1 Introduction

Many years ago, a generic, three-dimensional, time-dependent COMMIX computer program based on the novel porous media formulation for single phase with multicomponent (see Chapter 7) was developed. The computer program was then adopted throughout the world, and the novel porous media formulation has proven both promising and cost effective for many engineering applications. This book now presents the novel porous media formulation for multiphase flow conservation equations.

Multiphase flows consist of interacting phases that are dispersed randomly in space and in time. It is extremely difficult, if not impossible, to track down the interfaces between dispersed phases of multiphase flows. Turbulent, dispersed, multiphase flows can only be described statistically or in terms of averages [1], a fact that was not recognized during the early development of multiphase flow. Averaging procedures are necessary to avoid solving a deterministic multiboundary value problem with the positions of interfaces being a priori unknown. Additional complications arise from the fact that the flow system of interest often contains stationary and complex, solid, heat-generating and heat-absorbing structures. Although, in principle, the intraphase conservation equations for mass, momentum, and energy, as well as their associated initial and boundary conditions, can be written, the problem is far too complicated to permit detailed solutions. In fact, they are seldom needed in engineering applications. A more realistic approach is to express the essential dynamics and thermodynamics of such a system in terms of local volume-averaged quantities. This may be achieved by applying an averaging process, such as time, volume, or statistical averaging. The present work begins with local volume averaging, followed by time averaging. The whole process is called time-local volume averaging, or time-volume averaging.

1.1 Background information about multiphase flow

In earlier papers, the concept of a generalized porous media formulation was conceived [2]* and the local volume-averaged transport equations for multiphase flow were derived in regions containing stationary, heat-generating and heat-absorbing solid structures [3].* Further time averaging of these equations was presented in Refs. [4,5].* A significant step in the development of these averaged equations was the introduction of the concept of volume porosity and

* Directional surface porosity replaces directional surface permeability in Refs. [2–5].

directional surface porosities* associated with stationary and complex solid structures. For a given local volume v, which consists of volume occupied by the fluid mixture v_m and volume occupied by the stationary and complex structures v_w, the volume porosity is defined as the ratio of v_m/v. The directional surface porosities are defined as the ratio of free-flow surface area to the surface area in three principal directions. This concept greatly facilitates the treatment of flow and temperature fields in anisotropic media and enables computational thermal hydraulic analysis of fluids in a region containing complex structures.

Recently, however, it was noted that certain assumptions introduced in Refs. [4,5] regarding the decomposition of the point values of dependent variables, such as density, velocity, pressure, total energy, internal energy, and enthalpy, could be improved [6]. To this end, this book presents a set of time-volume-averaged conservation equations of mass, momentum, and energy for multiphase systems with stationary and complex solid internal structures. Advantage is taken of the use of volume porosity, directional surface porosities, distributed resistance, and distributed heat source and sink. The concept of directional surface porosities was first suggested by Sha [2], greatly improving resolution and modeling accuracy. The governing conservation equations for multiphase flow derived here by time-volume averaging are subject to the length-scale restrictions inherent in the local

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volume-averaging theorems [7,8] and the time-scale restrictions prescribed in Refs. [4–6].

Time averaging after local volume averaging is chosen over other forms of averaging for the following three reasons:

1. Local volume averaging has been successfully used in many laminar, dispersed multiphase flows, and local volume averaging theorems are well established. Because we are concerned here with turbulent multiphase flows in general and dispersed flows in particular, it is only natural to take the same conventional approach followed by time averaging.
2. Much of our instrumentation records a space average followed by a time average. A Bourdon tube pressure gauge displays an area-averaged pressure averaged over its response time. A hot wire anemometer gives an area-averaged response as a function of time. With a gamma beam, we have a volume-averaged reading over time. We believe that the dependent variables calculated from time-volume-averaged conservation equations are more simply related to the corresponding variables measured by experiment.
3. Within the framework of generalized multiphase mechanics, Soo [9] first suggested that different ranges of particle sizes, densities, and shapes are treated as different dynamic phases. We note that using Eulerian time averaging from the beginning will remove this distinction of dynamic phases. Simple time averaging leads to fractional residence time of a phase rather than volume

fraction of a phase. The fractional residence time of a phase becomes equal to the physical volume fraction only in the case of one-dimensional uniform motion of incompressible phases. Therefore, local volume averaging must precede time averaging.

