Descriptive Complexity, Canonisation,
and Definable Graph Structure Theory

Descriptive complexity theory establishes a connection between the computational complexity of algorithmic problems (the computational resources required to solve the problems) and their descriptive complexity (the language resources required to describe the problems).

This ground-breaking book approaches descriptive complexity from the angle of modern structural graph theory, specifically graph minor theory. It develops a ‘definable structure theory’ concerned with the logical definability of graph-theoretic concepts such as tree decompositions and embeddings.

The first part starts with an introduction to the background, from logic, complexity, and graph theory, and develops the theory up to first applications in descriptive complexity theory and graph isomorphism testing. It may serve as the basis for a graduate-level course. The second part is more advanced and mainly devoted to the proof of a single, previously unpublished theorem: properties of graphs with excluded minors are decidable in polynomial time if, and only if, they are definable in fixed-point logic with counting.

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PREFACE

This monograph evolved around the proof of a single theorem: *fixed-point logic with counting captures polynomial time on all graph classes with excluded minors*. The proof of this theorem heavily relies on structural graph theory, and the core question that needs to be addressed is how to make graph-theoretic concepts definable in logic. As many of those graph-theoretic concepts, for example, tree decompositions, are not invariant under isomorphisms and as isomorphism invariance is a prerequisite for being definable, the graph theory needs to be adapted. This leads to the definable graph structure theory presented in this monograph.

I started to work on this topic in 1997, a few years after I completed my PhD. At the time, I was mainly interested in finite model theory and especially in the main open problem of the area: the question of whether there is a logic that captures polynomial time. The results I had proved at the time were mostly "negative": counterexamples to nice conjecture and inexpressibility results, usually involving the construction of very complicated graphs and combinatorial structures. I felt a certain desire to prove a "positive" result for once, so I started to look at simpler structures, in the naive hope that on such structures the complicated counterexamples could be avoided and everything would work out nicely. To cut a long story short: it did, though only after a few complications and learning a lot of graph theory.

Jörg Flum encouraged me to present the material in a book rather than a series of technical papers, and I think this was a good idea. This book has greatly improved through the discussions I had with and comments and corrections I received from my colleagues. I am very grateful to all of them! In particular, I would like to thank Achim Blumensath, Reinhard Diestel, Jörg Flum, Frederik Harwath, Neil Immerman, Skip Jordan, Stephan Kreutzer, Martin Otto, Pascal Schweitzer, Wolfgang Thomas.