NOVEL POROUS MEDIA FORMULATION FOR MULTIPHASE FLOW CONSERVATION EQUATIONS

William T. Sha first proposed the porous media formulation in an article in *Nuclear Engineering and Design* in 1980, and later on with many improvements renamed it the novel porous media formulation (NPMF). The NPMF represented a new, flexible, and unified approach to solving real world engineering problems. Sha introduced a new concept of directional surface porosities and incorporated spatial deviation into the decomposition of all point dependent variables into the formulation. The former greatly improved resolution and modeling accuracy, and the latter made it possible to evaluate all interfacial integrals. A set of conservation equations of mass, momentum, and energy for multiphase flows via time-volume averaging has been rigorously derived for the first time. These equations are in differential-integral form, in contrast to a set of partial differential equations currently used. The integrals arise due to interfacial mass, momentum, and energy transfer.

Dr. William T. Sha is formerly a senior scientist at Argonne National Laboratory and the former director of the Analytic Thermal Hydraulic Research Program and the Multiphase Flow Research Institute. He has published more than 290 papers in the field of thermal hydraulics. He is the recipient of many awards, including the 2005 Technical Achievement Award from the Thermal Hydraulic Division (THD) of the American Nuclear Society (ANS). The highest award given by the THD, "for many outstanding and unique contributions to the field of two phase flow and nuclear reactor design and safety analyses through the development and application of novel computational technique for analyzing thermal hydraulic behavior and phenomena, the development of NPMF of conservation equations used in the COMMIX code, development of boundary fitted coordinates transformation method used in BODYFIT code." He also received the 2006 Glenn T. Seaborg Medal from ANS "for outstanding contributions in understanding multi-dimensional phenomena of natural circulation and fluid stratification in reactor components and systems during normal and off-normal reactor operating conditions" and the 2007 Samuel Untermyer II Medal from ANS "in recognition of pioneering work in the development of significant improvements in NPMF for multiphase flow with far reaching implications and benefits for water cooled reactor components and systems." Most recently he was given the 2008 Reactor Technology Award from ANS "for outstanding leadership and exceptional technical contribution for the U.S. Department of Energy's Industrial Consortium in developing computer codes for intermediate heat exchangers and steam generators of Liquid Metal Fast Breeder Reactors which are based on the NPMF."

Novel Porous Media Formulation for Multiphase Flow Conservation Equations

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Tokyo, Mexico City

Cambridge University Press
32 Avenue of the Americas, New York, NY 10013-2473, USA

www.cambridge.org
Information on this title: www.cambridge.org/9781107012950

First published 2011

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication data

Sha, William T.
Novel porous media formulation for multiphase flow conservation equations /
William T. Sha.
p. cm
Includes bibliographical references and index.
ISBN 978-1-107-01295-0 (hardback)
1. Multiphase flow – Mathematical models. 2. Conservation laws (Mathematics)
I. Title.
TA357.5.M84S52 2011
532′.56–dc22 2011009810

ISBN 978-1-107-01295-0 Hardback

This book is dedicated to

My Parents

Mr. and Mrs. C. F. Sha, and particularly with great affection to my mother, Yunei Gee Sha, whose love and advice have inspired me to obtain the best education, work hard, and contribute to society.

My Wife

Joanne Y. Sha for understanding that I have been working very hard and have not had much time for her. I am deeply grateful she has helped me for so many years.

My Daughters and Son

Ms. Andrea E. Sha Hunt and her husband, Gregory L. Hunt

Dr. Beverly E. Sha and her husband, Dr. Thomas E. Liao, and granddaughter, Grace A. Liao

Professor William C. Sha and his wife, Shawna Suzuki Sha, and grandsons, Samuel Sha and Walter Sha

My Friends

The late Professors B. T. Chao and S. L. Soo for collaborating tirelessly and working with me for more than 25 years. Their contributions are acknowledged.

Cambridge University Press
978-1-107-01295-0 - Novel Porous Media Formulation for Multiphase Flow Conservation Equations
William T. Sha
Frontmatter
More information

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Foreword

Dr. William T. Sha's longstanding technical achievements and outstanding contributions in the nuclear reactor field are well known both in the United States and abroad. As the director of the Argonne National Laboratory (ANL), I had the privilege of working with Dr. Sha for more than a decade during which he markedly enhanced the reputation of ANL's international reactor programs as the director of the Analytical Thermal Hydraulic Research Program and Multiphase Flow Research Institute. Over many years, his rare combination of analytical rigor and creative insight allowed him to earn international recognition as a leader in the field of thermal hydraulics in both theoretical formulation and reactor design and safety analysis.

His recent work on novel porous media formulation for multiphase flow conservation equations is the subject of this book. "A set of conservation equations of mass, momentum, and energy for two-phase and multiphase flows via time-volume averaging has been rigorously derived for the first time. These equations are in differential-integral form, in contrast to a set of partial differential equations used currently. The integrals arise due to interfacial mass, momentum, and energy transfer." This is an important discovery that will have far reaching implications for both academic and industrial applications. The recent tragic accident at the Fukushima Nuclear Reactor in Japan, which is a boiling-water reactor involving two-phase or multiphase flows, makes the subject of this book even more timely and important.