Local volume averaging has been used extensively for analyzing porous media flows [7,8,10–27]. Volume averaging leads naturally to volume fraction of phases, whereas a priori time averaging yields their fractional residence time. The thermodynamic properties of a fluid, such as density and specific heat, are cumulative with volume fraction but not with fractional residence time, which becomes identical to volume fraction only in the special case of one-dimensional, incompressible flow (see Chapter 6).

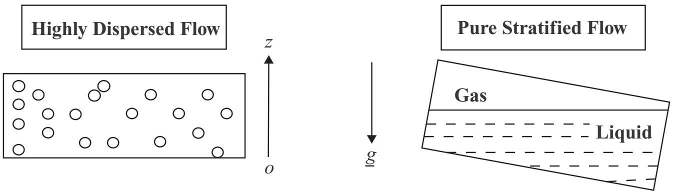
The usual approach for single-phase flow is to define for every dependent variable its time average at each point in space. This is feasible but not applicable for turbulent, dispersed multiphase flow in which interface configurations may be moving randomly with time across the point under consideration.

It is awkward to define time-averaged variables in the immediate neighborhood of the phase interface when the phase interface itself is subject to turbulent fluctuations and may be moving back and forth across a particular point during the period for which the time average is defined [1]. Several disadvantages associated with time averaging as the basis of analysis for multiphase flows were pointed out by Reynolds [28] in his review of Ishii's [29] book. Drew [15] and Ishii [29,30] were among the first to recognize the importance of interfacial mass, momentum, and energy balance for two-phase or multiphase flows.

The intuitive answer is that we can describe these multiphase flows only in some average sense, as though each phase occupied all available space of fluids in local volume averaging. We advocate retaining the usual equations of motion appropriate for each phase but using only local volume or time-volume averages of these equations. Closure relations are required because information is lost in the averaging process. However, the approach is general, limited only by our ability to devise these closure relations [1].

1.2 Significance of phase configurations in multiphase flow

The configuration of phases plays a major role in determining the dynamics of multiphase flows and the concomitant heat and mass transport processes when they occur. This is illustrated in Fig. 1.1 for the two extreme cases of highly dispersed flow and ideally stratified flow, which by definition has a plane interface. Figure 1.1 is largely self-explanatory. Given the defining relation for mixture density ρ_m [Eq. (1.2.1)] and for mixture velocity $\underline{U}_m$ [Eq. (1.2.2)], it is easy to show that $\rho_m \underline{U}_m^2$ and $\sum_k \alpha_k \rho_k \underline{U}_k^2$ are not the same. It is also easy to demonstrate that if the Bernoulli relationship for an ideal mixture in highly dispersed flow is written as Eq. (1.2.5), then that for the individual phase must be given by Eq. (1.2.6). Clearly, the form of the Bernoulli equation depends on the configuration of the phases. The Bernoulli equation for other



- Gradient of volume fraction
  $\nabla \alpha \propto \nabla$ (conc.) — $\nabla \alpha$ is of no significance
- Finite diffusivity, D
- Diffusion velocity, $D\nabla\alpha$

  } Concept of diffusivity is irrelevant
- Wave propagation
  Existence of speed of sound in the mixture — No common speed of sound
  Common characteristics — Individual characteristics of phases
  Transfer of inertia force across interface — Plane interface; no inertia force transferred across interface
- Bernoulli relationships for steady, incompressible, inviscid, one-dimensional flow

$$\rho_m = \sum_k \alpha_k \rho_k, \qquad k = 1, 2, \ldots, \tag{1.2.1}$$

where $\alpha_k \rho_k$ is density of phase k based on mixture volume

$$\rho_m U_m = \sum_k \alpha_k \rho_k U_k \tag{1.2.2}$$

Clearly,
$$\rho_m U_m^2 \neq \sum_k \alpha_k \rho_k U_k^2 \tag{1.2.3}$$

$$P_m = \sum_k \alpha_k P_k \tag{1.2.4}$$

Ideal Mixture

$$(1/2)\, \rho_m U_m^2 + P_m + \rho_m g z = \text{Constant} \tag{1.2.5}$$

Individual Phase

$$(1/2)\, \rho_k U_k^2 - (1/2)\, \rho_k (U_k - U_m)^2 + P_k + \rho_k g z = \text{Constant} \tag{1.2.6}$$

Fig. 1.1. Significance of phase configurations in multiphase flows.

systems, such as bubbly flow, annular wavy flow with dispersed liquid, intermittent flow, and stratified wavy flow, are far more complex, representing cases intermediate between the highly dispersed flow and ideally stratified flow.

