

#### **Principles of Applied Statistics**

Applied statistics is more than data analysis, but it is easy to lose sight of the big picture. David Cox and Christl Donnelly draw on decades of scientific experience to describe usable principles for the successful application of statistics, showing how good statistical strategy shapes every stage of an investigation. As one advances from research or policy questions, to study design, through modelling and interpretation, and finally to meaningful conclusions, this book will be a valuable guide. Over 100 illustrations from a wide variety of real applications make the conceptual points concrete, illuminating and deepening understanding. This book is essential reading for anyone who makes extensive use of statistical methods in their work.





## Principles of Applied Statistics

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### **Preface**

Statistical considerations arise in virtually all areas of science and technology and, beyond these, in issues of public and private policy and in everyday life. While the detailed methods used vary greatly in the level of elaboration involved and often in the way they are described, there is a unity of ideas which gives statistics as a subject both its intellectual challenge and its importance.

In this book we have aimed to discuss the ideas involved in applying statistical methods to advance knowledge and understanding. It is a book not on statistical methods as such but, rather, on how these methods are to be deployed. Nor is it a book on the mathematical theory of the methods or on the particular issue of how uncertainty is to be assessed, even though a special feature of many statistical analyses is that they are intended to address the uncertainties involved in drawing conclusions from often highly variable data.

We are writing partly for those working as applied statisticians, partly for subject-matter specialists using statistical ideas extensively in their work and partly for masters and doctoral students of statistics concerned with the relationship between the detailed methods and theory they are studying and the effective application of these ideas. Our aim is to emphasize how statistical ideas may be deployed fruitfully rather than to describe the details of statistical techniques.

Discussing these ideas without mentioning specific applications would drive the discussion into ineffective abstraction. An account of real investigations and data with a full discussion of the research questions involved, combined with a realistic account of the inevitable complications of most real studies, is not feasible. We have compromised by basing the discussion on *illustrations*, outline accounts of problems of design or analysis. Many are based on our direct experience; none is totally fictitious. Inevitably there is a concentration on particular fields of interest!



x Preface

Where necessary we have assumed some knowledge of standard statistical methods such as least-squares regression. These parts can be skipped as appropriate.

The literature on many of the topics in the book is extensive. A limited number of suggestions for further reading are given at the end of most chapters. Some of the references are quite old but are included because we believe they retain their topicality.

We are grateful to the many colleagues in various fields with whom we have worked over the years, particularly Sir Roy Anderson, through whom we met in Oxford. It is a special pleasure to thank also Manoj Gambhir, Michelle Jackson, Helen Jenkins, Ted Liou, Giovanni Marchetti and Nancy Reid, who read a preliminary version and gave us very constructive advice and comments.

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