I have been most impressed by the depth of Dr. Sha's technical knowledge in the area of thermal hydraulics of nuclear reactors. Most importantly, he has always been at the cutting edge of innovation and shares his knowledge with fellow workers, thus advancing the state of the art of thermal hydraulics. His enthusiasm and zest for technical challenges was amazing. It was a real pleasure to work with him.

Dr. Alan Schriesheim, Director Emeritus
Argonne National Laboratory
Member, National Academy of Engineering

Foreword

When I was the manager of reactor physics in the Westinghouse Atomic Power Division [later called the Pressurized Water Reactor (PWR) Division], Dr. William T. Sha worked for me and was instrumental in our development of the first multi-dimensional integral calculation of nuclear-thermal-hydraulic interaction named THUNDER code for the commercial PWRs. The reactivity feedbacks due to thermal-hydraulics, including local subcooled and bulk boiling, control rod insertion, dissolved boron poison in the moderator, and fuel pellet temperature (Doppler effect) were explicitly accounted for. We were then designing Yankee Rowe, Connecticut Yankee, Edison Volta, and Chooz 1. He was clever, indefatigable, and a great asset in our development of the THUNDER codes (WCAP-7006, 1967) and designing these reactors. Plants based on this design are now found in more than half of the world's nuclear power plants. This code represented a quantum jump in design and performance of PWRs when it was successfully completed in 1967.

Once again, Dr. Sha demonstrates innovation and lays the theoretical foundation to develop the novel porous media formulation for multiphase flow conservation equations. The starting point of the novel porous media formulation is Navier-Stokes equations and their interfacial balance equations; the local-volume averaging is performed first via local-volume-averaged theorems, followed by time averaging. A set of conservation equations of mass, momentum, and energy for multiphase systems with internal structures is rigorously derived via time-volume averaging. This set of derived conservation equations has three unique features: (1) the internal structures of the multiphase system are treated as porous media formulation – it greatly facilitates accommodating the complicated shape and size of the internal structures; (2) the concept of directional surface porosities is introduced in the novel porous media formulation and greatly improves modeling accuracy and resolution; and (3) incorporation of spatial deviation for all point dependent variables make it possible to evaluate interfacial mass, momentum, and energy transfer integrals. The novel porous media formulation represents a unified approach for solving real world multiphase flow problems.

Dr. Wm. Howard Arnold
Retired Vice President of Westinghouse Electric Corp.
Member, National Academy of Engineering
Member, U.S. Nuclear Waste Technical Review Board
(Presidential Appointee)

Foreword

Dr. William (Bill) T. Sha is insightful, inventive, and the epitome of professionalism in his technical work.

I have had extensive contacts with Bill Sha, first at Argonne National Laboratory, and later at the U.S. Nuclear Regulatory Commission's Office of Nuclear Regulatory Research. In the latter capacity, I was charged, following the accident at TMI-2, with developing and executing a plan of reactor safety research focused on severe accidents. A major problem facing us was that of knowing whether, when, and how a badly damaged nuclear reactor core could be cooled by natural convection. The obvious problem was that the coolant flow paths were not readily described analytically, even if we knew precisely what they were. We turned to Bill Sha for help with this problem. The response, in a refreshingly short time, was the COMMIX code.

Dr. Sha is the father of the COMMIX code. The code employs the (then) new porous media formulation pioneered by Dr. Sha. The formulation used concept of volume porosity, directional surface porosities, distributed heat source and sink, and distributed resistance and allowed computational analysis of complex geometries critical to power reactor safety analysis. Both conventional porous media and continuum formulations are subsets of this formulation. Dr. Sha's formulation represents a flexible and unified approach to computational fluid dynamics and heat transfer for solving practical engineering problems. Exemplifying Bill's insight, the COMMIX code has proved to be useful for a wide range of engineering design and analysis problems not limited to reactor safety.

The COMMIX code is widely used in the United States and internationally. It has received great attention because of its unique capabilities and features for analyzing inherently multidimensional phenomena such as fluid stratification, natural circulation, coupling effects between reactor core and upper and lower plenums, and so forth. Many foreign countries such as Germany, France, the United Kingdom, Italy, Finland, Japan, China, and South Korea have requested and adopted the COMMIX code and its formulation.

Dr. Sha and his group had the broad range of knowledge, skill, and inventiveness to solve critical engineering problems and acted on behalf of USNRC for an independent verification of the design and performance of passive containment cooling systems of AP-600 or 1000. His novel porous media formulation for multiphase flow conservation equations made significant contributions in the area of reactor design and safety analysis.

Dr. Charles Kelber
Retired Judge
United States Nuclear Regulatory Commission (USNRC)