theory. By developing the theory of seismic imaging from basic principles, they relate the different models of seismic propagation, reflection, and imaging – thus providing links to reflectivity-based imaging on the one hand, and to nonlinear seismic inversion on the other. Full, three-dimensional algorithms are incorporated for scalar, acoustic, and elastic wave equations.

The comprehensive and physically complete linear imaging foundation developed in this volume presents new results at the leading edge of seismic processing for target location and identification. The book serves as a fundamental guide to seismic imaging principles and algorithms, and their foundation in inverse scattering theory, for today's seismic processing practitioners and researchers. It is a valuable resource for geoscientists wishing to understand the basic principles of seismic imaging, for scientific programmers with an interest in imaging algorithms, and for theoretical physicists and applied mathematicians seeking a deeper understanding of the subject. It will also be of interest to researchers in other related disciplines such as remote sensing, non-destructive evaluation and medical imaging.

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Preface and acknowledgments

In exploration seismology, a man-made energy source (on or near the Earth's surface for land exploration, or within the ocean water column in the case of marine off-shore exploration) generates a wave that propagates down into the Earth. When the downward propagating wave encounters a rapid change in Earth properties, a portion of the wave is reflected upward and another part is transmitted below the rapid variation and continues propagating down and deeper into the subsurface. The spatial locations of rapid variations in Earth properties are typically called "reflectors". The reflected wavefields that ultimately return to the Earth's surface, or, in the marine case, arrive near the air–water boundary, are recorded by large numbers of wavefield sensors. The collection of these recorded wavefields constitutes seismic reflection data. The objective of exploration seismology is to use the recorded data to make inferences about the subsurface that are relevant to determining the location and quantity of hydrocarbons. Among types of subsurface information that are useful in hydrocarbon prediction are (1) the spatial locations of any rapid variations in Earth properties, loosely called "locating reflectors", and (2) the sign and size of changes in specific Earth properties at those locations. The former of these two goals is called "imaging" or "migration", and the latter is typically called "inversion".

In this book, the first of a two-volume set, we present both the basic concepts behind, and the relationship between, wave-equation migration and inversion. In Volume I, relationships and algorithms between recorded data and the seismic image are constrained to be linear, and the relationship between the seismic image and the changes in Earth mechanical properties across the image is assumed to be linear, as well. For seismic imaging, this linearity implies that (1) the velocity model of the Earth is known between the measurement surface(s) where the energy sources and the recording sensors reside and the seismic image at depth, and (2) that an adequate wave theoretical model is available to back-propagate the waves accurately through that velocity model. For seismic inversion and target

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identification, the linearity implies (1) that all subsurface mechanical properties that influence the amplitude and phase of a wave propagating between the measurement surface(s) and the seismic image are known, and (2) that at the migrated image both the changes in Earth properties and the reflection angles being considered are small. The second of these two objectives requires a high-end form of amplitude-preserving back-propagation with a generalization and extension of the seismic imaging condition, and this has resulted in an evolution and merging of the originally distinct processes of migration and inversion into migration–inversion. The linearity also assumes that multiply reflected events (called multiples) have either been removed or can be ignored. The linearity assumptions in Volume I are behind all current leading-edge seismic processing algorithms employed in the petroleum industry, and are broadly assumed by research programs within industry research laboratories and within most leading academic consortia supported by the petroleum industry. To move beyond the constraints imposed by linearity is the province of Volume II.

Exploration geophysics has evolved over the years, with contributions from several disciplines. Before digital computers, seismic processing was analog and mechanical. Early digital processing was single-channel, amenable to signal-processing concepts imported from electrical engineering. As computational power improved, multichannel processing was introduced, and the field began to attract the attention of physicists and applied mathematicians. Wave-equation processing methods were introduced, and seismic imaging began to be recognized as a physical inverse scattering problem.

We believe that the best way to glean information from seismic data is to bring to bear the tools of inverse scattering theory. Linear inverse scattering theory, although relatively simple, is physically sound (within the limits of its approximation). That linear theory provided, for the first time, the framework to put imaging and inversion on a single firm and consistent multidimensional footing, which naturally led to the concept and methods of migration-inversion, and the mechanism for the necessary merging of two fields, migration and AVO (amplitude variation with offset). Each of these fields had previously acted (and sometimes still behave) as though the other was unnecessary or perhaps did not even exist. Filling that unfortunate and harmful chasm between two useful methodologies was a first and important conceptual and practical contribution brought to exploration seismology by inverse scattering theory.

Inverse scattering theory, as applied to seismic data, has been around for a while, and is well represented in the seismic literature. However, this is the first textbook that provides this comprehensive level of depth and completeness to the linear theory, as well as the broader nonlinear concepts and algorithms.

In Volume I, the introductory chapters present a number of different imaging techniques, algorithmically very different but sharing (and deriving from) a common set of imaging principles. For a deeper understanding of seismic imaging, we choose not to solve for specular reflectivity. Rather, we introduce the concept of scattering potential and invoke the Born approximation. We also introduce the reflectivity function as a derived quantity, defined by its relation to the scattering potential. Thus defined, the reflectivity function has a value at each subsurface point, whether or not a specular reflector exists at that point. Imaging operators invert the Born equation for either the scattering potential or the reflectivity function. At points where specular reflectors exist, the reflectivity function reduces to the specular reflectivity. Formulation of the imaging problem in terms of the scattering potential gives up nothing in terms of algorithms or capability, rather gains physical respectability while giving access to the toolbox of scattering theory. Treating a scattering potential as the generator or source of the reflection data allows imaging of smooth surfaces as well as, for example, diffractors and pinchouts, automatically and without a-priori information on which type of wave generator is at play, and without the need for interpreter intervention or two separate imaging methods/models.

Our approach is largely three-dimensional, viewing two-dimensional seismic acquisition and processing as a restriction of the general case. Primary focus is on the elastic wave equation, though with slight adjustments the formulas are also applicable to the acoustic and scalar wave equations. In addition to seismic migration, other imaging processes such as data continuation and residual migration are included in this volume. Among other new contributions, the volume provides the first linear inverse for the elastic isotropic wave equation in a three-dimensional Earth. No attempt, however, has been made to incorporate all modern migration algorithms – there are just too many. Most attention has been paid to f - k algorithms, on the one hand, and asymptotic imaging on the other. Other subjects not dealt with include anisotropic and lossy media, ray tracing algorithms, and determination of background velocities. Throughout the text the emphasis is on understanding how the mathematical physics speaks to the relationship between the recorded seismic wavefield, the source that generated the wave, and the properties of the Earth that the wave has experienced in its history, and staying focused on how that understanding can be used for the purposes of exploration seismology.

We would like to thank ConocoPhillips for support, for encouragement, and for permission to publish this textbook. We are deeply grateful to Sven Treitel for his continuous encouragement of this project, and for his positive interest and reinforcement as this book was being written. We would like to thank the M-OSRP sponsors, and NSF CMG award DMS-0327778 and DOE BES award DE-FG02-05ER15697 for their strong and constant support of this writing project. We thank

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