1.3 Need for universally accepted formulation for multiphase flow conservation equations

The current status of multiphase flows with heat transfer is lacking focus and is confusing. The confusion arises from (1) the failure to recognize that the approach to turbulent, dispersed multiphase flows can be described only in terms of averages or statistically; and (2) the fact that because constitutive relations vary with different sets of conservation equations, we have been using many different sets of conservation equations resulting from different formulations such as time averaging, time-volume averaging, volume-time averaging, two-fluid models, and multifluid models. Unless the same set of conservation equations is used, this confusion will persist. The first order of business, then, is to develop a sound, generic, and consistent formulation for rigorously deriving a set of multiphase conservation equations that is equivalent to the Navier-Stokes equations for single phase.

The novel porous media formulation for multiphase flow conservation equations described in this book is the answer. The formulation starts with the local volume averaging of the Navier-Stokes equations and interfacial balance relations of each phase via local volume-averaging theorems that are well established and *theoretically sound*. Time averaging is performed on both the local volume-averaged conservation equations and interfacial balance relations. The time-volume-averaged conservation equations for multiphase flows are in differential-integral form and are in contrast to a set of partial differential equations currently in use. The novel porous media formulation is also *generic*, with the flexibility to include or not the high-frequency fluctuation variables of α_k', v_k', and A_k' based on the need from physics of the problem under consideration. Moreover, the formulation is *consistent*: it derives multiphase flow conservation equations based on a single formulation. For example, if we need to retain α_k' and A_k', then we can follow the same procedure to derive another set of multiphase conservation equations based on the same formulation.

The benefits resulting from the idea of a universally accepted formulation to derive the multiphase flow conservation equations are enormous. The effort and expense in developing constitutive relations no longer need to be repeated for each specific problem. Any developed constitutive relations will deposit in the pool, which can be used for the same or similar problems.

In summary, the unique features of the novel porous media formulation for multiphase flow conservation equations are as follows:

1. The time-volume-averaged multiphase conservation equations are derived in a region that contains stationary and solid internal structures. The fluid–structure interactions are explicitly accounted for via both heat capacity effects during a transient and its additional fluid resistance due to the presence of the structures. This set of time-volume-averaged equations is particularly suitable for numerical analysis with a staggered grid computational system (see Appendix A).

2. Most engineering problems involve many stationary complex shapes and sizes structures whose distributed resistance is impossible to quantify accurately. The concept of directional surface porosities was derived naturally through local volume average; it reduces the sole reliance on an empirical estimation of distributed resistance and provides flexibility to develop a numerical simulation model of a real world engineering system. This concept thus improves resolution and accuracy in modeling results.
3. Introducing spatial deviation of point values of the dependent variables makes it possible to evaluate or approximate interfacial integrals.
4. In deriving the time-volume-averaged continuity equation, the time-volume-averaged interfacial mass generation rate of phase k per unit volume is (see Chapter 5)

$$\gamma_v \alpha_k^t \langle \Gamma_k \rangle = -v^{-1} \int_{A_k}^{t} \langle \rho_k (\underline{U}_k - \underline{W}_k) \rangle \cdot \underline{n}_k dA. \quad (1.3.1)$$

This interfacial mass generation rate of phase k per unit volume will directly appear in the time-volume-averaged momentum and energy equations, plus additional interfacial momentum and energy transfer integrals that are intuitively expected (see Chapter 5). It gives some confidence that the derived time-volume-averaged momentum and energy equations for multiphase flows are in good order. We note that Eq. (1.3.1) does not appear in the time averaging formulation.